Lecture 19: Chapter 33, November 8 2005

Energy in Inductor

$$P_{elec} = I\Delta V, \qquad \Delta V_L = -L\frac{dI}{dt}$$

$$P_{elec} = I\Delta V_L = -LI\frac{dI}{dt}$$

$P_{elec} < 0$ since the current is losing energy

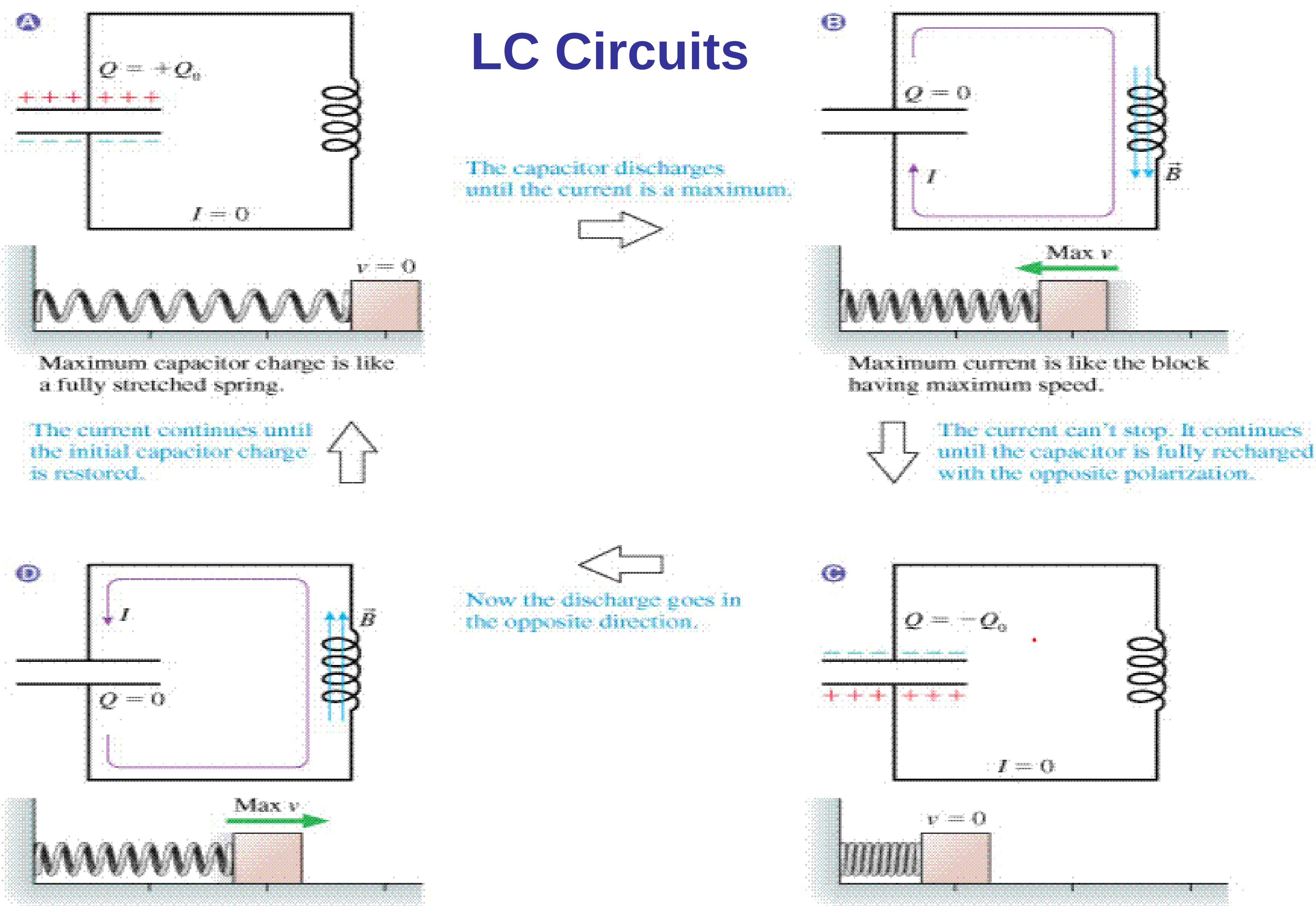
This energy is transfered to L:

$$\frac{du_L}{dt} = +LI\frac{dI}{dt}, \qquad \text{assume } u_L = 0 \text{ at } I = 0$$

