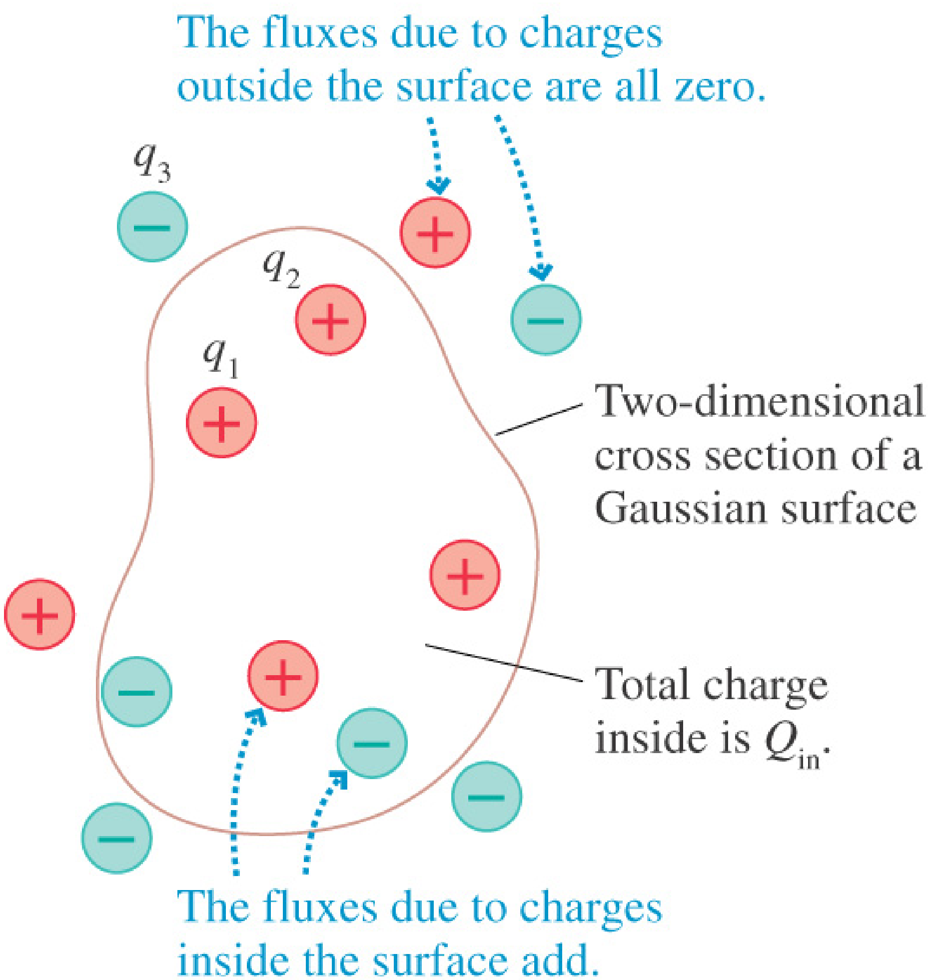
Lecture 5: Chapter 27, September 8 2005

One of the Maxwell's equations: Gauss's Law

$$\Phi_e = \oint_{surface} \vec{E} d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$



