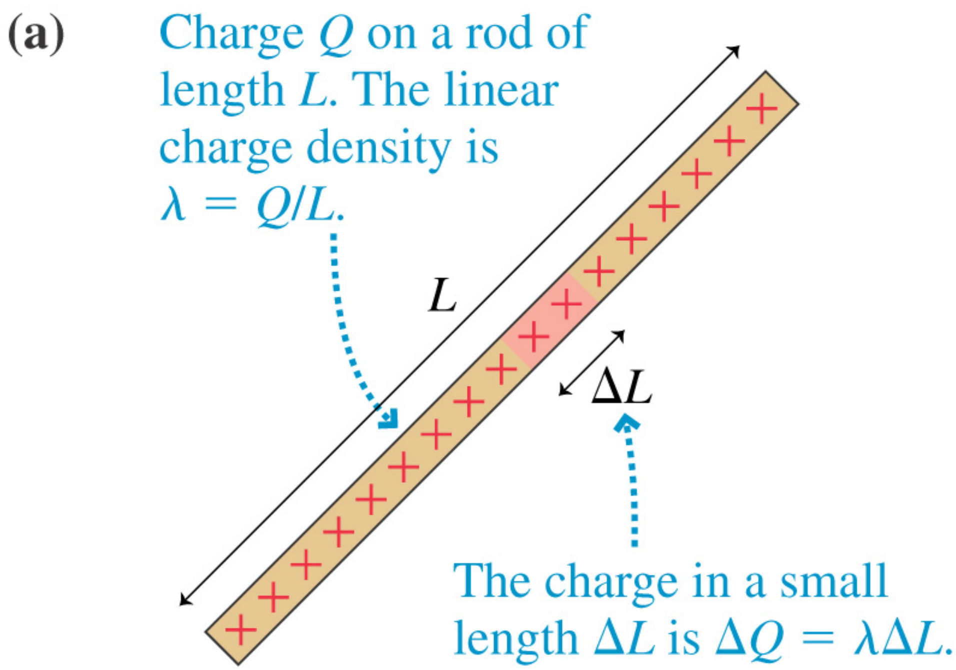
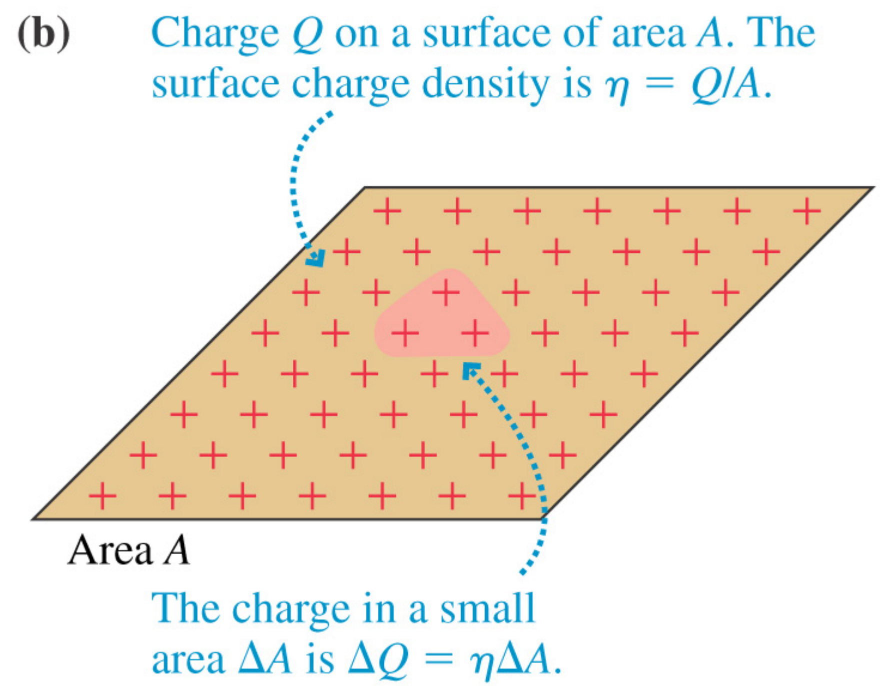
Lecture 4: Chapter 26, September 6 2005

