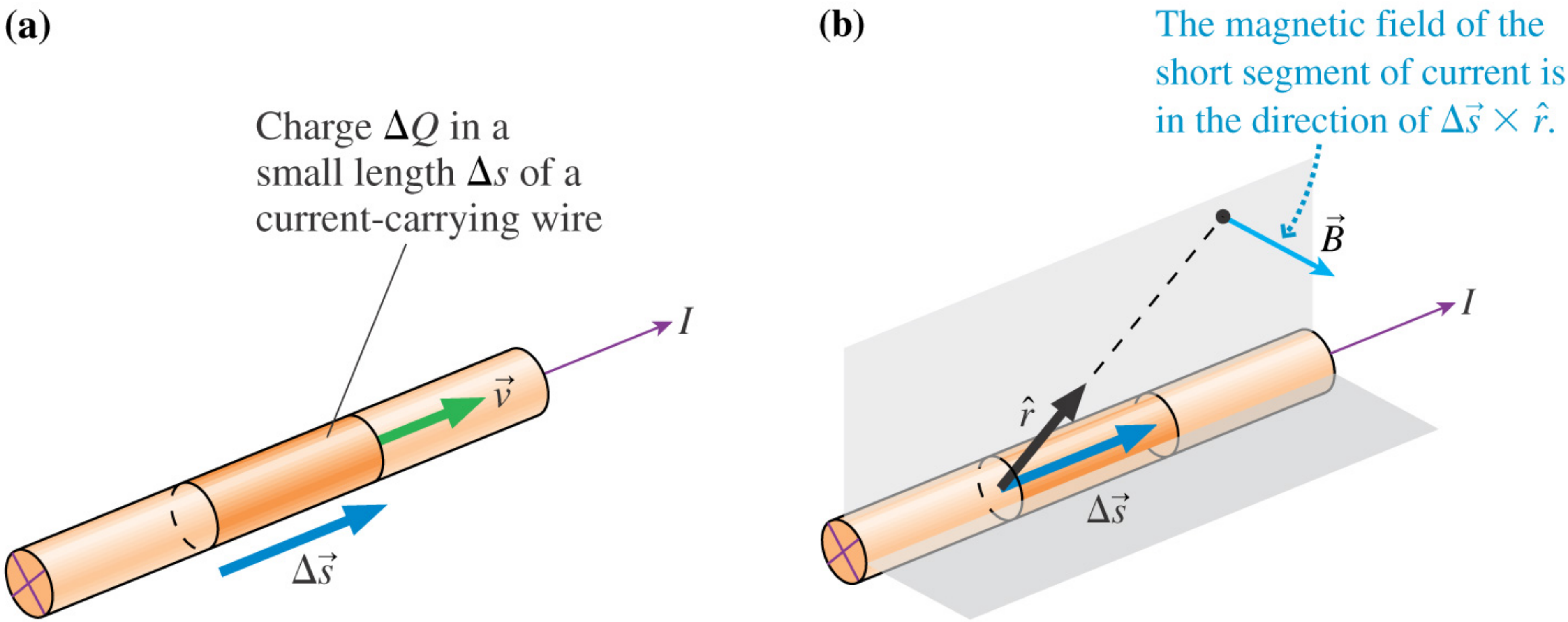
Lecture 15: Chapter 32, October 25 2005

Magnetic Field of a Current



$(\Delta Q)v = \Delta Q(\Delta s/\Delta t) = (\Delta Q/\Delta t)\,\Delta s = I\,\Delta s$ – *current-length element*

Magnetic field of a very short segment of current:

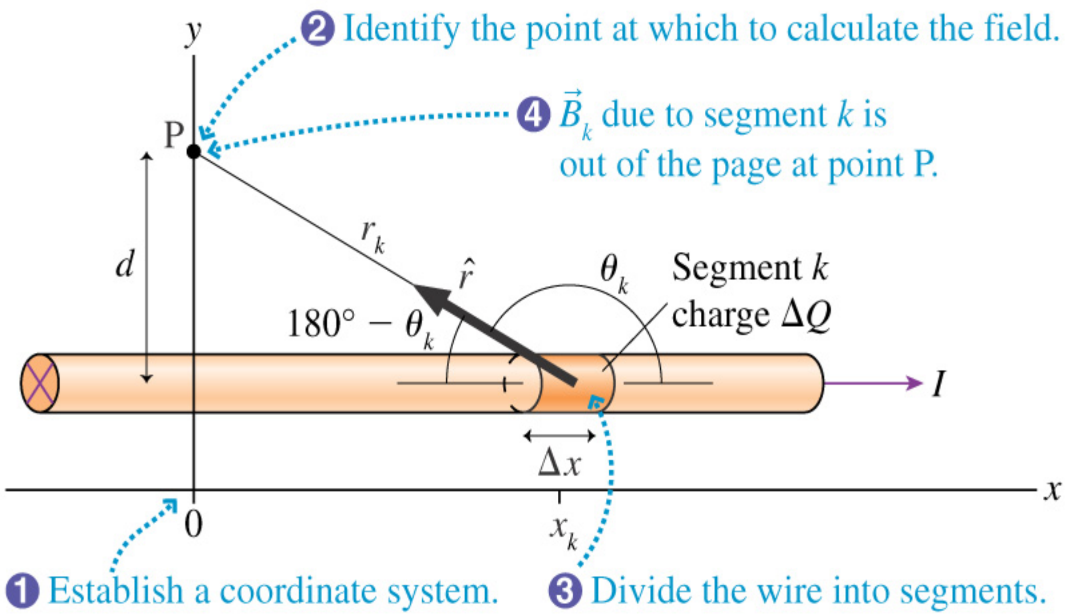
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s} \times \hat{r}}{r^2}$$

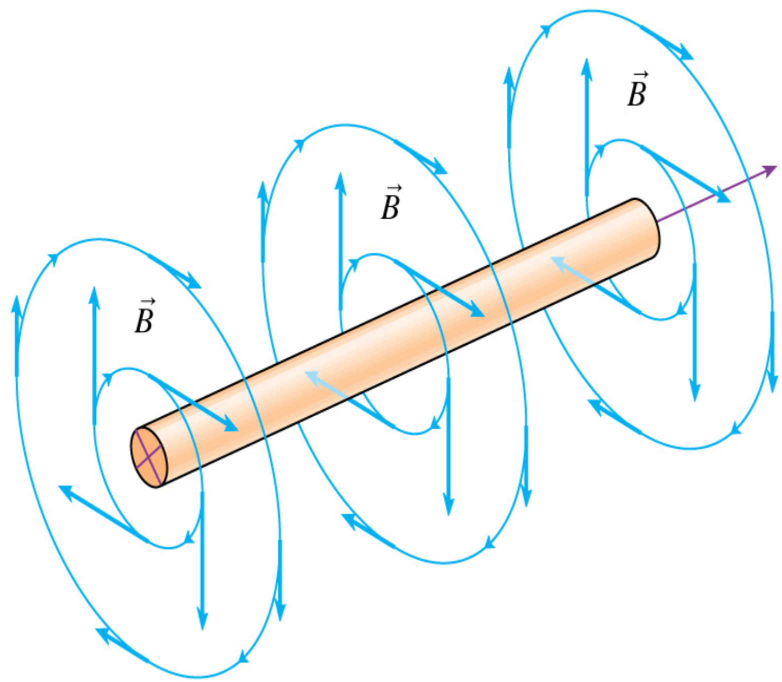
The magnetic field of a long, straight wire

Two right hand rule techniques to determine direction of ***B***

- Dividing the wire into small segments and finding directions of $\Delta\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{s}\times\hat{r}}{r^2}$

- Applying right hand rule to the entire wire





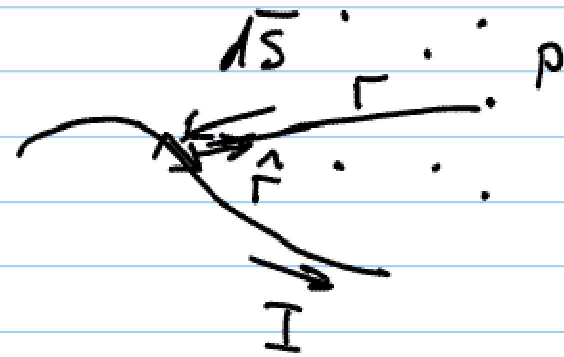
Calculating the magnetic field of a long, straight wire

