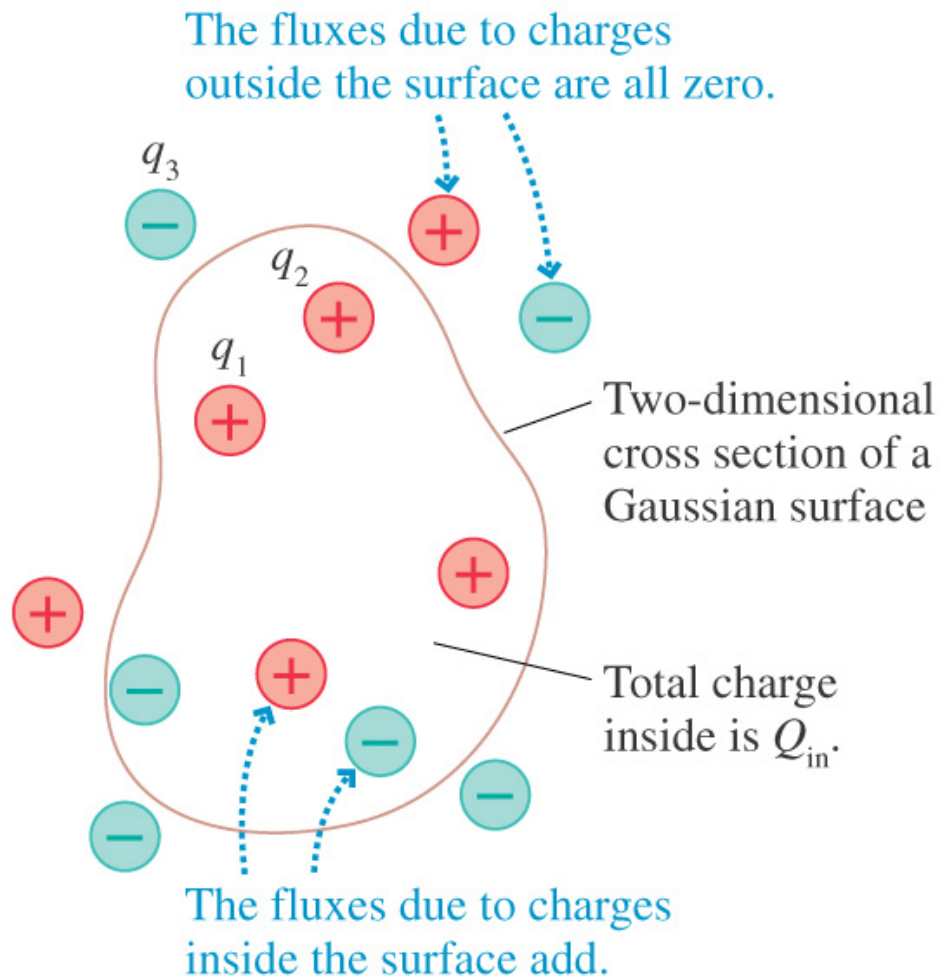


Lecture 5: Chapter 27, September 8 2005

One of the Maxwell's equations: Gauss's Law

$$\Phi_e = \oint_{\text{surface}} \vec{E} d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$



Powerful tool of finding \mathbf{E} for highly symmetric charge distributions:

- i) Spherical
- ii) Cylindrical
- iii) Plane.

Requires knowledge of new concepts:

Symmetry

Flux (Φ_e)

Gaussian Surface

Taking surface integral

Main trick: Move \mathbf{E} through the integration due to symmetry

Symmetry



Original cylinder



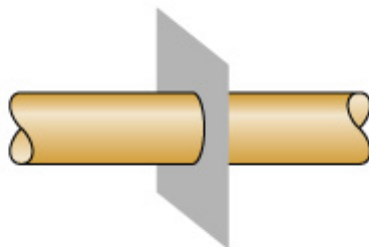
Translation parallel to the axis



Rotation about the axis



Reflection in plane containing the axis



Reflection perpendicular to the axis

The charge distribution is **symmetric** if there is a group of *geometrical transformations* that don't cause any *physical change*:

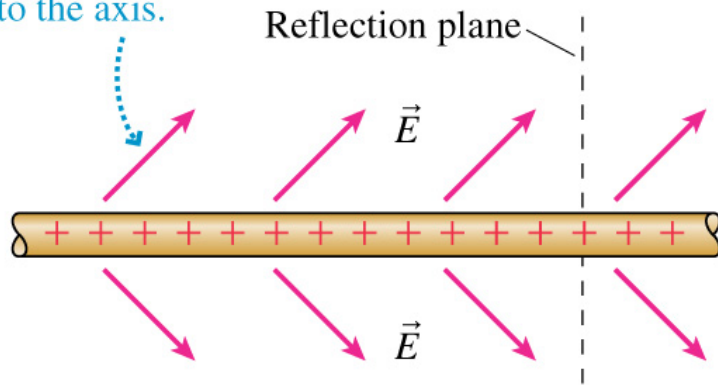
- Translation
- Rotation about an axis (rotation axis)
- Reflection the charge in a mirror (symmetry plane)

Why is this symmetry important?

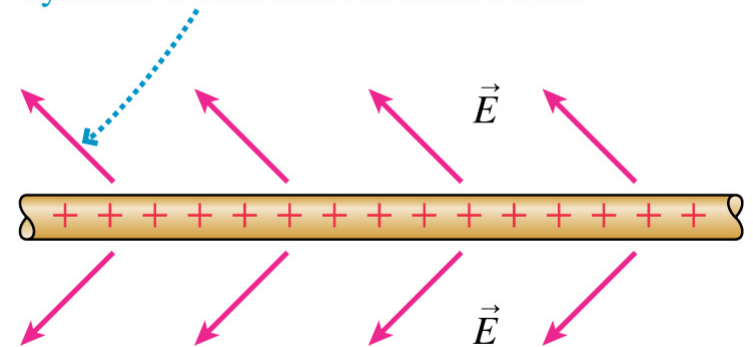
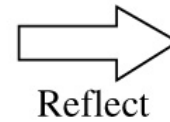
Because the symmetry of electric field must match the symmetry of charge distribution

Is this field distributions possible?

- (a) Is this a possible electric field of an infinitely long charged cylinder? Suppose the charge and the field are reflected in a plane perpendicular to the axis.



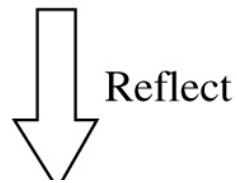
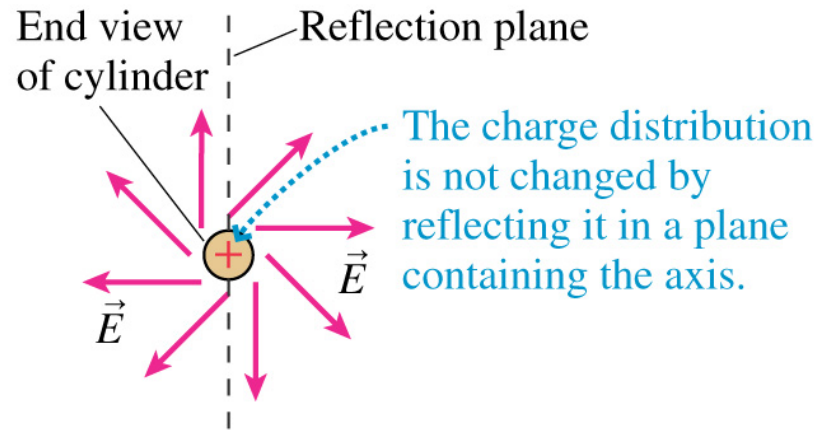
- (b) The charge distribution is not changed by the reflection, but the field is. This field doesn't match the symmetry of the cylinder, so the cylinder's field can't look like this.



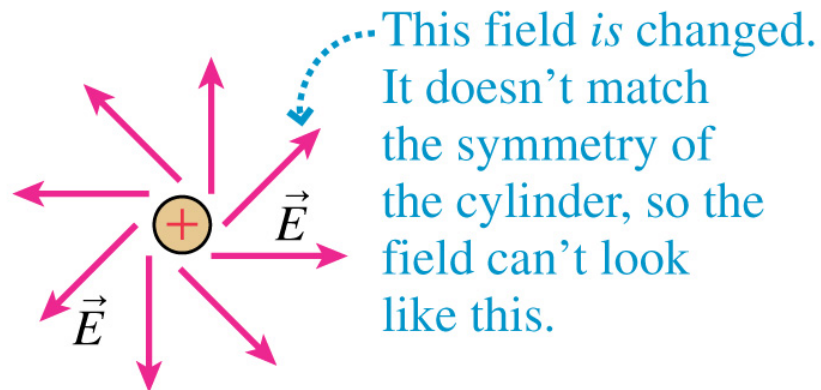
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Is this field distributions possible?