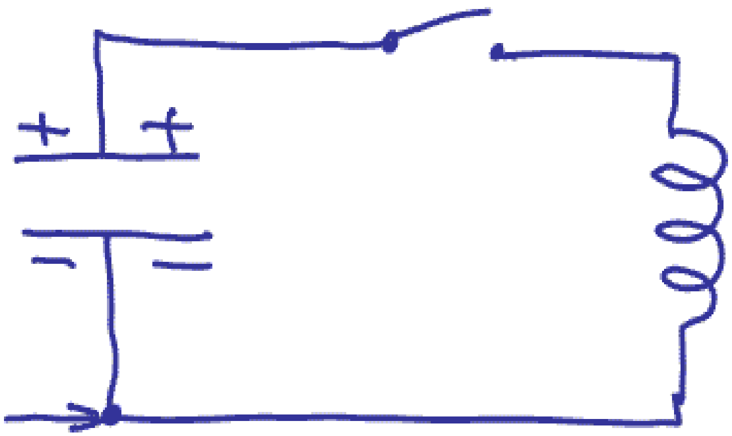
Compare to C:

$$\boxed{u_L = L\int_0^I I\,dI = \frac{1}{2}LI^2} \longleftrightarrow u_C = \frac{1}{2}C(\Delta V)^2$$

LC Circuits


A
Q = +Q0
I = 0
v = 0
Maximum capacitor charge is like a fully stretched spring.
The capacitor discharges until the current is a maximum.
B
Q = 0
I
B
Max v
Maximum current is like the block having maximum speed.
The current can't stop. It continues until the capacitor is fully recharged with the opposite polarization.
C
Q = −Q0
I = 0
v = 0
Now the discharge goes in the opposite direction.
D
I
B
Q = 0
Max v
The current continues until the initial capacitor charge is restored.

$\omega - ?$



$\Delta V_c + \Delta V_L = 0$ — Loop rule

$$\begin{cases} \Delta V_c = \dfrac{Q}{C} \\ \Delta V_L = -L\dfrac{dI}{dt} \end{cases}$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0,$$

