Lecture 20: Paragraphs 34.4-34.8 fromChapter 34, November 22 2005

Maxwell's Equations

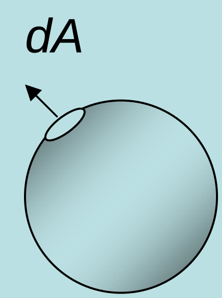
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \text{ Gauss's law}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \text{ Gauss's law for magnetism}$$

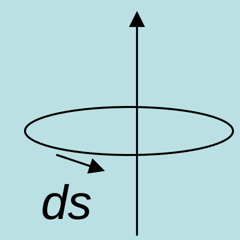
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \text{ Faraday's law}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} + \boxed{\epsilon_0 \mu_0 \frac{d\Phi_e}{dt}} \quad \text{Ampère-Maxwell law}$$

Surface integrals:

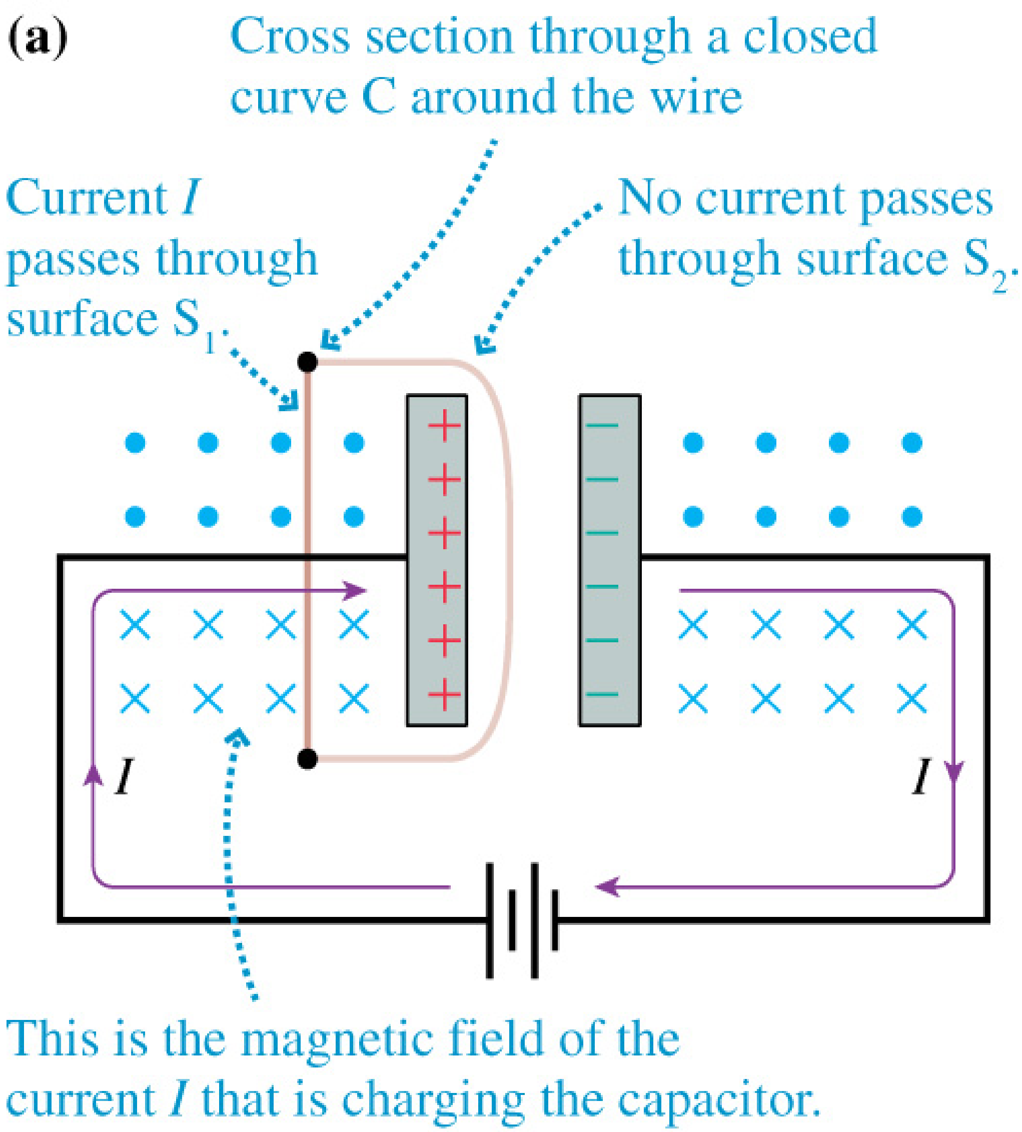


Line integrals



The equations have been studied except the term in the red box.

Displacement Current



Previously we introduced Ampere's law:

$$\oint B ds = \mu_0 I_{through}$$

Any surface bounded by C

- Ampere's law says that we can consider ***any*** surface bounded by curve C to calculate $I_{through}$.

- However the result is different for surfaces S_1 and S_2. Why is this?

Displacement Current (continuing)

The carriers are stopped at the electrode, but the electric field (and its flux) physically presents in the gap

$$\Phi_e = EA,\_but\_E = Q/\varepsilon_0 A - plane\_parallel\_capacitor$$

$$\Phi_e = \frac{Q}{\varepsilon_0 A} A = \frac{Q}{\varepsilon_0}$$

$$\frac{d\Phi_e}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} = \frac{I}{\varepsilon_0}$$