The Electric Field of a Continuous Charge Distribution

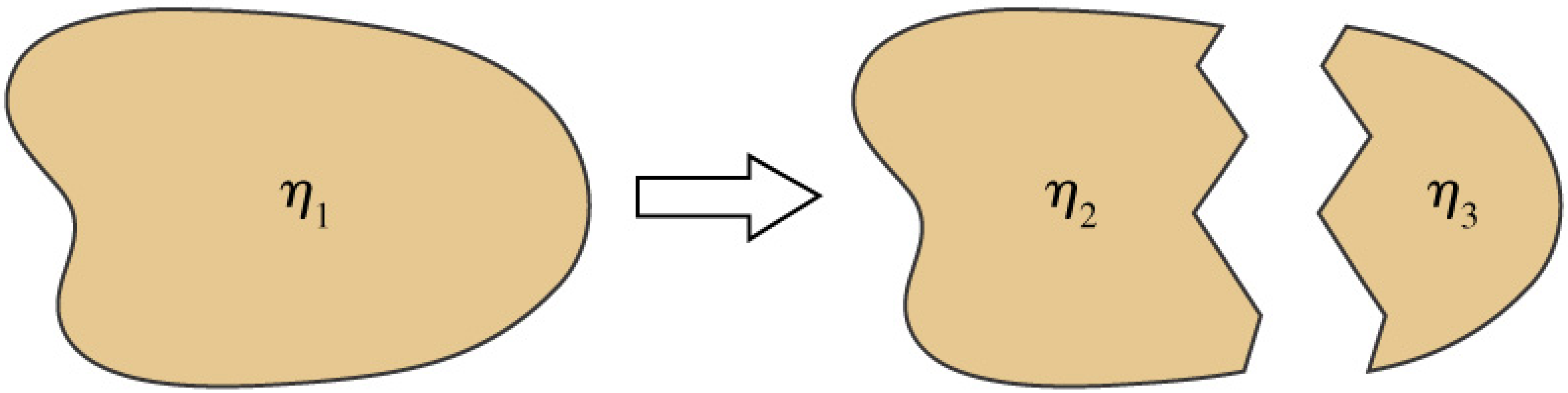




Linear charge density:
$\lambda = Q/L$

Surface charge density:
$\eta = Q/A$ (In most of other texts it is σ)

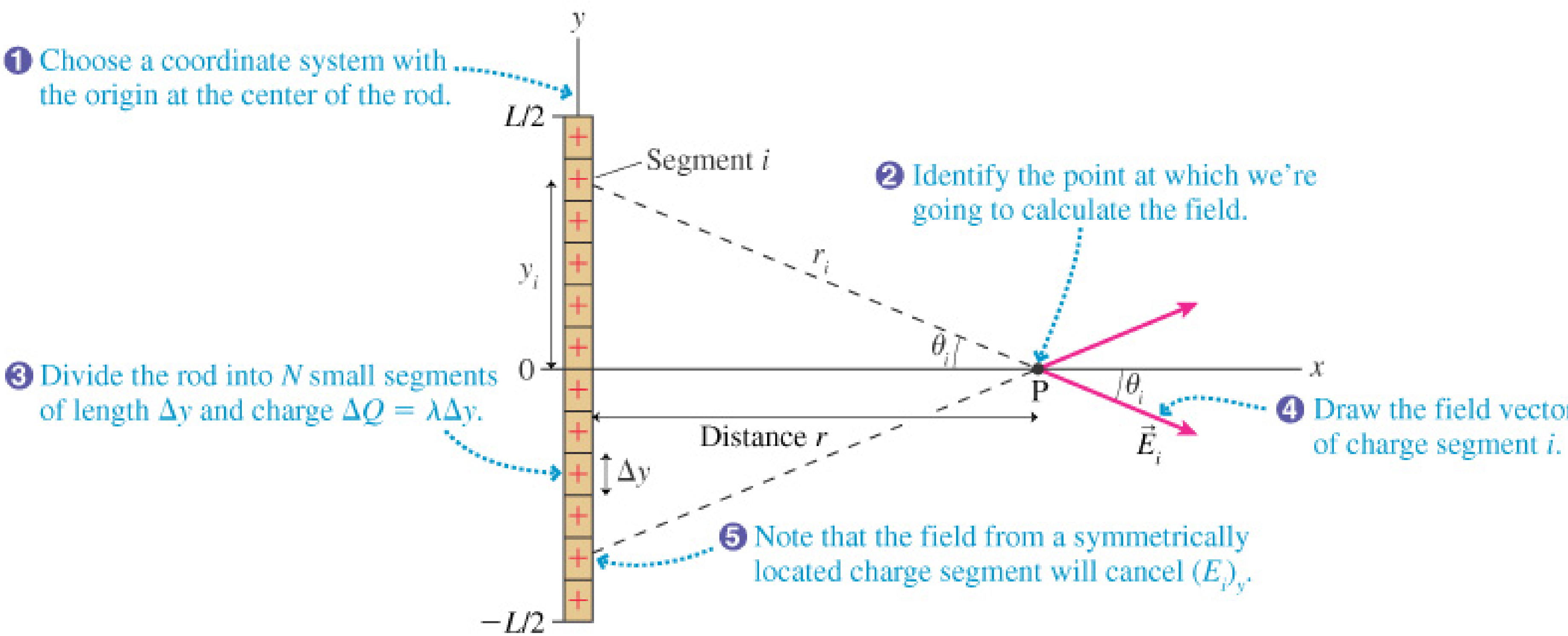
A piece of plastic is uniformly charged with surface density η_1. After braking into pieces, rank in order, from largest to smallest, the surface charge densities η_1 to η_3.