but $I = \dfrac{dq}{dt}$

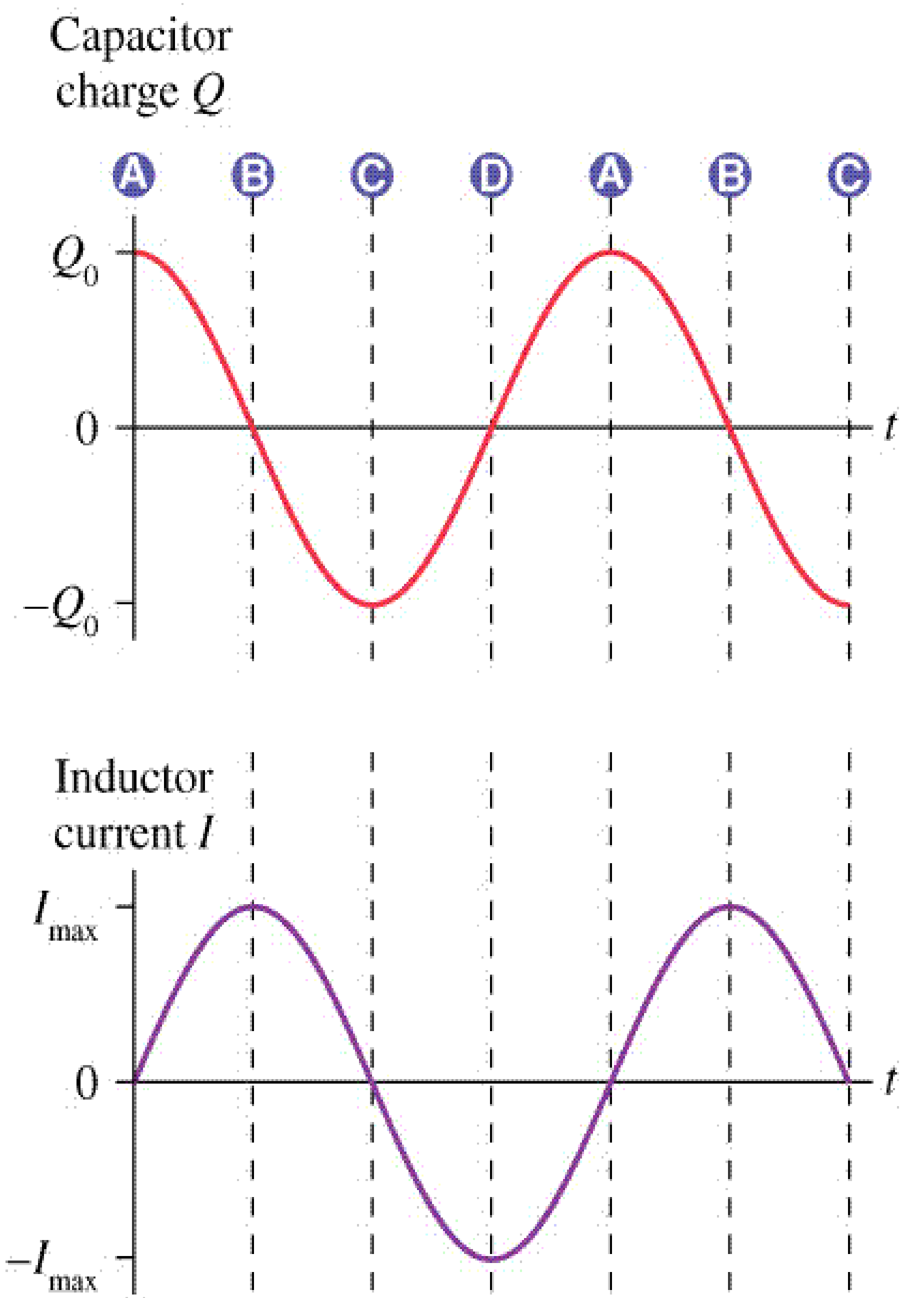
q - current through the circuit

$dq = -dQ$

Q - current on the capacitor

$$\boxed{\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0}$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$



$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\begin{cases} x \rightarrow Q \\ \frac{k}{m} \rightarrow \frac{1}{LC} \end{cases}$$

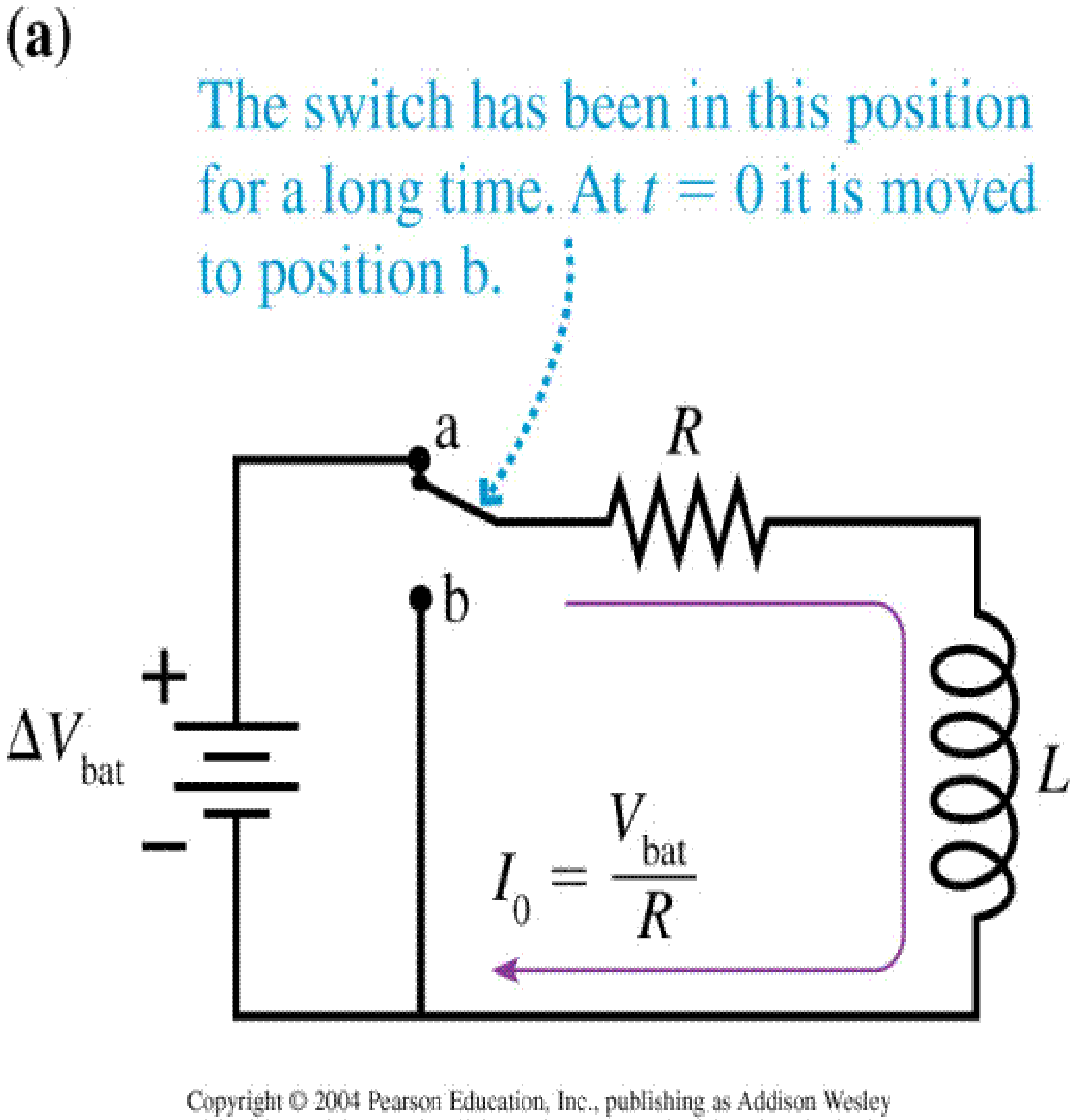
$$x = X_0 \cdot \cos\omega t$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\boxed{Q(t) = Q_0 \cdot \cos\omega t \qquad \omega = \sqrt{\frac{1}{LC}}}$$