$$I_{disp} = \varepsilon_0 \frac{d\Phi_e}{dt}$$

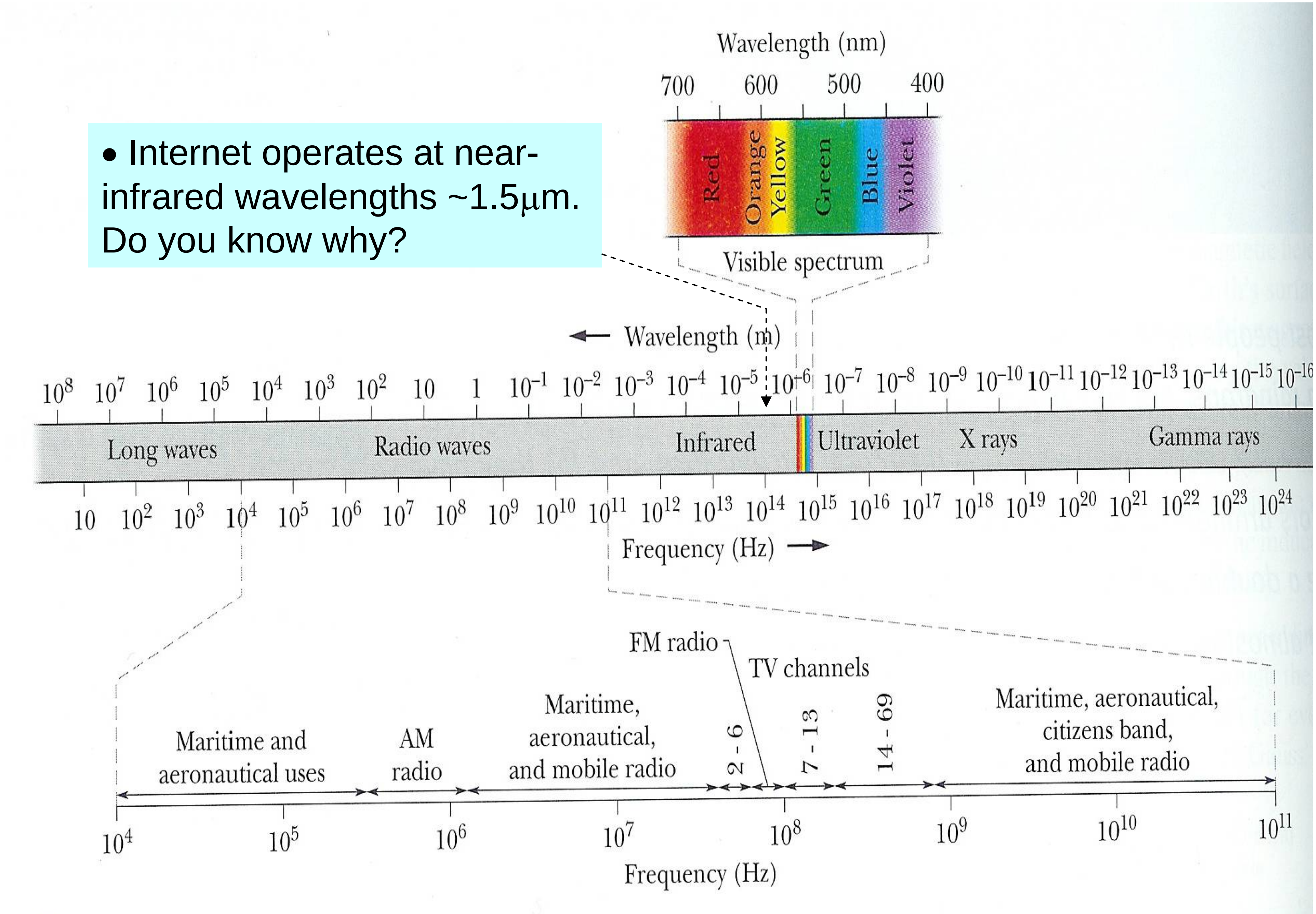
If we introduce this “current” when Ampere’s Law will be correct for any S_2

$$\oint \vec{B} d\vec{s} = \mu_0 (I_{through} + I_{disp}) = \mu_0 \left(I_{through} + \varepsilon_0 \frac{d\Phi_e}{dt} \right)$$

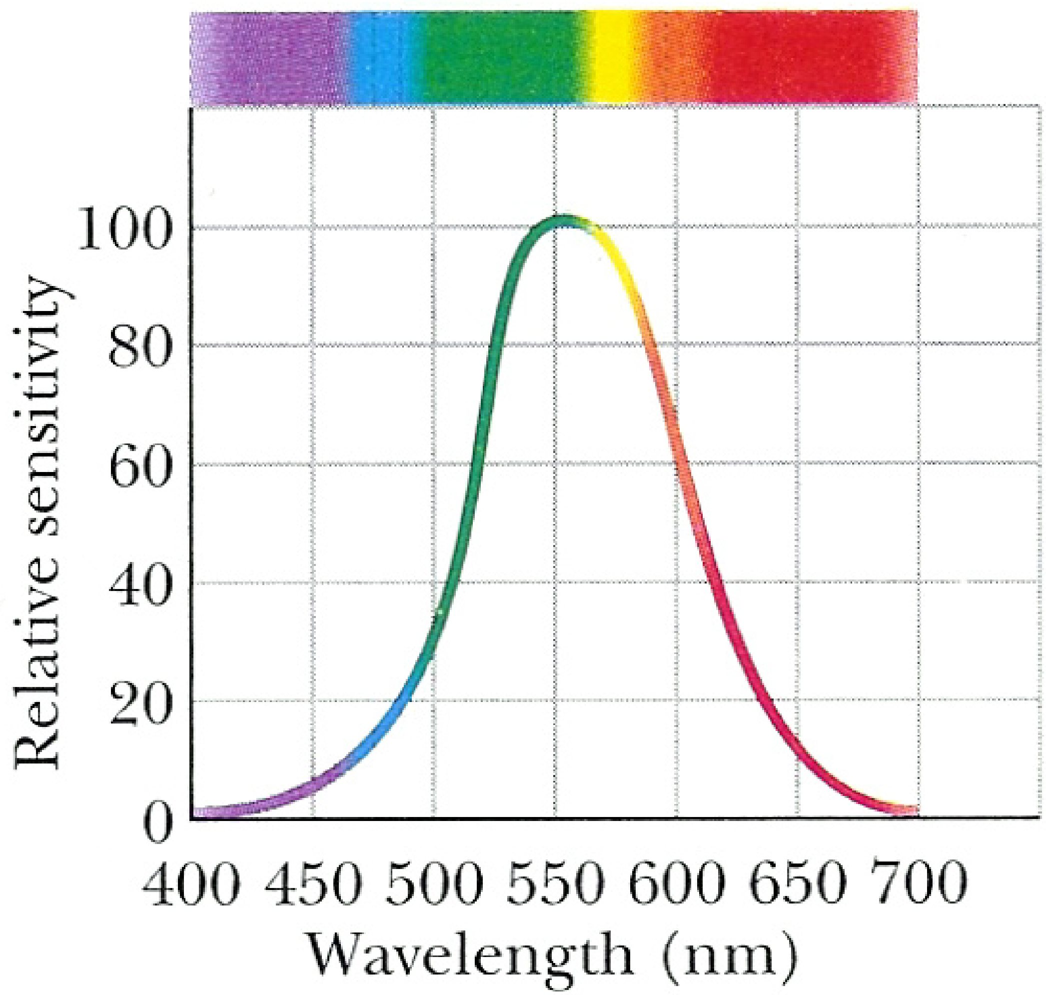
- Displacement current creates same B as real current, but it does so with a changing flux rather than a flow of charge

Maxwell's Rainbow

• Internet operates at near-infrared wavelengths ~1.5μm. Do you know why?