Is plastic dielectric or metal? Why is this important?

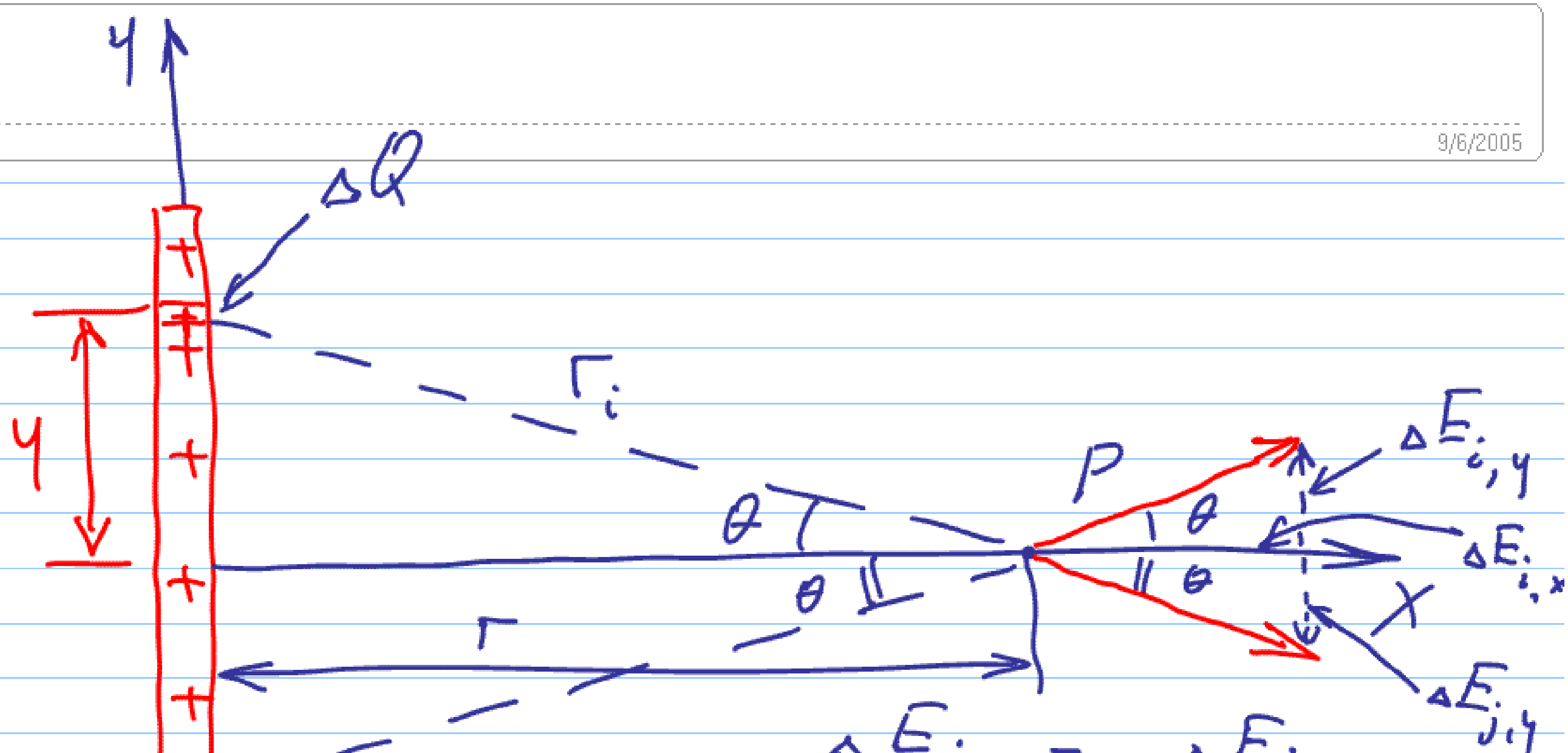
Problem Solving Strategy

Example: Electric field of a line of charge



- Divide the total charge Q into small pieces of charge ΔQ
- Use Coulomb law and draw the el. field ($\boldsymbol{E}_i$) for one or two small pieces of charge
- Look for symmetries of the charge distribution that simplify the fields
- Find $\boldsymbol{E}_{net} = \Sigma \boldsymbol{E}_i$, express an integral and integrate it component by component

Note Title 9/6/2005



$$\Delta E_{i,y} = -\Delta E_{j,y}.$$

$$\underbrace{\sum_i \Delta E_i = 0}\ \text{— nice}$$

$$\Delta E_{i,x} = |\Delta E_i| \cdot \cos\theta =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q_i \cdot \cos\theta}{r_i^2} = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q_i}{r^2 + y_i^2} \underbrace{\frac{r}{\sqrt{r^2 + y_i^2}}}_{\cos\theta} =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q_i \cdot r}{(r^2 + y_i^2)^{3/2}}.$$

$$E_{net,x} = \sum_i \Delta E_{i,x} = \frac{r}{4\pi\varepsilon_0} \sum_i \frac{\Delta Q_i}{(r^2 + y_i^2)^{3/2}}$$

$\Sigma \to \int$

$$= \frac{r}{4\pi\varepsilon_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\lambda \cdot dy}{(r^2 + y^2)^{3/2}} =$$

$\lambda \cdot dy \longleftrightarrow \Delta Q_i$

$$\boxed{\int \frac{dy}{(r^2 + y^2)^{3/2}} = \frac{y}{r^2 (r^2 + y^2)^{1/2}}}$$

$$= \frac{\cancel{y}\cdot\lambda}{4\pi\varepsilon_0}\;\frac{y}{r\cancel{x}\,(r^2+y^2)^{1/2}}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} =$$

$$\frac{\lambda}{4\pi\varepsilon_0 r}\left(\frac{L/2}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}} - \frac{\left(-\frac{L}{2}\right)}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}}\right) =$$