Check (substitute)

LR Circuits



$$\Delta V_L = -L \frac{dI}{dt}$$

$$\Delta V_R + \Delta V_L = 0$$

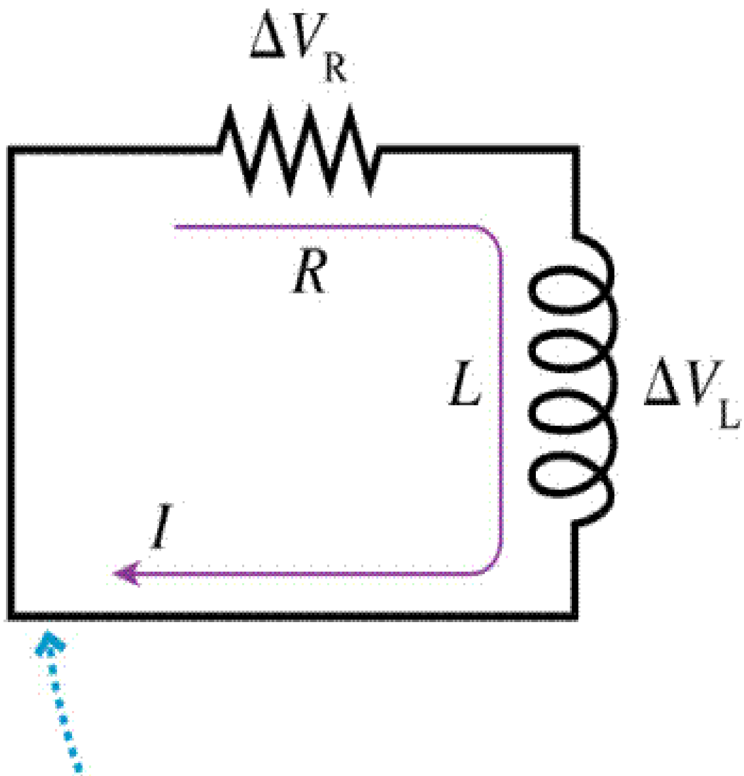
Loop rule

$$\boxed{\begin{aligned} \Delta V_R &= -IR \\ \Delta V_L &= -L \frac{dI}{dt} \end{aligned}}$$

$$-IR - L\frac{dI}{dt} = 0$$

$$\frac{dI}{I} = -\frac{R}{L}\,dt = -\frac{dt}{L/R}$$

(b)



This is the circuit with the switch in position b. The inductor prevents the current from stopping instantly.