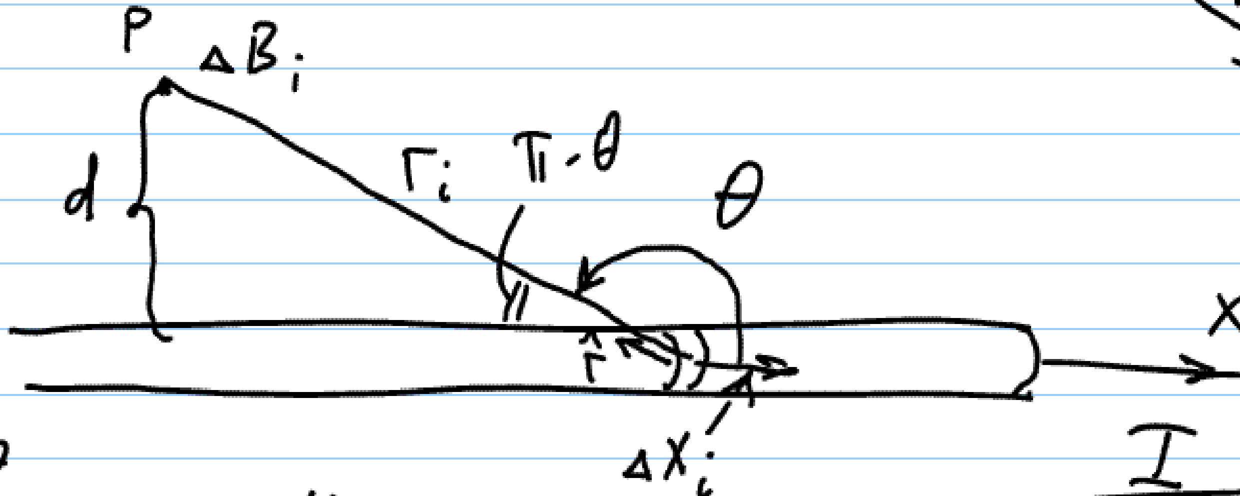
Note Title 3/28/2005

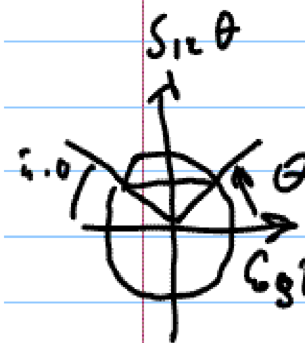
1. $\bar{B}$ from a long straight wire
2. Ampere's Law

current-length element

$$d\bar{B} = \frac{\mu_0}{4\pi}\,\frac{I\cdot d\bar{s}\times\hat{r}}{r^2}$$







$$\cos\theta\,\Delta B_i = \frac{\mu_0}{4\pi}\,\frac{I\cdot\Delta x_i\cdot 1\cdot\sin\theta}{x_i^2+d^2}$$

$$\boxed{d\bar{s}\times\hat{r} = \Delta x_i\cdot|\hat{r}|\cdot\sin\theta}$$

$$\boxed{\sin\theta = \sin(\pi-\theta) = \frac{d}{\sqrt{x_i^2+d^2}}}$$

$$= \frac{\mu_0}{4\pi} \frac{I \cdot \Delta x_i \cdot d}{(x_i^2 + d^2)^{3/2}}$$

$\Delta x_i \rightarrow dx$

$x_i \rightarrow x$

$$dB = \frac{\mu_0}{4\pi} \frac{I \cdot d}{(x^2 + d^2)^{3/2}} dx$$

$$B = \int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} \frac{I \cdot d}{(x^2 + d^2)^{3/2}} \cdot dx =$$

$$= \frac{\mu_0 I \cdot d}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + d^2)^{3/2}} = \frac{\mu_0 \cdot I \cdot \cancel{d}}{4\pi} \left. \frac{x}{d^{\cancel{2}}(x^2 + d^2)^{1/2}} \right|_{-\infty}^{\infty}$$

$$\boxed{\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 (x^2 + a^2)^{1/2}}}$$

Tangent to a circle around the wire in the right-hand direction:

$$\vec{B}_{wire} = \frac{\mu_0}{2\pi} \frac{I}{d}$$

Exploring the symmetry between electrostatics and magnetism:

- In *electrostatics* we have two equivalent laws relating source of field (*q*) to the field (***E***): (i) differential (Coulomb) and (ii) integral (Gauss).
- In *magnetism* we have a differential law (Biot-Savart) relating the source of field (*q**v***) to the field (***B***). How about an integral law?