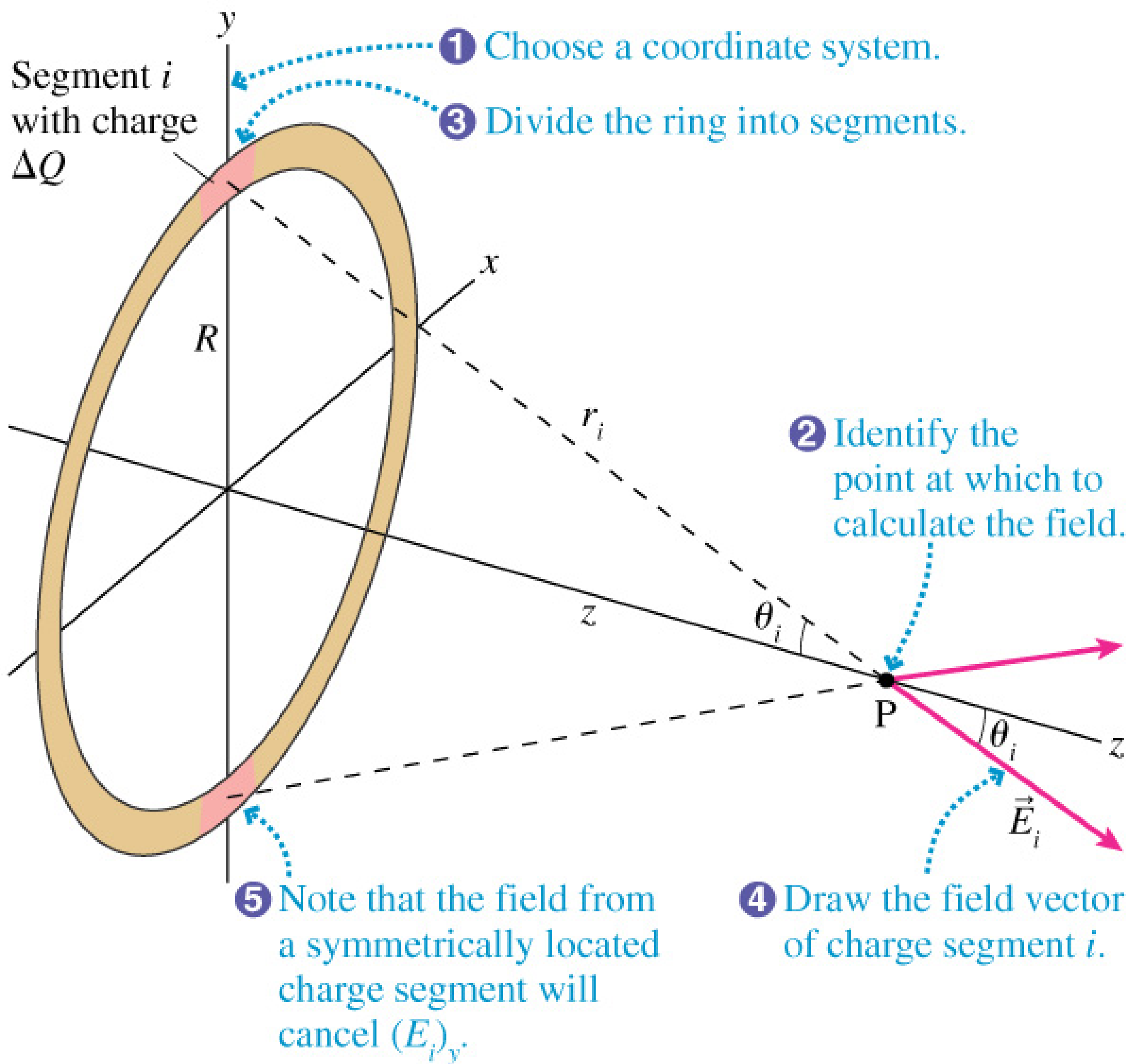
$$\boxed{\lambda\cdot L = Q}$$

$$= \frac{\lambda}{4\pi\varepsilon_0\cdot r}\;\frac{L}{\sqrt{r^2+\left(\frac{L}{2}\right)^2}} = \frac{Q}{4\pi\varepsilon_0}\;\frac{1}{r\cdot\sqrt{r^2+\left(\frac{L}{2}\right)^2}}$$

$$Q = \lambda\cdot L$$

$$E_{net,x}(L\to\infty) = \frac{\lambda\cdot\cancel{L}}{4\pi\varepsilon_0}\;\frac{1}{r\;\frac{\cancel{L}}{2}} = \frac{2\lambda}{4\pi\varepsilon_0\cdot r}$$

The Electric Field of a Ring of Charge



See solution in Chapter 26, page 829

Distance dependencies of some distribution of charges

$$E_{\text{line}} = \lim_{L \to \infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

An Infinite Line of Charge

• Line of charge is infinite in one dimension in 3-D space ⇒Q is infinite

$$E_{point} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

A point Charge

• *Point* charge is totally localized and *Q* (or *q*) is finite

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3} \quad \text{(on the axis of an electric dipole)}$$