(a)

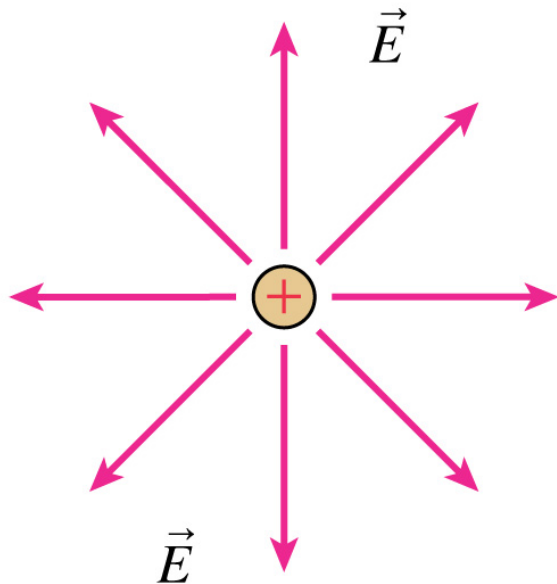


(b)



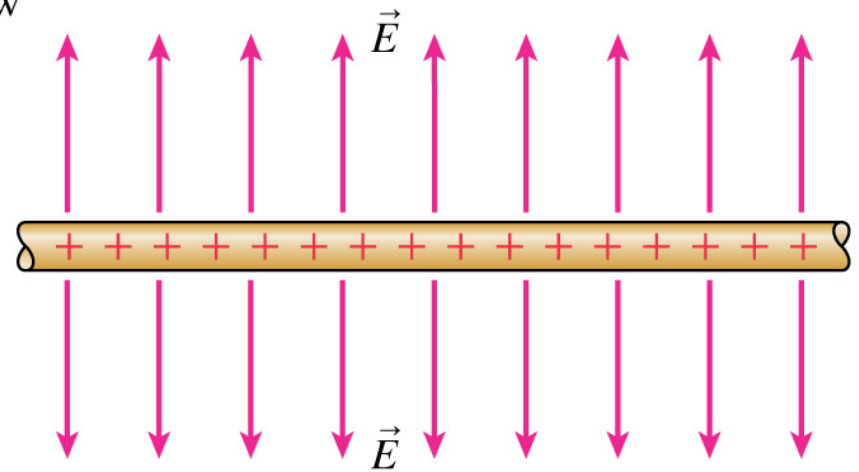
Is this field distributions possible?

End view



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Side view



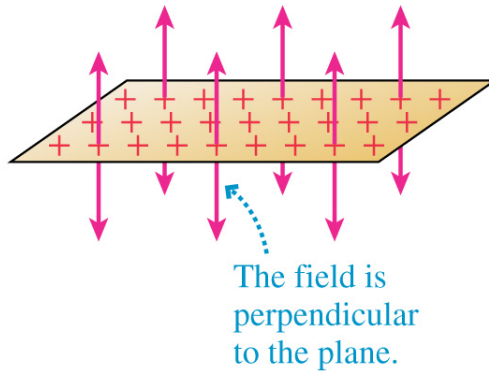
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This is the only shape for the electric field that matches the symmetry of the charge distribution