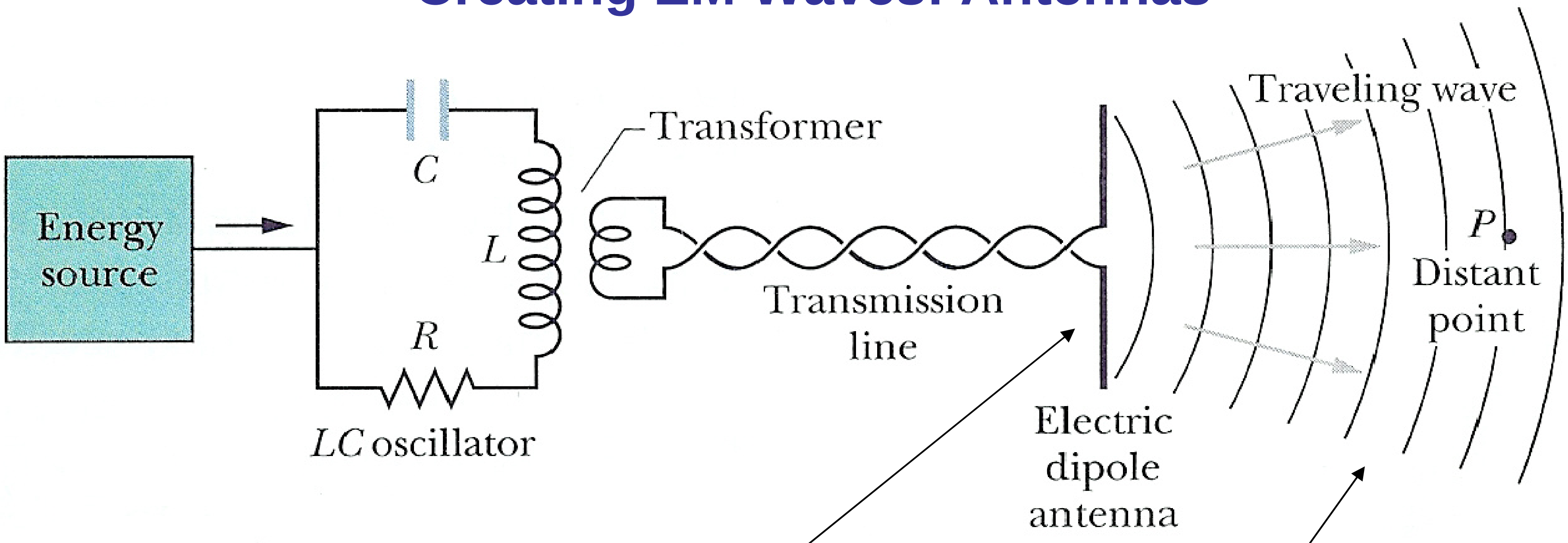


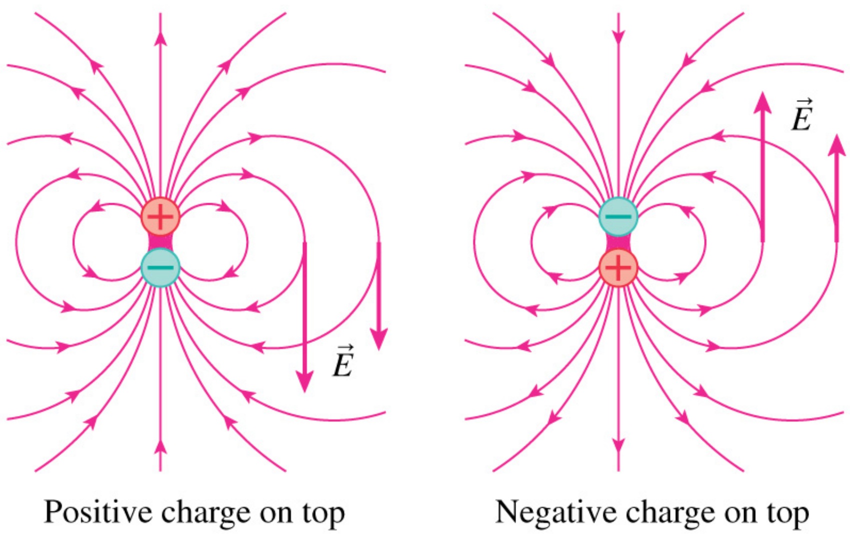
Visible Light

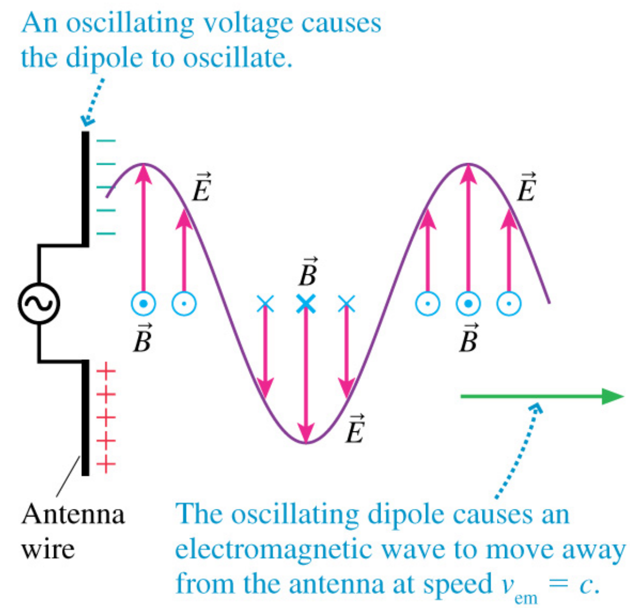


- Eye sensitivity to visible light is maximal for a green color
- Visible light occupies a very short range of wavelengths