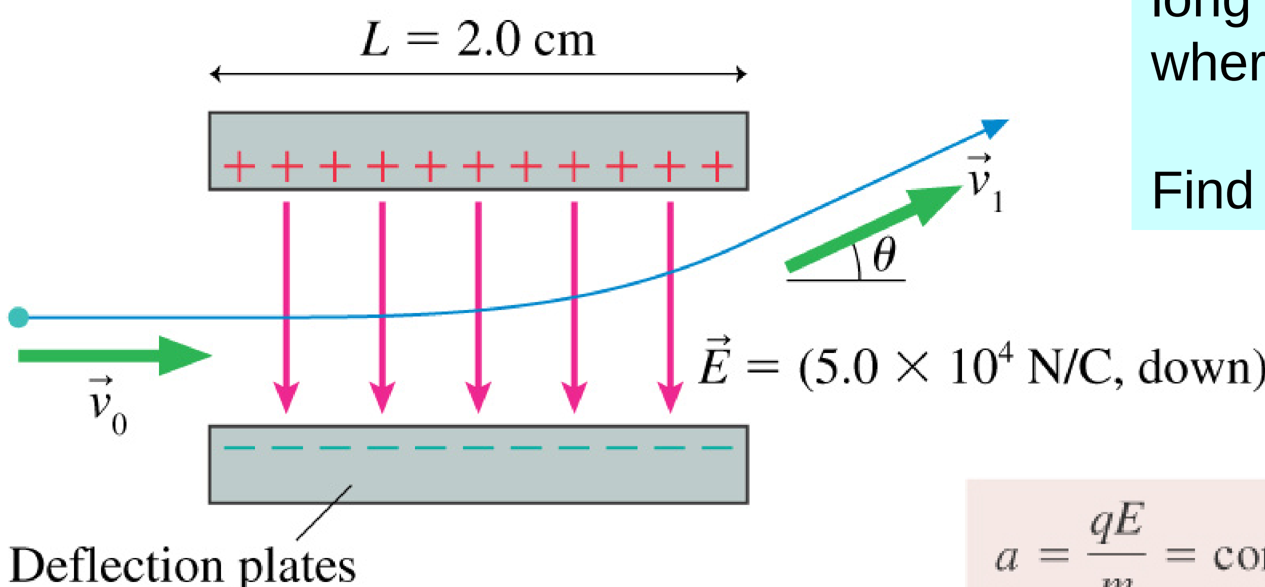
A Dipole

• *Dipole* has zero net charge ⇒ Q=0, but ***p*** is finite

These dependencies have similar structure but different distance (~r^n where n=1,2,3) dependencies. Any ideas why is this?

How electric fields acts on charges

Deflecting an Electron Beam



An electron gun creates a beam of electrons moving horizontally: $v_0 = 3.34\ 10^7$ m/s. The electrons enter a 2.0-cm-long gap between electrodes where E = 5.0 10^4 N/C.

Find the deflection angle θ.

$$a = \frac{qE}{m} = \text{constant} \qquad (26.31)$$

Solution

Along x-axis we have motion with constant velocity: $v_x = v_0$

This allows determining time-of-flight: $\Delta t = L/v_x$

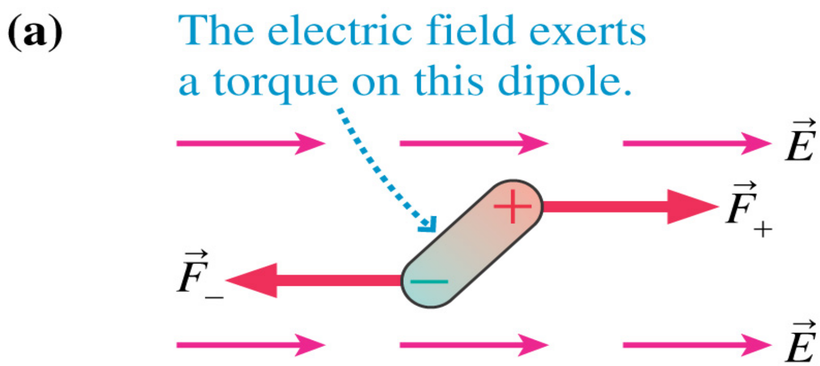
Along y-axis we have motion with constant acceleration:
$a=F/m$, where $F = eE$