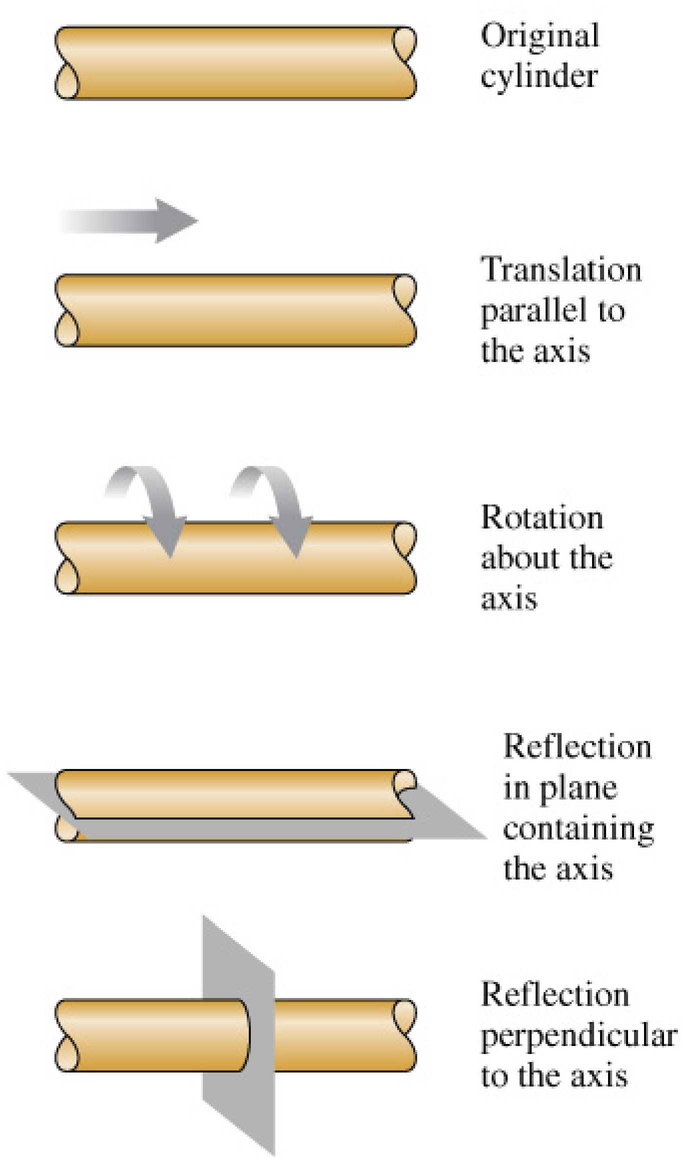
Powerful tool of finding ***E*** for highly symmetric charge distributions:

i) Spherical
ii) Cylindrical
iii) Plane.

Requires knowledge of new concepts:
Symmetry
Flux (Φ_e)
Gaussian Surface
Taking surface integral

Main trick: Move ***E*** through the integration due to symmetry

Symmetry



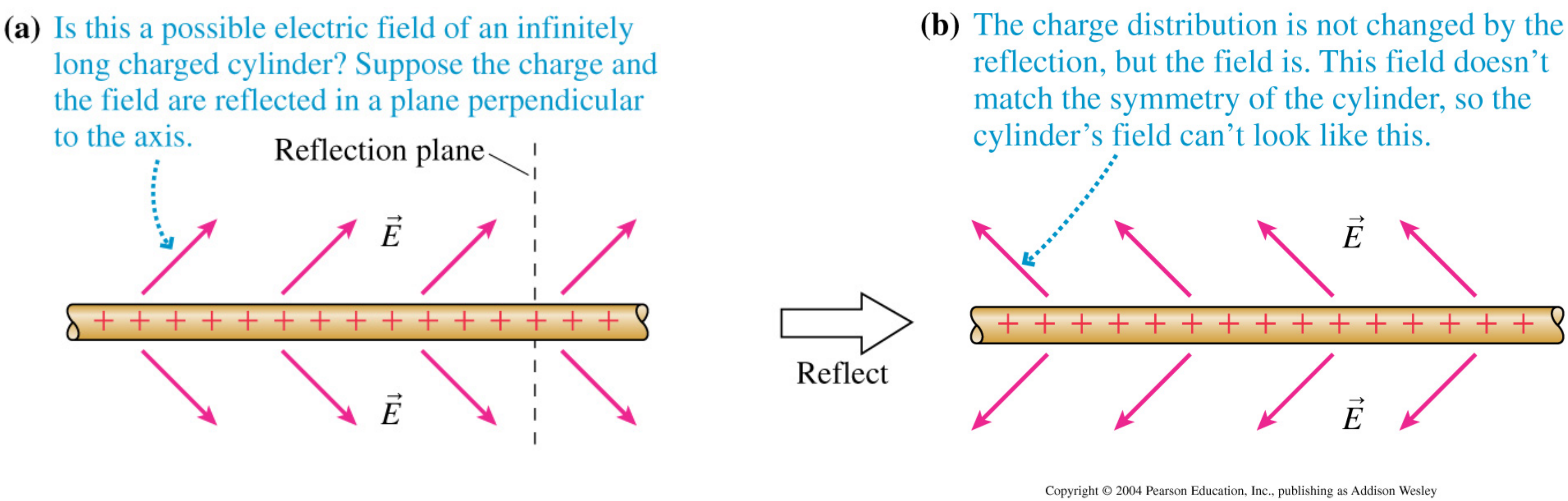
The charge distribution is **symmetric** if there is a group of *geometrical transformations* that don't cause any *physical change*:

- Translation
- Rotation about an axis (rotation axis)
- Reflection the charge in a mirror (symmetry plane)

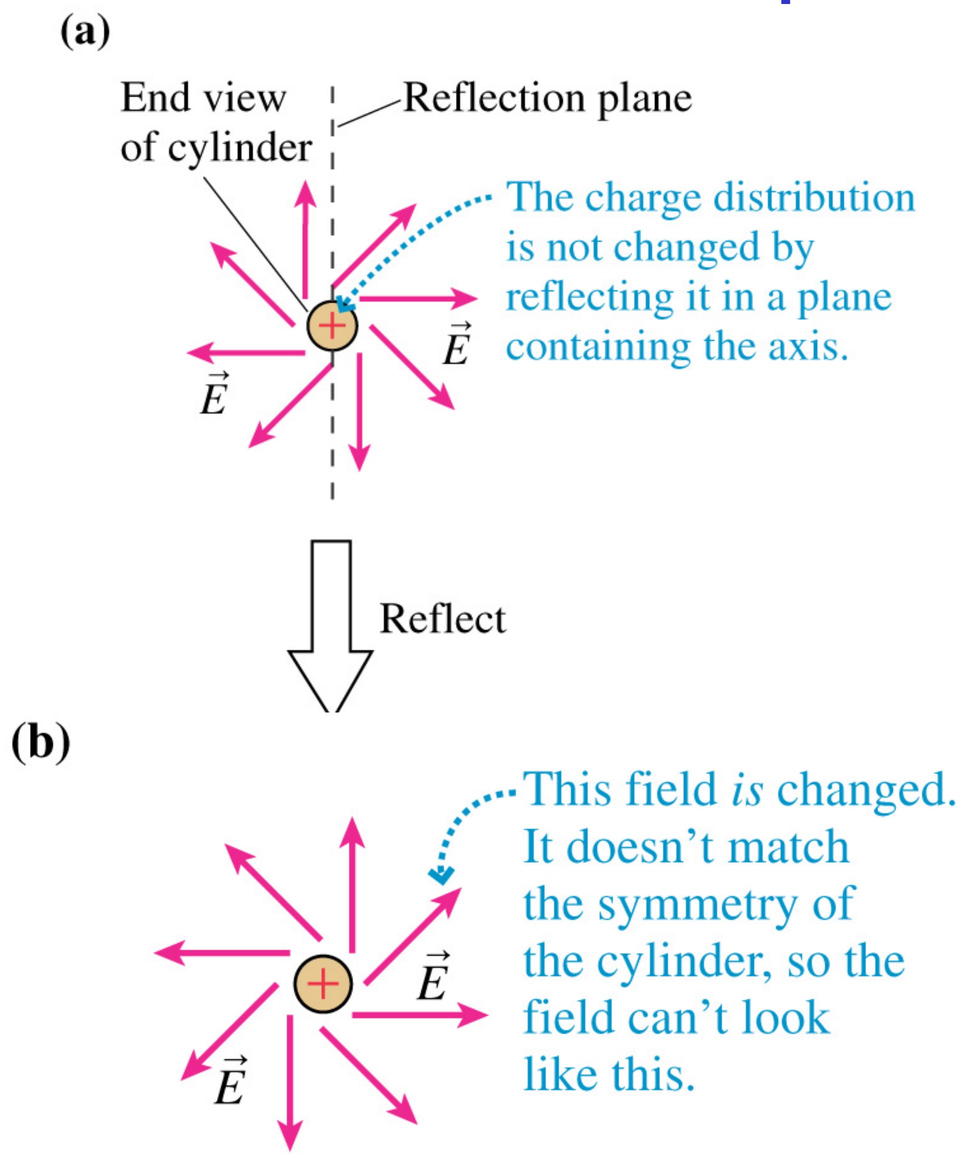
Why is this symmetry important?