Three Fundamental Symmetries

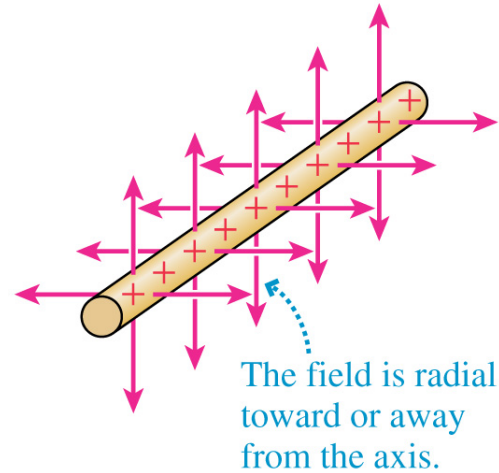
Basic symmetry

Planar symmetry



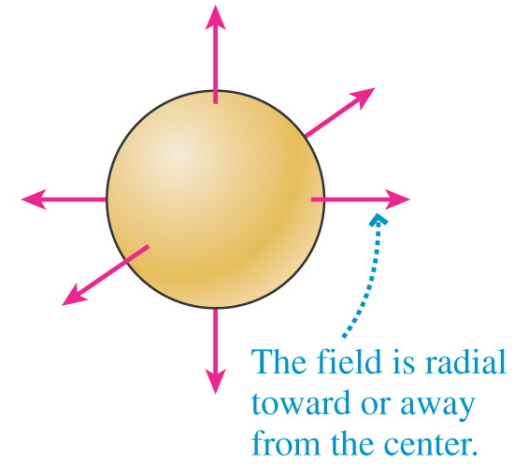
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Cylindrical symmetry



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Spherical symmetry



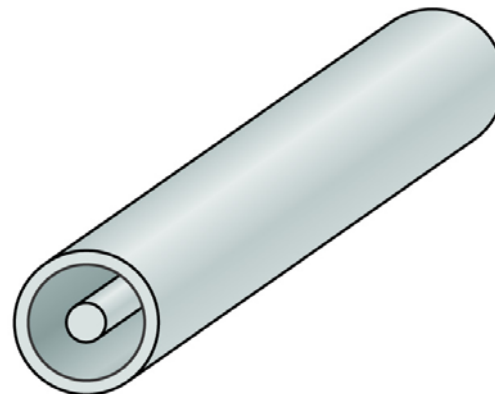
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More Complex example



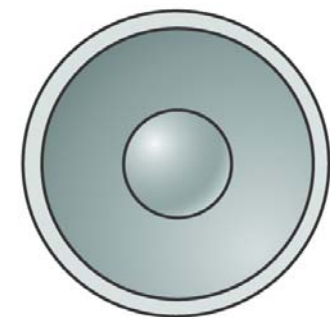
Infinite parallel-plate capacitor

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Coaxial cylinders

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Concentric spheres

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What symmetry can provide and what it can't

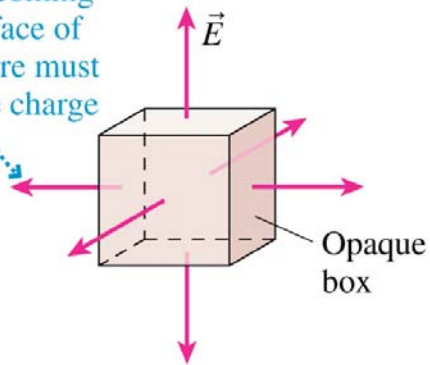
- Can provide **shape** of E
- Can't provide **strength** and **distance dependence** $E(r)$
- This is the place for Gauss's Law: knowing the shape to find $E(r)$

$$\Phi_e = \oint_{\text{surface}} \vec{E} d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

What is the flux Φ_e ?

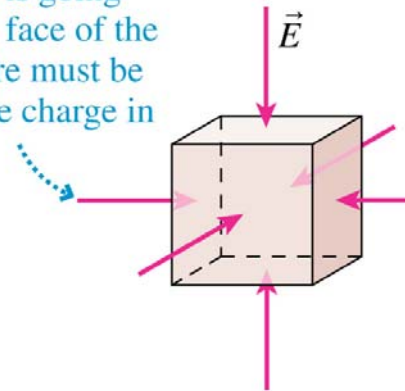
Can you figure out what is in the box?

- (a) The field is coming out of each face of the box. There must be a positive charge in the box.



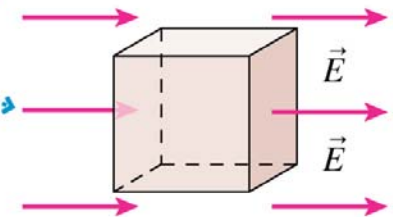
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- (b) The field is going into each face of the box. There must be a negative charge in the box.