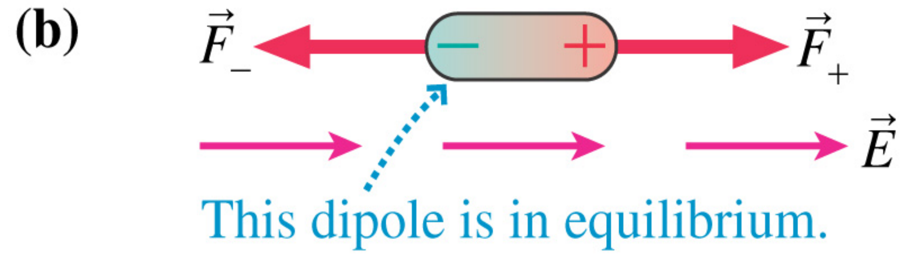
This allows determining vertical component of the resultant velocity:

$$v_y = a\, \Delta t = eEL/mv_0$$

Finally, $\theta = \tan^{-1} (v_y/v_x)$

How electric field acts on a dipole





• ***E*** is *not* the field of the dipole, but, instead, is a field to which the dipole is responding!
$\boldsymbol{F}_+ = +q\boldsymbol{E}$ and $\boldsymbol{F}_- = -q\boldsymbol{E}$

• These forces are equal but opposite, thus:
$\boldsymbol{F}_{net} = \boldsymbol{F}_+ + \boldsymbol{F}_- = 0$

• Because two forces are ***not aligned***, the field causes the dipole to ***rotate***.

•What is the torque on this dipole

• Two forces are ***aligned***.

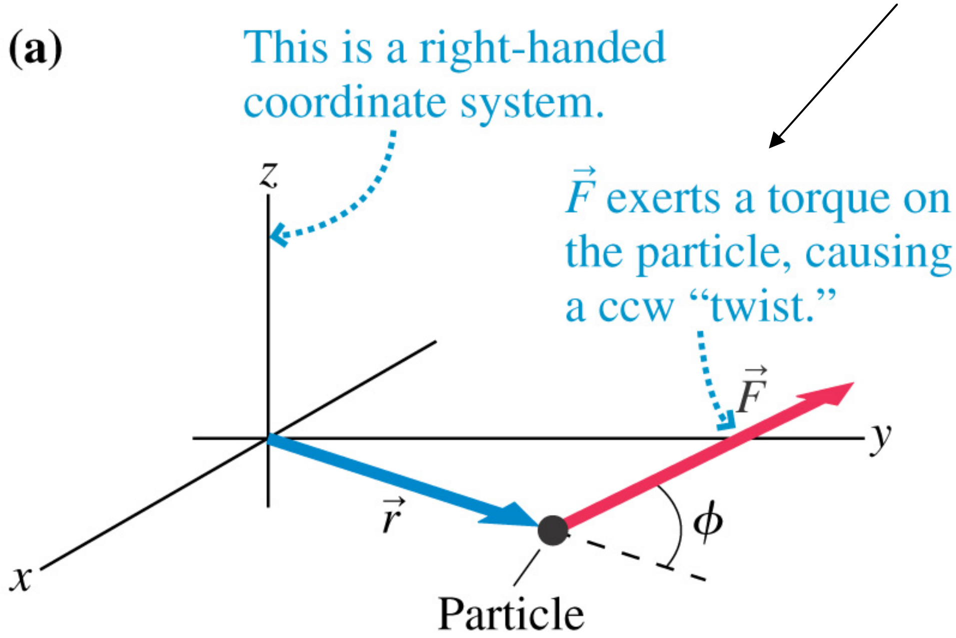
•What is the torque on this dipole?