Because the symmetry of electric field must match the symmetry of charge distribution

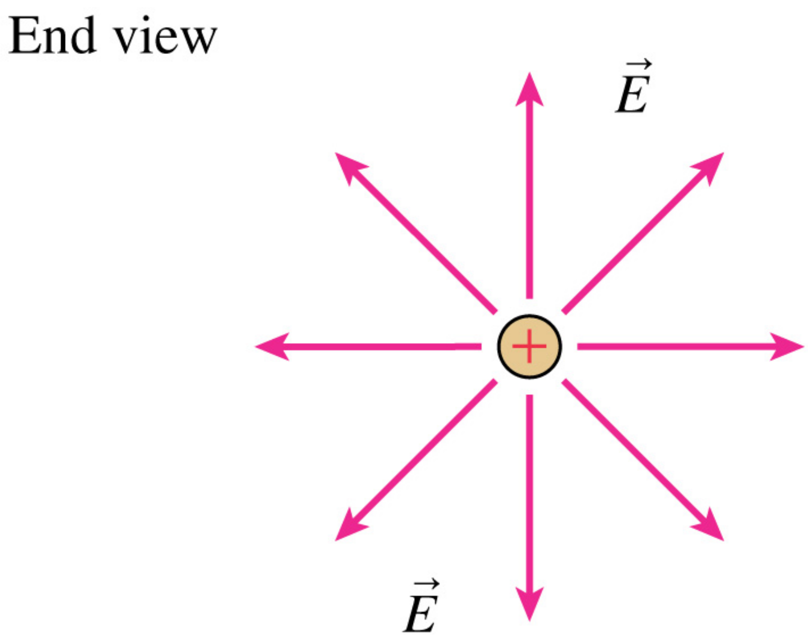
Is this field distributions possible?

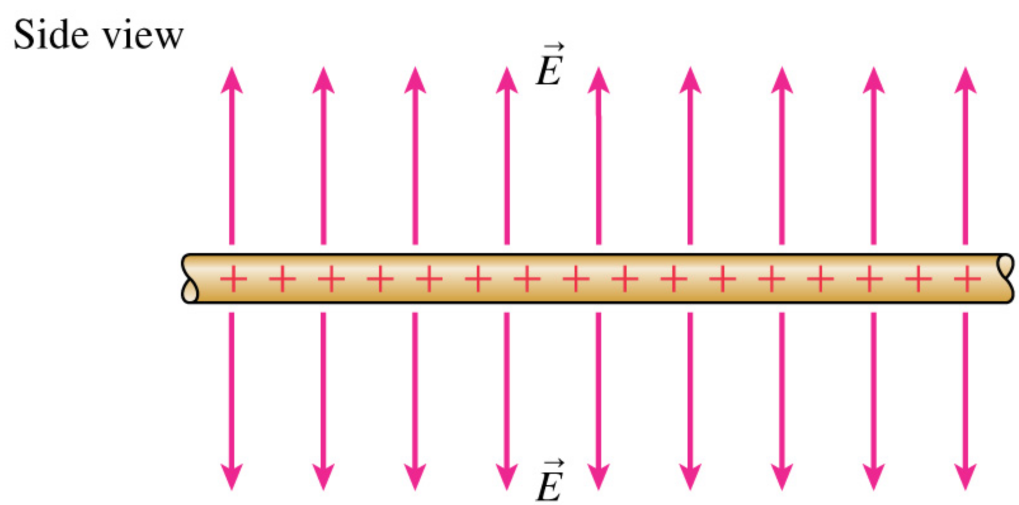


Is this field distributions possible?