It is represented by Ampere's Law.

$$\begin{cases} \bar{E} = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{q\cdot\hat{r}}{r^2} \\[2ex] d\bar{B} = \dfrac{\mu_0}{4\pi}\,\dfrac{I\cdot d\bar{s}\times\hat{r}}{r^2} \end{cases}$$

Gauss' Law:

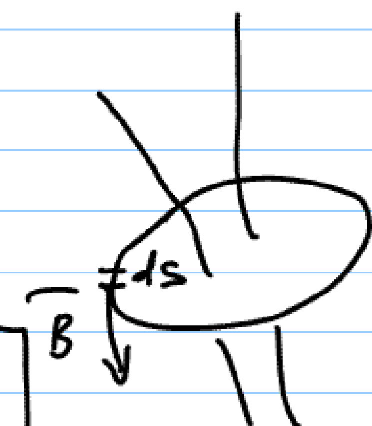
$$\Phi_e = \frac{Q_{enc}}{\varepsilon_0}.$$

$$\Phi_e = \int_{\text{Surface}} \bar{E}\cdot d\bar{A}$$

In magnetism it is

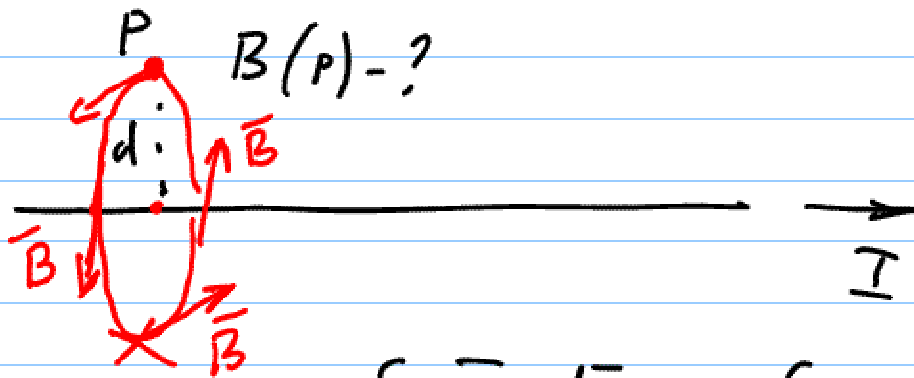
Ampere's Law

$$\boxed{\oint_{Line} \bar{B}\cdot d\bar{s} = \mu_0 I_{through}}$$



Let us show that in the case of ***B*** created by the long straight wire these laws (Biot-Savart's and Ampere's) are equivalent

$\bar{B}\cdot d\bar{S} = B\cdot dS\cdot \cos\theta$, θ – angle between B and dS



$$\boxed{B = \frac{\mu_0 I}{2\pi d}}$$

$$\oint_{Line} \bar{B}\cdot d\bar{S} = \oint_{Line} B\cdot dS\cdot \cos\theta = \oint_{Line} B\cdot dS =$$

$$\boxed{\theta - ? \qquad \theta = 0 \Rightarrow \cos\theta = 1}$$

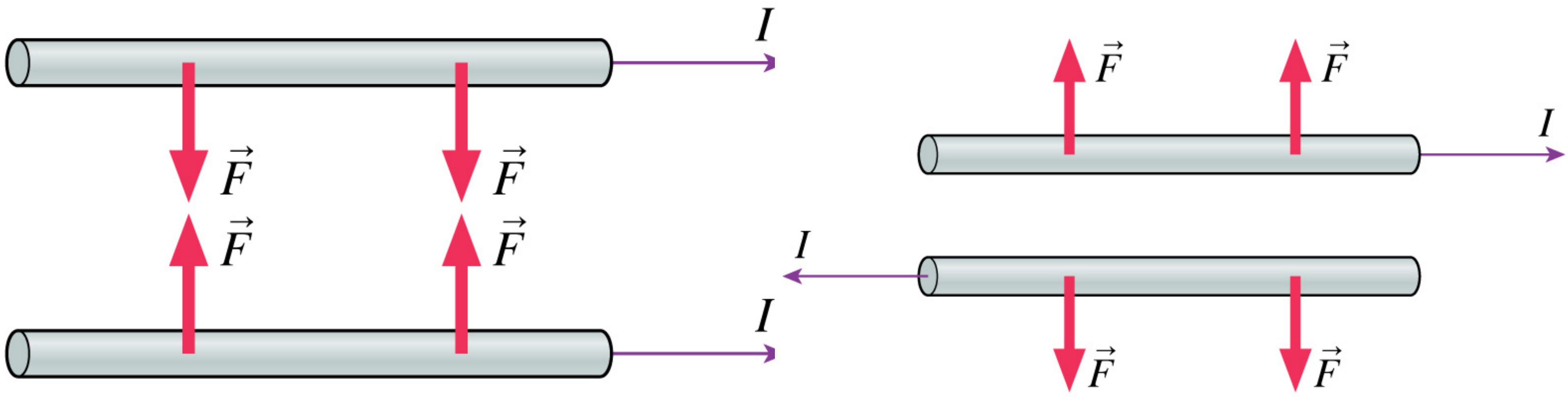
$$= B \oint_{Line} dS = \frac{\mu_0 I}{\cancel{2\pi d}}\, \cancel{2\pi d} = \mu_0 I$$