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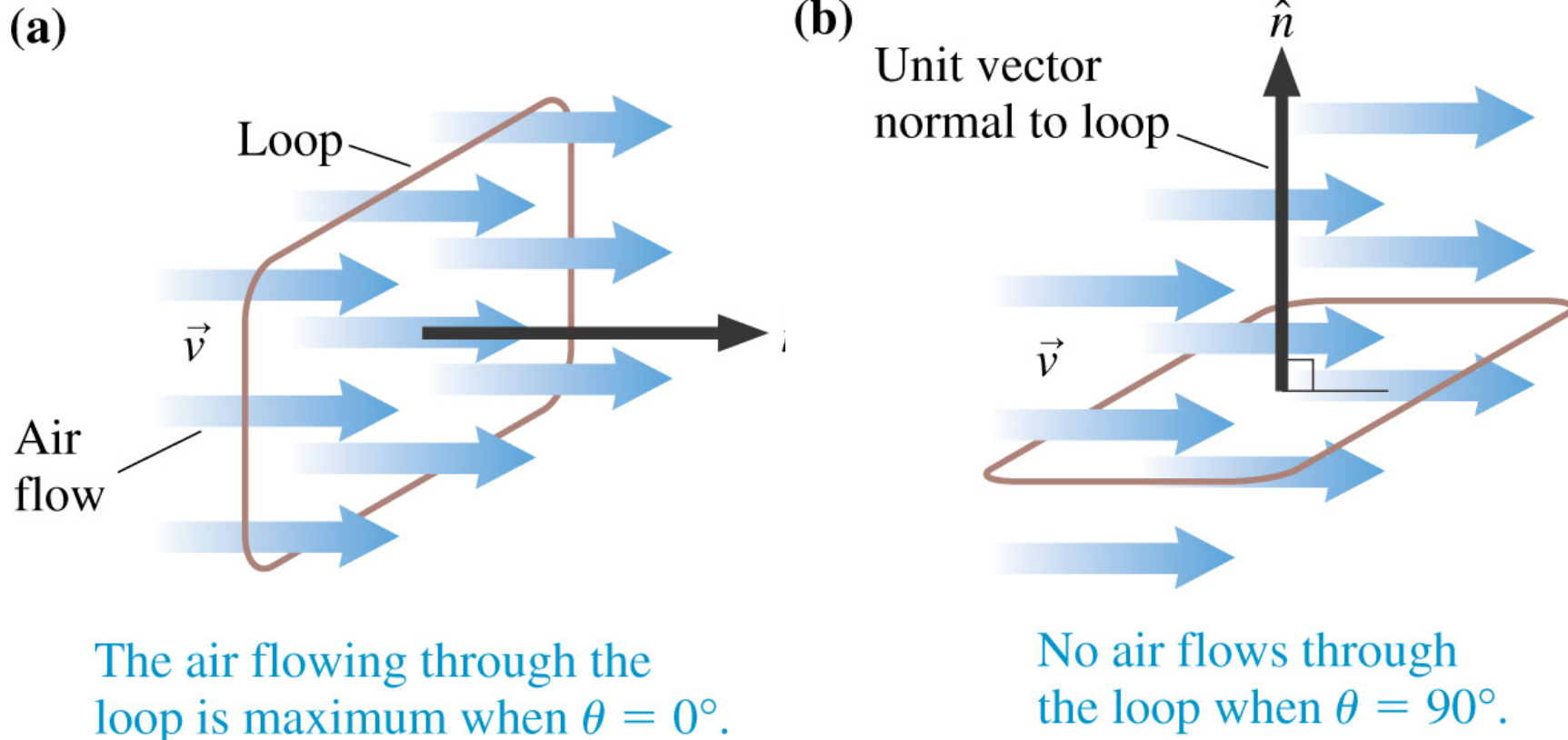
- (c) A field passing through the box implies there's no net charge in the box.



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- Is there any charge inside the box?
- Is it positive or negative?

The Basic Definition of Flux



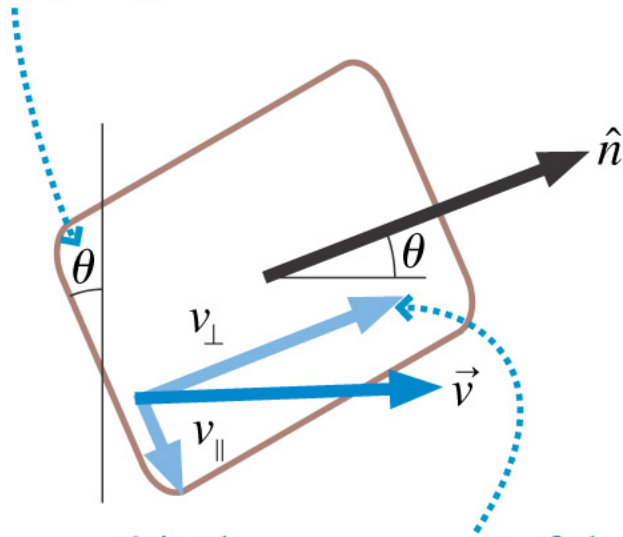
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Only $v_{\perp} = v \cos \theta$ carries air through the loop
Flux: volume of air flowing through the loop each second (m^3/s):
$$\Phi = v_{\perp} A = v A \cos \theta$$