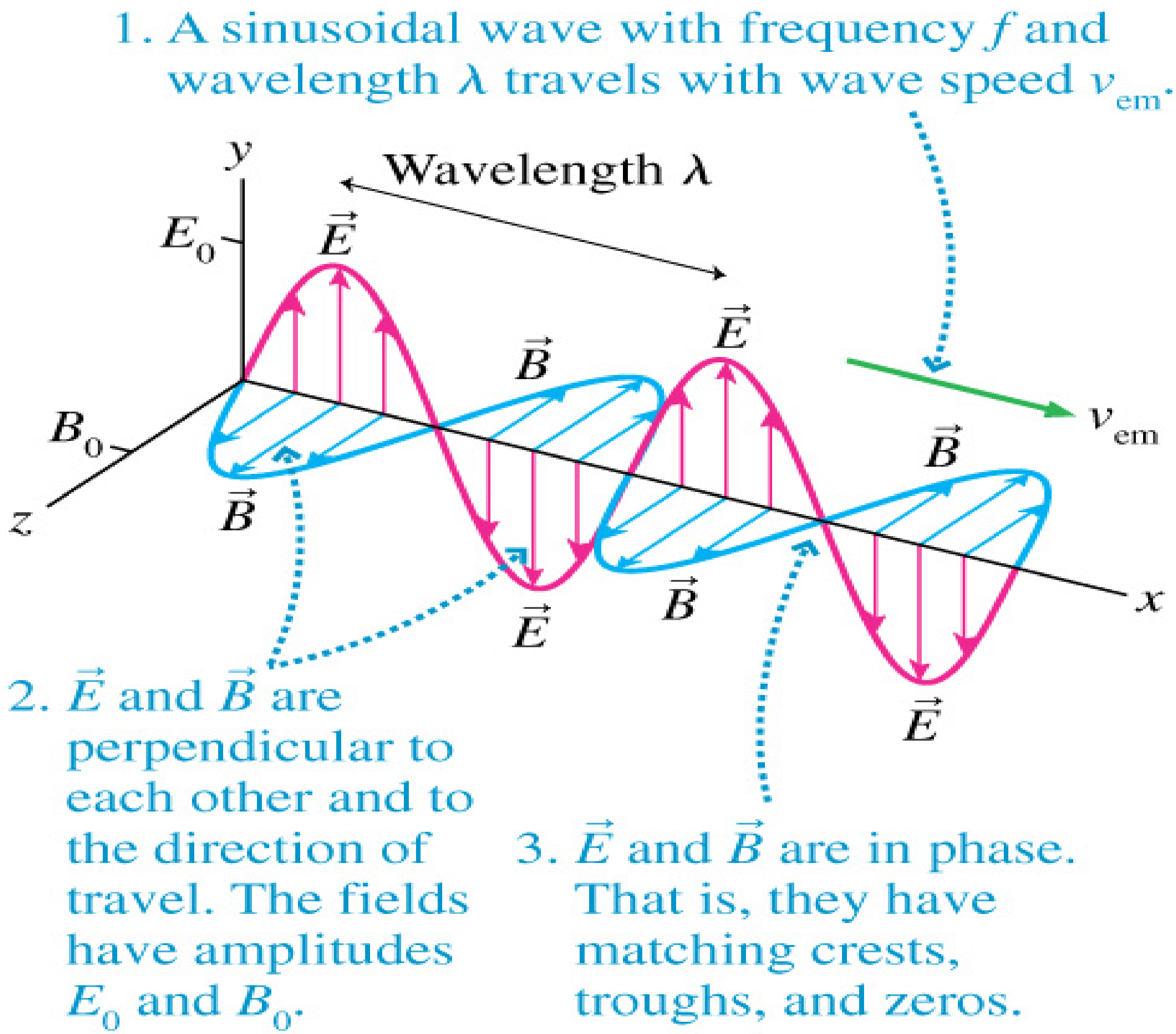
Creating EM Waves: Antennas







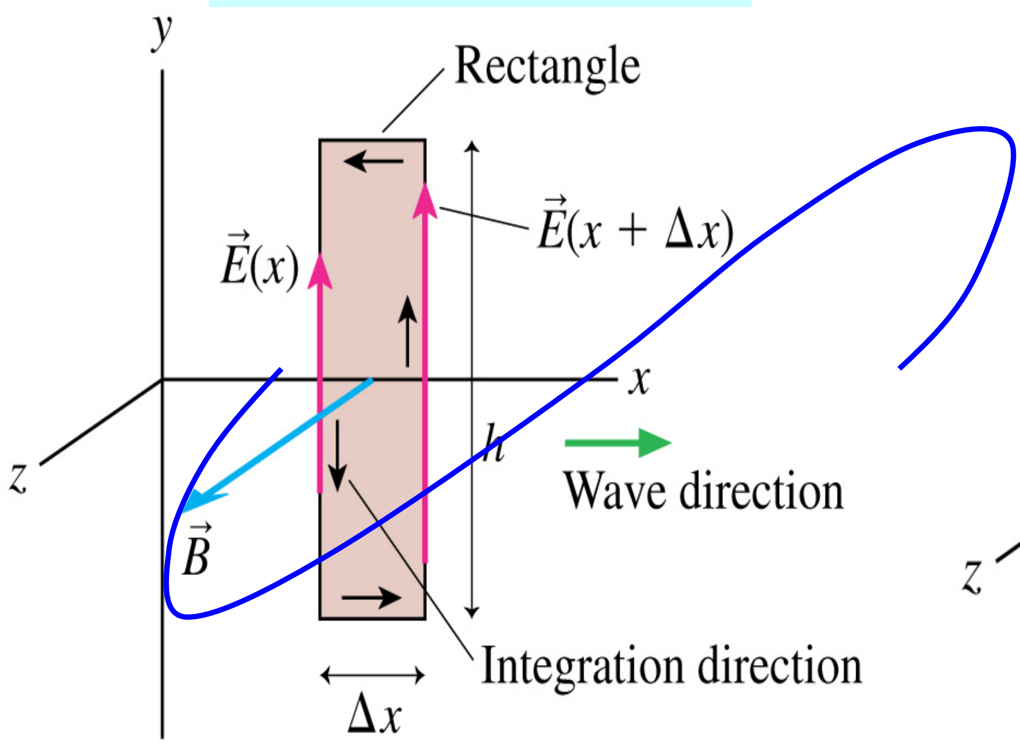
Propagation of EM Waves



Two Major Laws Describing EM Waves

Faraday Law

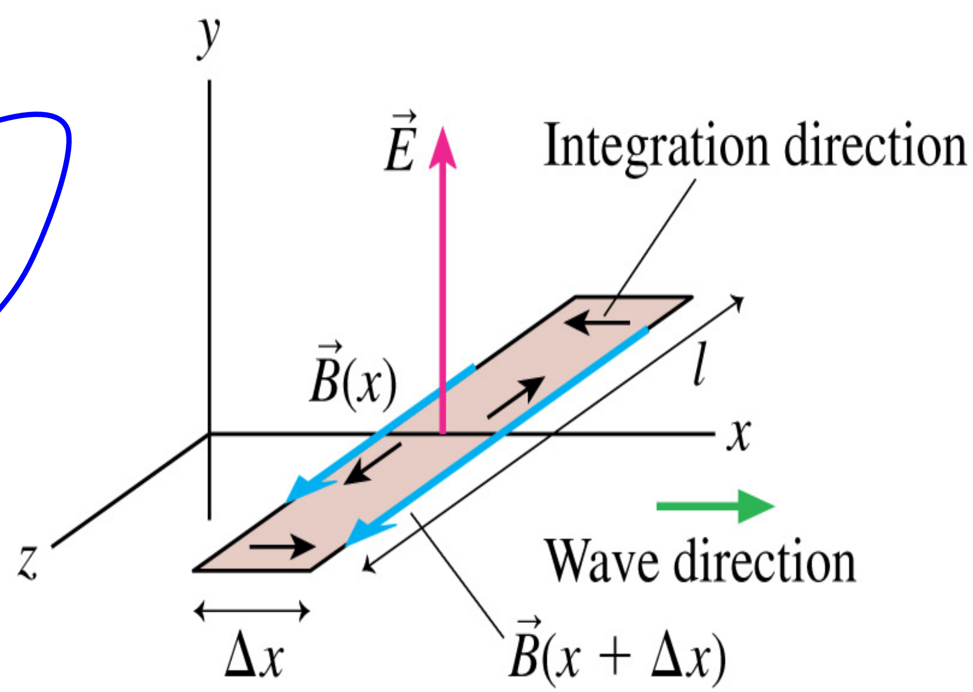
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$



Changing *B* creates *E*

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{s} = \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt}$$



Changing *E* creates *B*

The Speed of Light

Two Equations should be satisfied at the same time, see Ch. 34:

$$E_0 = (\lambda f)B_0 = v_{em}B_0$$