– Checked it works perfectly for a wire

- Any shape of the loop
- Use total current through

The Magnetic Field is Created by Moving Charges, but once it is generated how and on what does it act?

Ampere's Experiments: Forces between currents



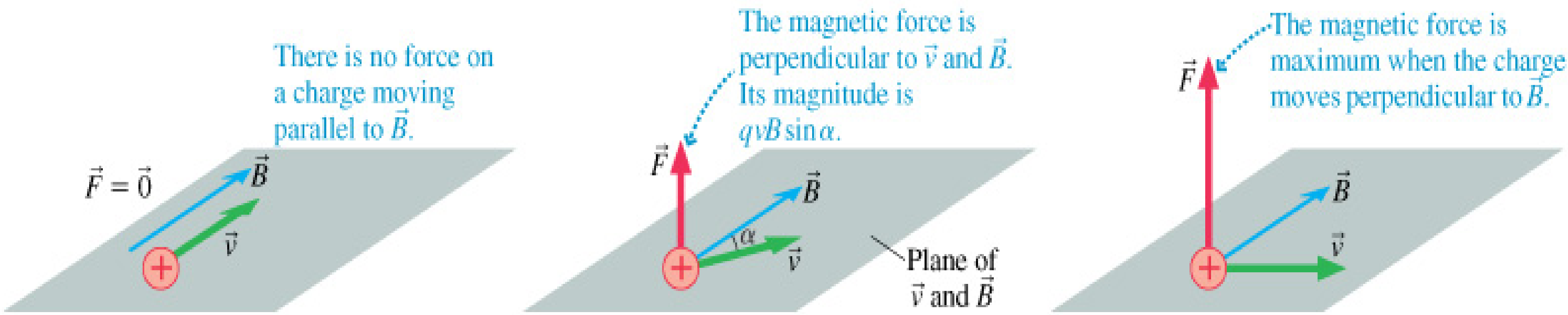
"Like" currents attract.

"Opposite" currents repel.

- Each current creates a magnetic field.
- This field ***B*** exerts a force ***F*** on the second current.