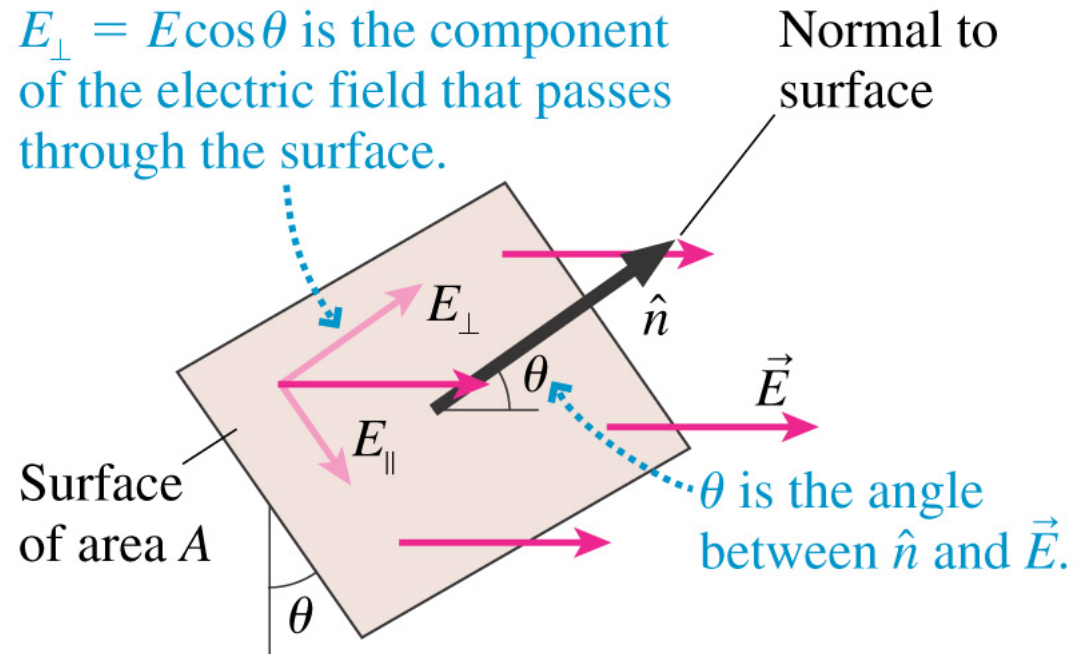
Electric Flux Φ_e

(c) The loop is tilted by angle θ .



$v_{\perp} = v \cos \theta$ is the component of the air velocity perpendicular to the loop.

$E_{\perp} = E \cos \theta$ is the component of the electric field that passes through the surface.



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Definition of the **electric flux**: $\Phi_e = E_{\perp} A = EA \cos \theta$

Let us define an **area vector**: $\vec{A} = A\hat{n}$

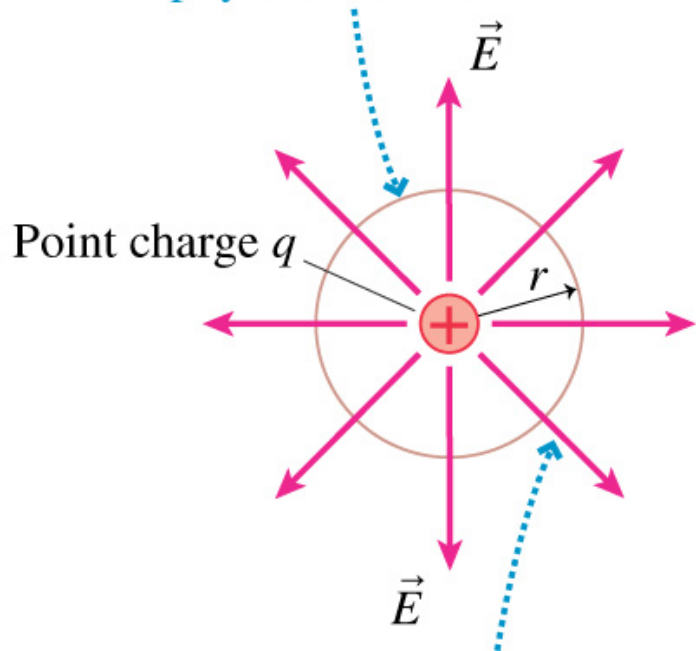
Then it can be represented as a **scalar (or dot) product**