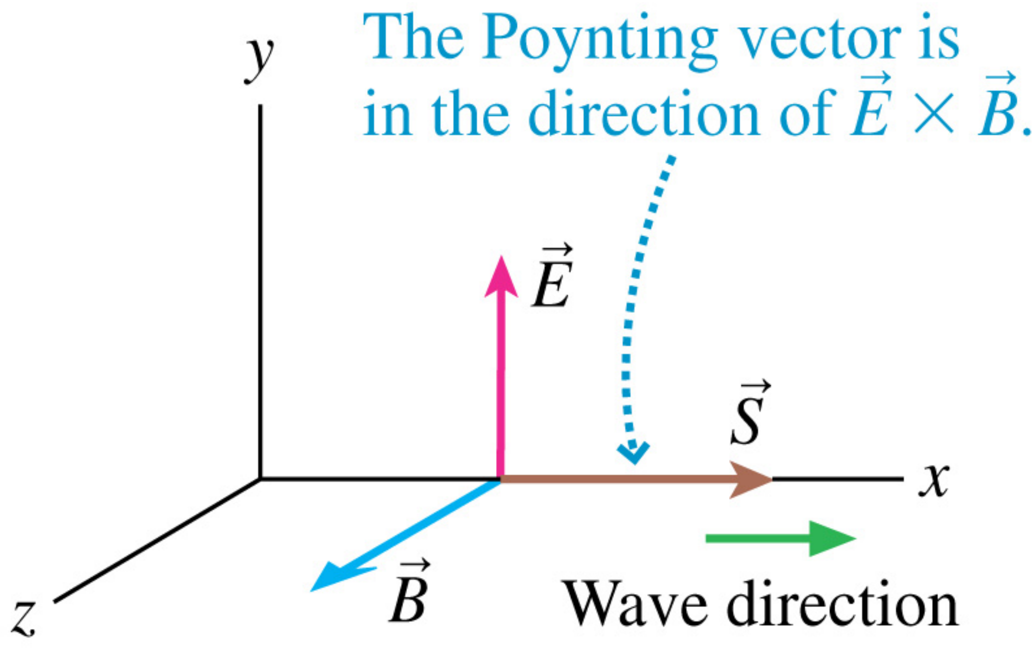
$$E_0 = B_0/(\varepsilon_0\mu_0 v_{em})$$

This means that:

$$v_{em} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3.00 \times 10^8 \text{ m/s} = c \qquad (34.45)$$

No other speed will satisfy Maxwell's equations

Energy and Intensity



$$S = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$Since\_\vec{E} \perp \vec{B}\_and\_E = cB$$

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

$$I = \frac{P_{source}}{4\pi r^2}$$

$$I = \frac{P}{A} = S_{avg} = \frac{1}{2c\mu_0} E_0^2 = \frac{c\epsilon_0}{2} E_0^2 \qquad (34.47)$$