Conclusion: Magnetic field acts on moving charges

The magnetic Force on a Moving Charge



$F = 0$

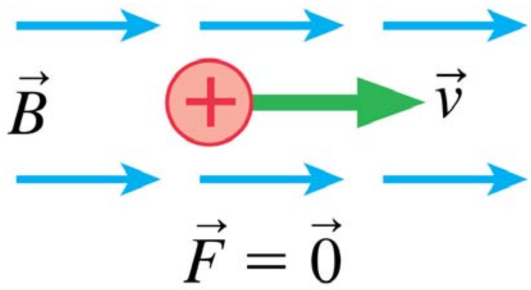
$F = qvB\sin\alpha$

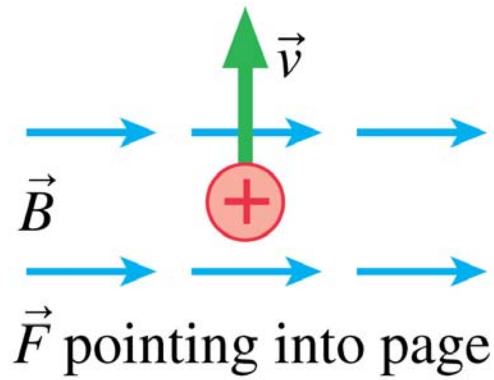
$\alpha = 90^0$
$F = qvB$ - Max

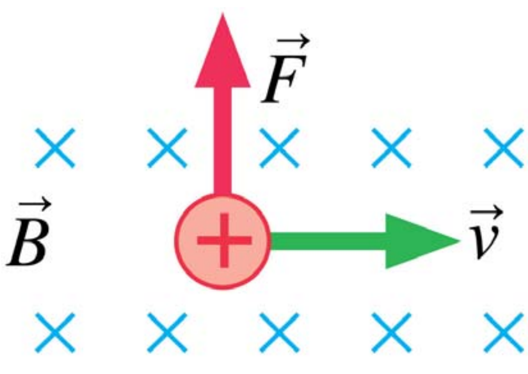
This is characteristic of a vector product $\Rightarrow$

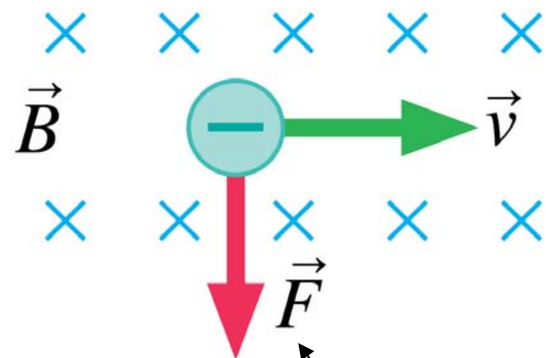
$$\vec{F} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, right - hand)$$

Direction of the magnetic Force



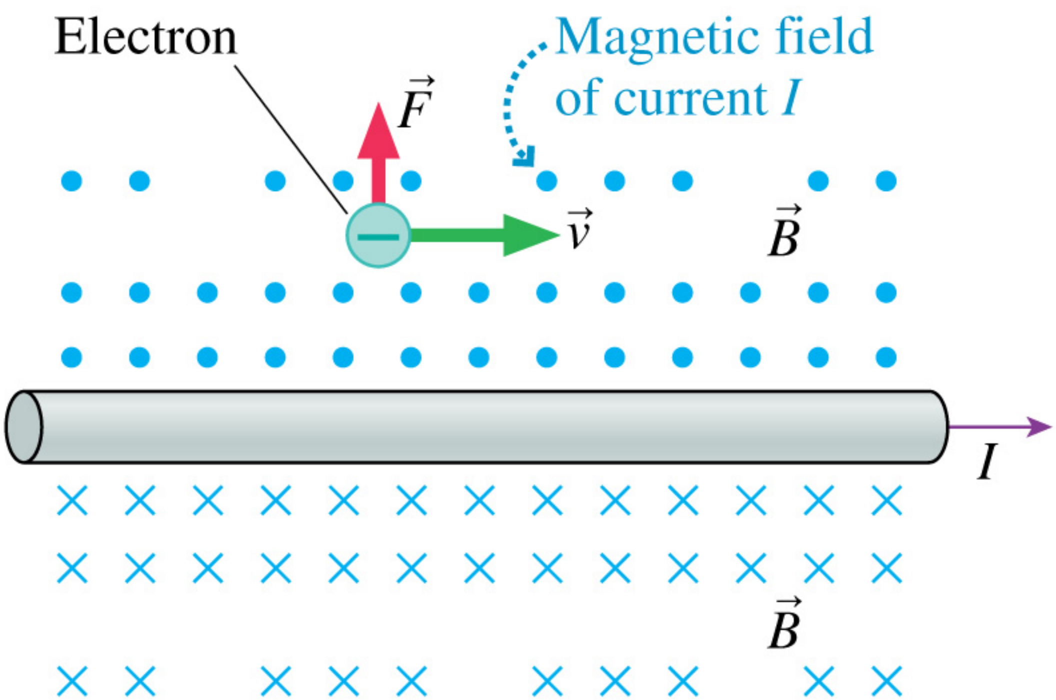






- Only ***moving*** charges experience ***B***.
- No force on charge moving parallel to ***B***.
- The force is perpendicular to both ***v*** and ***B***
- **The force on negative charge is in the opposite direction**

Problem for discussion: How to solve?



1. How to find direction and magnitude of ***B***?
2. How to find direction of ***v***×***B***?
3. How to find direction and magnitude of ***F***?

End of Lecture 15
Reading: Chapter 32
HW7 and HW8