Start at $t=0 \rightarrow I_0$

Finish at $t \rightarrow I(t)$

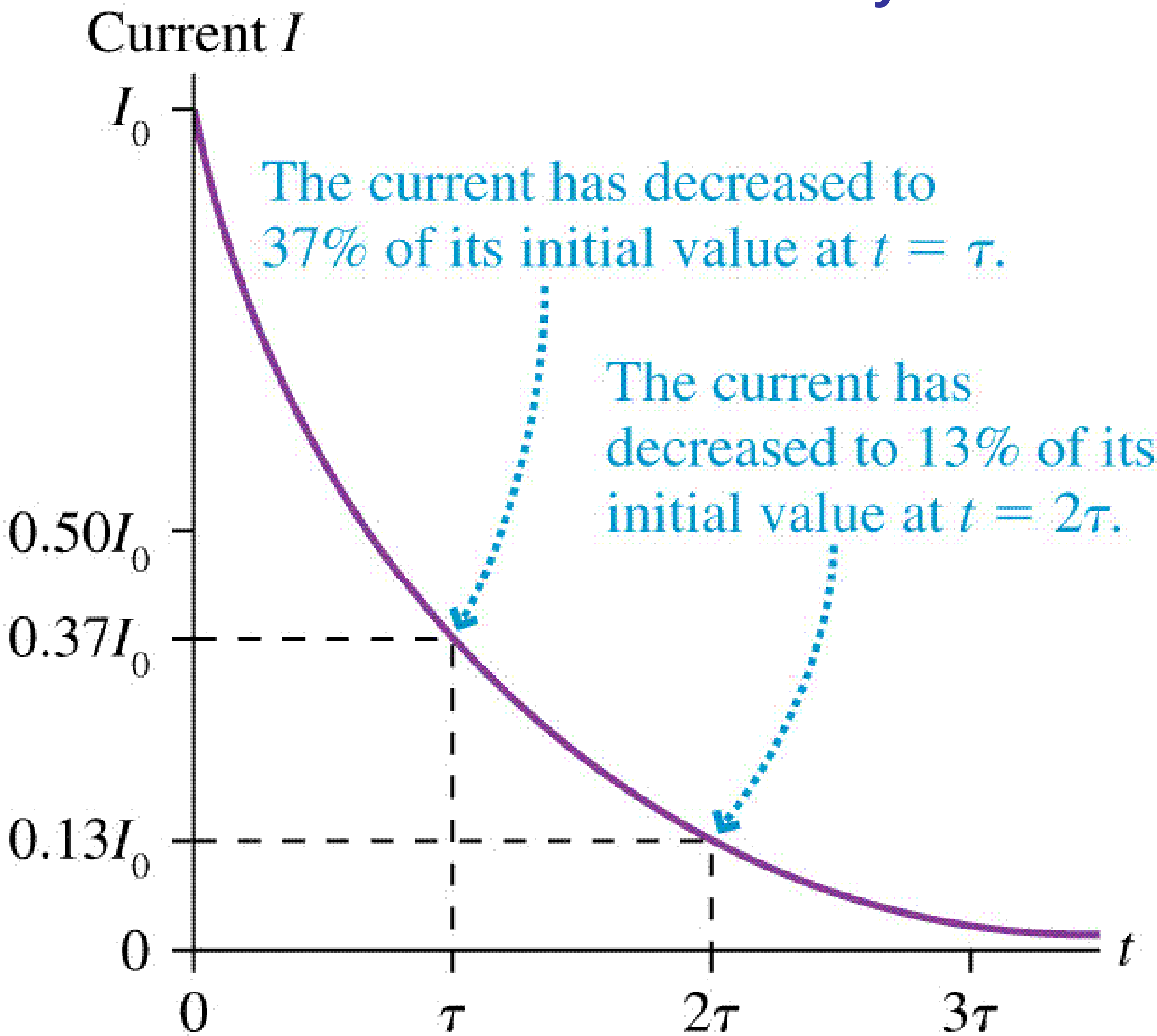
$$\int_{I_0}^{I(t)} \frac{dI}{I} = -\frac{1}{(L/R)} \int_0^t dt$$

$$\ln I \Big|_{I_0}^{I(t)} = \ln I - \ln I_0 = \ln \frac{I}{I_0} = -\frac{t}{(L/R)}$$

$$e^{\ln \frac{I}{I_0}} = e^{-\frac{t}{(L/R)}}$$

$$\frac{I}{I_0} = e^{-\frac{t}{\tau}}, \quad \tau = \frac{L}{R}$$

The Current Decay in LR Circuit



$$I(t) = I_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R}.$$

End of Lecture 19
Reading: Entire Chapter 33
Review for Quiz 9
HW9