- At any point the Pointing vector represents the direction of wave
- The magnitude of *S* represents the rate of energy transfer per unit area of the wave. It oscillates at extremely high frequency.
- Intensity is averaged energy transfer, $I = S_{avg}$

Radiation Pressure

$$\Delta p = \frac{energy\_absorbed}{c}$$ ← Without proof

Newton's second law:

$$F = \frac{\Delta p}{\Delta t} = \frac{(energy\_absorbed)/\Delta t}{c} = \frac{P}{c}$$

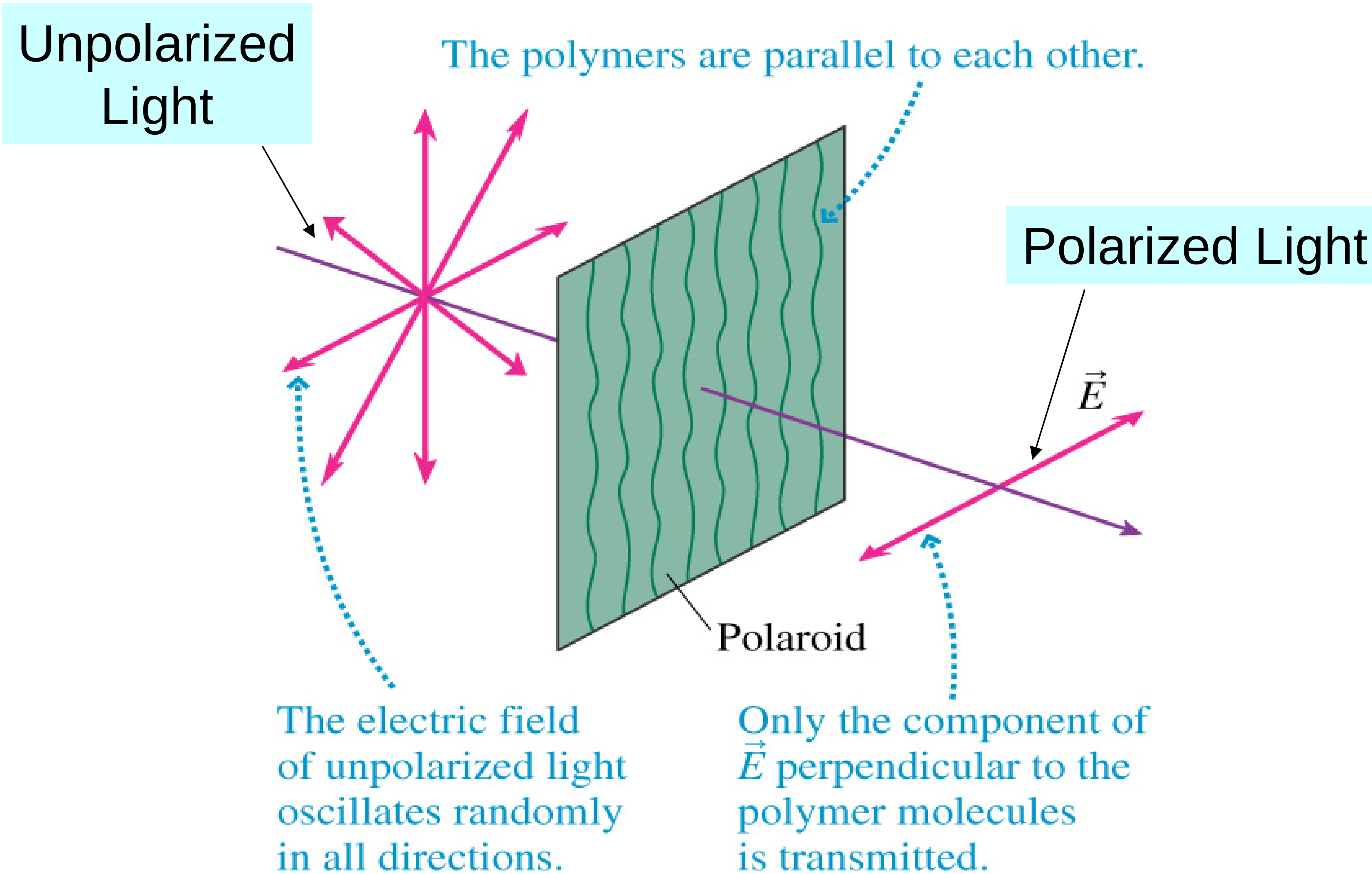
$$Force\_per\_unit\_area:$$

$$Radiation\_pressure$$