$$\Phi_e = \vec{E} \cdot \vec{A}$$

Electric flux of constant electric field

Gauss's Law is equivalent to Coulomb's Law

Cross section of a Gaussian sphere of radius r . This is a mathematical surface, not a physical surface.



The electric field is everywhere perpendicular to the surface *and* has the same magnitude at every point.

Consider flux:

$$\Phi_e = \oint_{\text{surface}} \vec{E} d\vec{A} = EA_{\text{sphere}}$$

Use Coulomb's law:

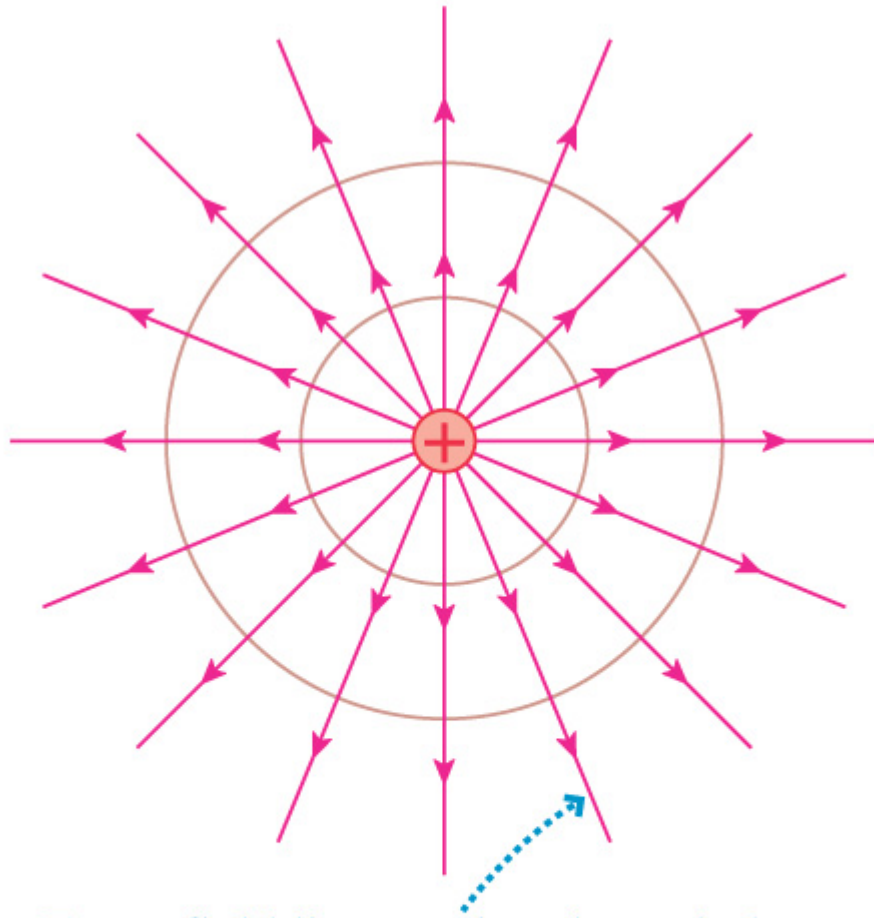
$$E(r) = \frac{q}{4\pi\epsilon_0 r^2}$$

Obtain Gauss's law:

$$\Phi_e = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

This however is not a proof of the Gauss's law

Flux is independent of the Surface Shape and Radius



Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

The flux through any closed surface surrounding a point charge q is

$$\Phi_e = \oint_{\text{surface}} \vec{E} d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

This is a consequence of the inverse-square force law

End of Lecture 5
Reading: Entire Chapter 27
Home Work 2 in Mastering Physics