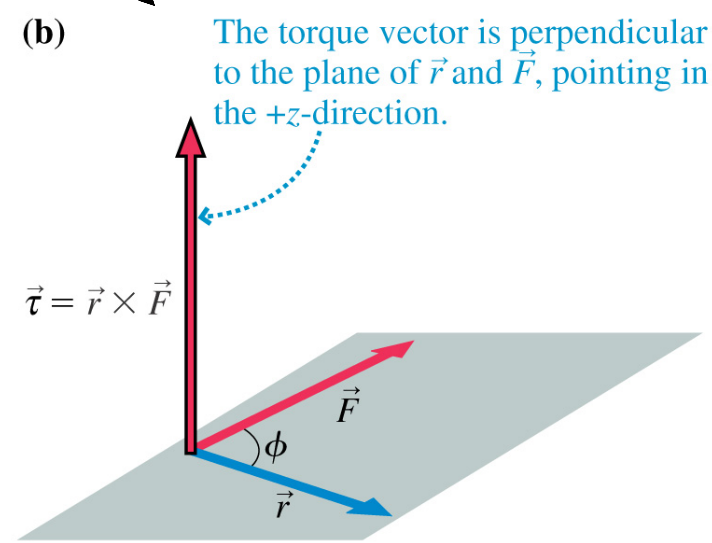
To remind about rotational motion

Striking Analogy

Newton's law for straight motion	Newton's law for rotational motion
Newton's law for straight motion: $\boldsymbol{a} = \boldsymbol{F}/m$	Newton's law for rotational motion: $\alpha = \tau_{net}/I$
$\boldsymbol{a}$ - acceleration	α - angular acceleration
$\boldsymbol{F}$ – Force	$\tau = \boldsymbol{r} \times \boldsymbol{F}$ - torque
m - mass	$I = \Sigma m_i r_i^2$ – moment of inertia

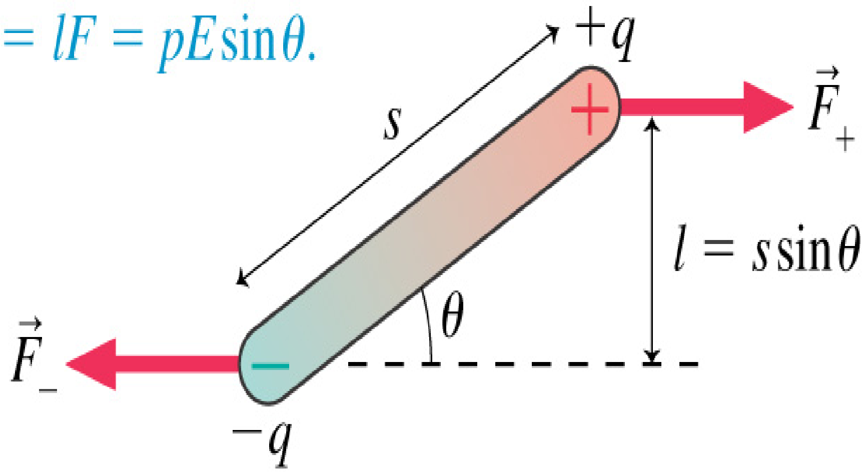
Torque (For a review see Chapter 13)



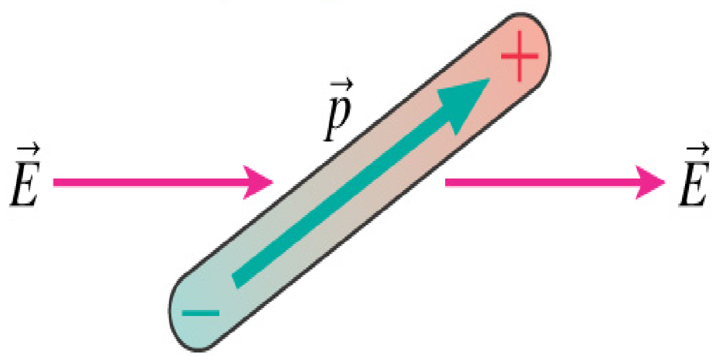


How to calculate the torque on a dipole?

The torque due to a couple is $\tau = lF = pE\sin\theta$.



In terms of vectors, $\vec{\tau} = \vec{p} \times \vec{E}$.



Torque (τ) is a product of F with the distance (l) between the forces lines (Chapter 13):

$$\tau = Fl = (qE)(s\ \sin\theta) = pE\sin\ \theta$$

End of Lecture 4
Reading: Entire Chapter 26
Review for Quiz 2
Home Work 1 and 2 in Mastering Physics