$$p_{rad} = \frac{F}{A} = \frac{(P/A)}{c} = \frac{I}{c}$$

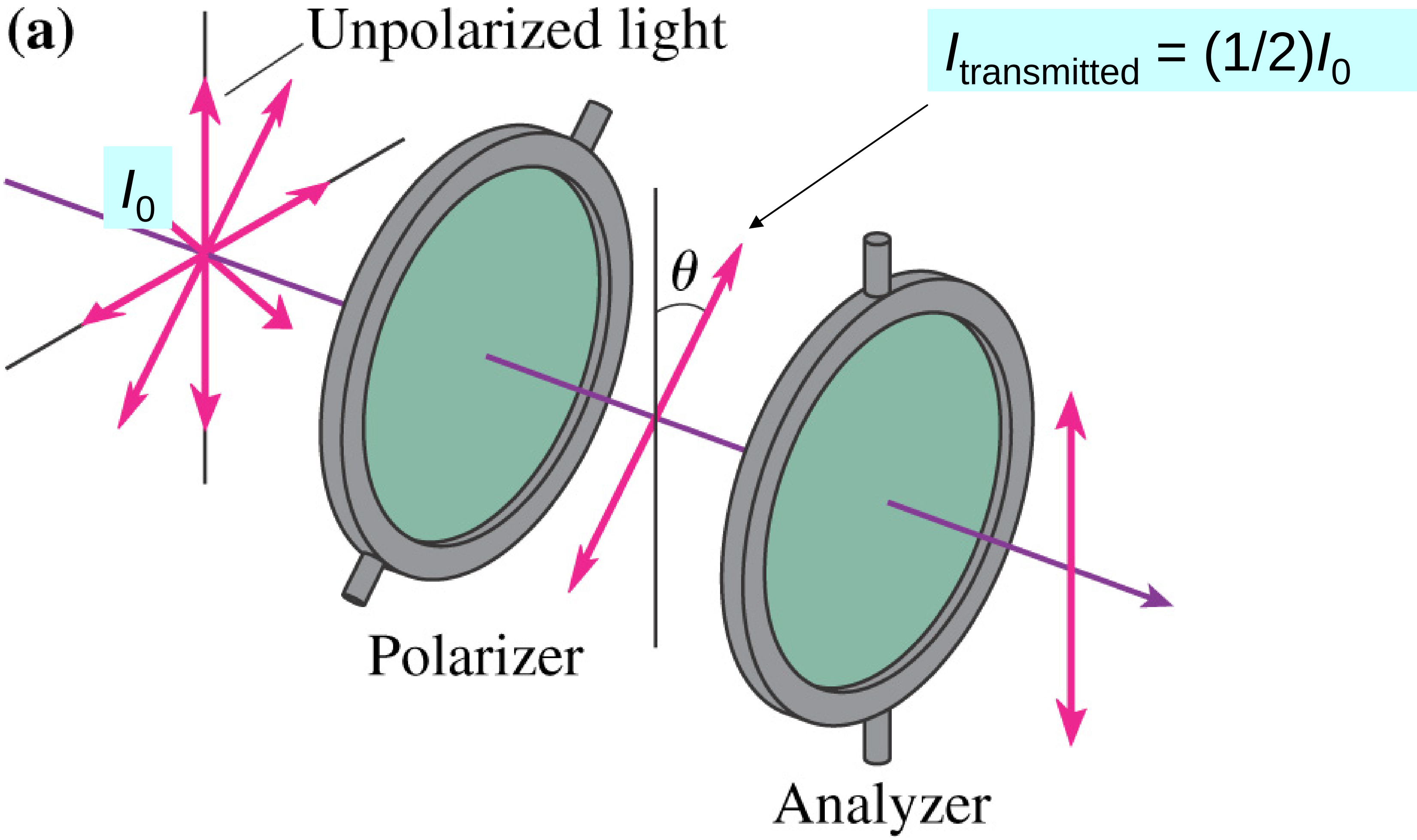
- Transfer of momentum from light to the object
- For totally reflecting object the pressure is doubled: $p_{\mathrm{rad}} = 2I/c$

Polarization



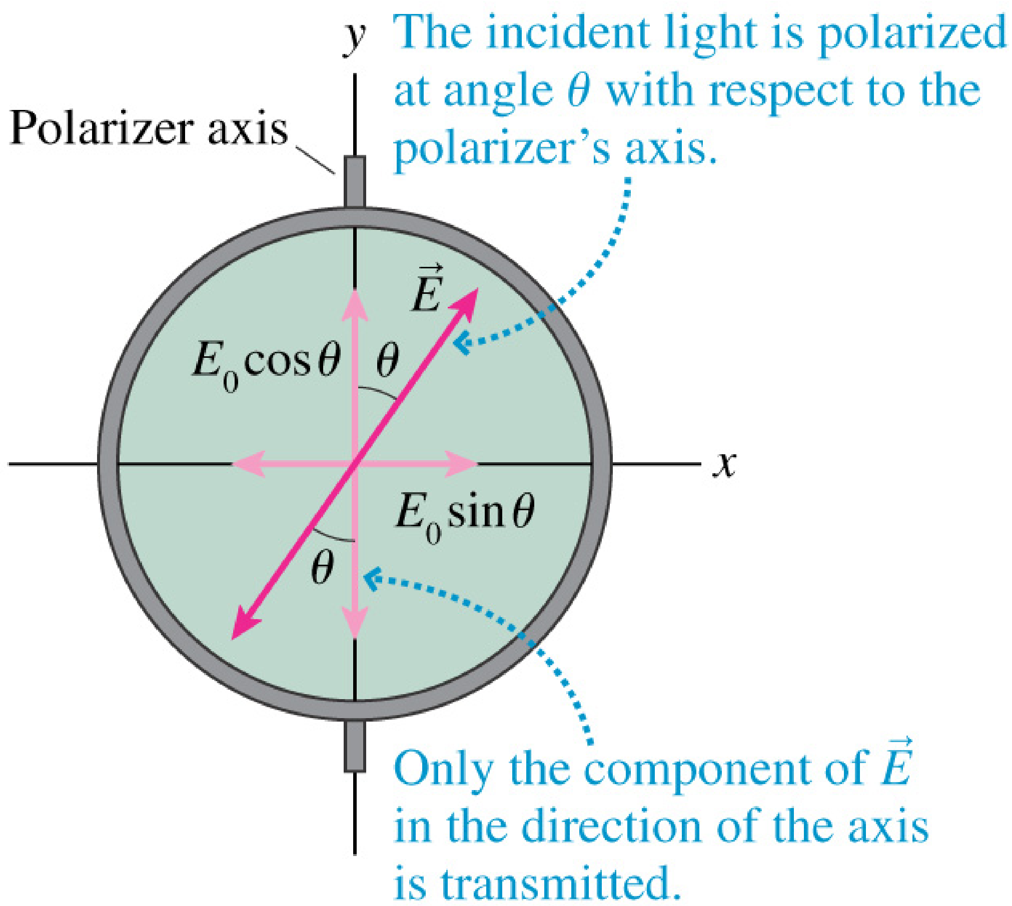
- Usual sources, sun light, lumps, etc., are unpolarized
- Lasers produce polarized light
- Unpolarized light can be polarized using polaroid

First Malus's Law: Initially unpolarized light



- If incident light is unpolarized the intensity of the beam transmitted through the polarized is $I_0/2$.

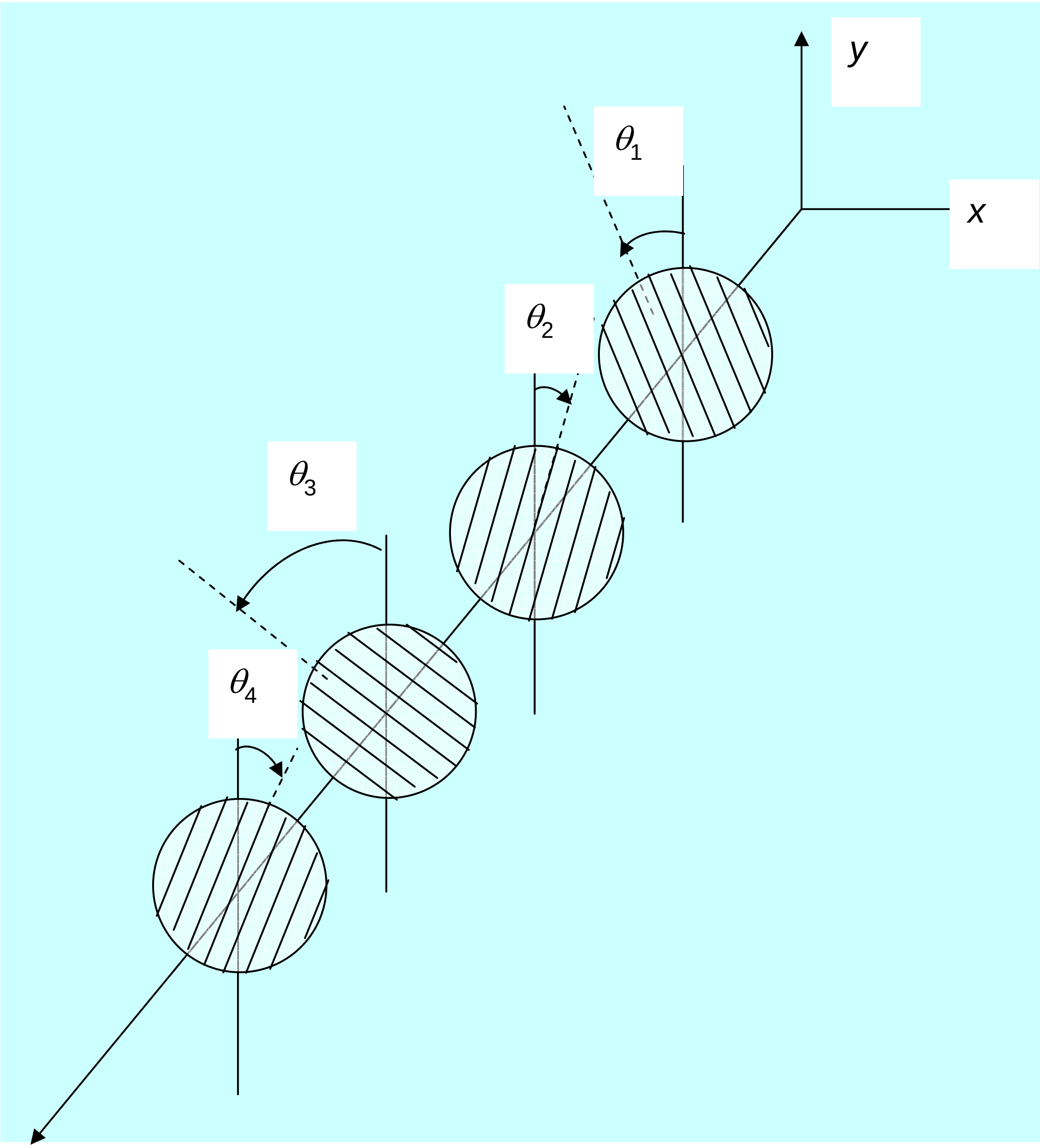
Second Malus's Law: Incident Light is already polarized



$$I_{\text{transmitted}} = I_0 \cos^2\theta \qquad \text{(incident light polarized)} \qquad (34.52)$$

• This law is a consequence of the fact that *I* ~ (squared component of ***E*** in the direction of polarizer)

Example of the Problem with Multiple Polarizers



In figure, initially unpolarized light is sent through four polarizing sheets. The first and third sheets have polarizing directions making angles of $\theta_1 = 30^0$ and $\theta_3 = 60^0$ (measured counterclockwise) with the direction of the *y*-axis. The second and forth sheets have polarizing direction making angles of $\theta_2 = 15^0$ and $\theta_4 = 25^0$ (clockwise) with the direction of the y-axis.

What percentage of the initial intensity is transmitted by the system of the four polarizing sheets?

Solution

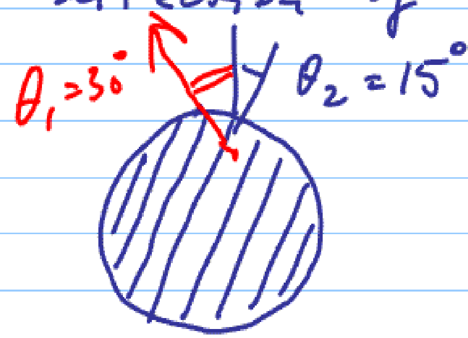
If incident light is unpolarized:

$$I_1 = \frac{1}{2} I_0$$

If incident light is polarized:

$$I_i = I_{i-1} \cos^2 \theta_i$$ (Malus's Law)

θ - angle between direction of polarization of incident wave and direction of polarizer



$$\theta = \theta_1 + \theta_2$$

$$I_2 = I_1 \cdot \cos^2(\theta_1 + \theta_2) =$$

$$= I_1 \cdot \cos^2 45^\circ = \frac{I_1}{2}.$$

$$I_3 = I_2 \cdot \cos^2(\theta_2 + \theta_3) = I_2 \cdot \cos^2 75^\circ =$$

$$= I_2 \cdot 0.067$$

$$I_4 = I_3 \cdot \cos^2(\theta_3 + \theta_4) = I_3 \cdot \cos^2 85^\circ =$$

$$= I_3 \cdot 7.6 \cdot 10^{-3}.$$

$$I_4 = I_2 \cdot 0.067 \cdot 7.6 \cdot 10^{-3} = \frac{I_1}{2} \cdot 0.067 \cdot 7.6 \cdot 10^{-3} =$$

$$= 0.25 \cdot 6.7 \cdot 10^{-2} \cdot 7.6 \cdot 10^{-3} I_0 = 1.27 \cdot 10^{-4} I_0.$$

Transmission: $I_4 = 0.0127\% \, I_0$

End of Lecture 20

Reading: Paragraphs 34.4-34.8 from Chapter 34

HW11