Is this field distributions possible?



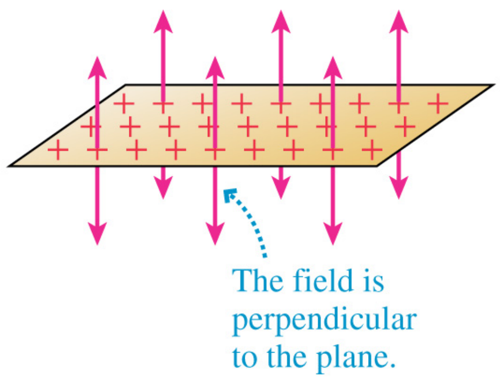


This is the only shape for the electric field that matches the symmetry of the charge distribution

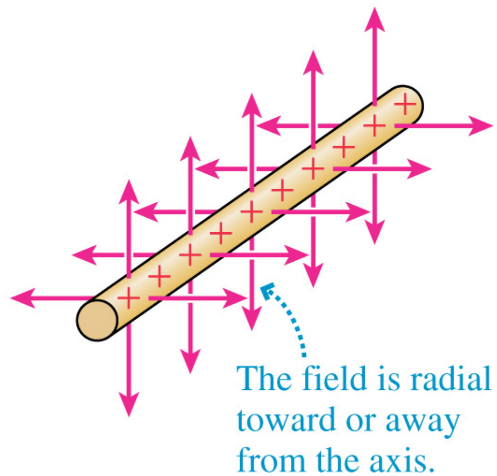
Three Fundamental Symmetries

Basic symmetry

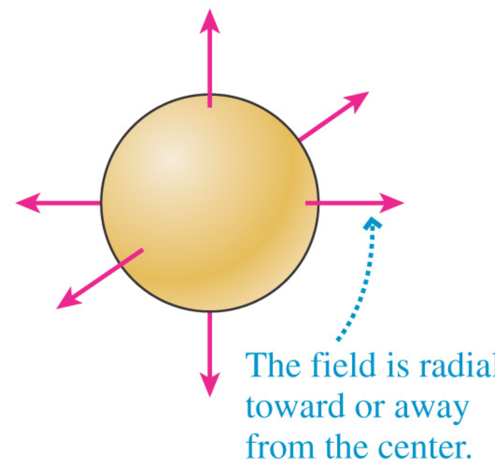
Planar symmetry



Cylindrical symmetry

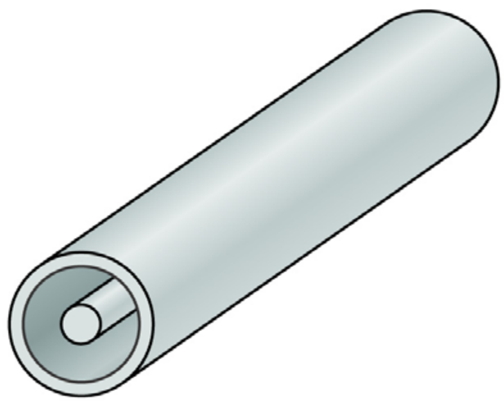


Spherical symmetry

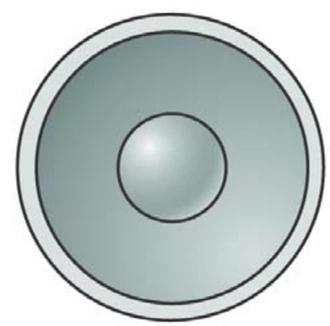


More Complex example

Infinite parallel-plate capacitor



Coaxial cylinders



Concentric spheres

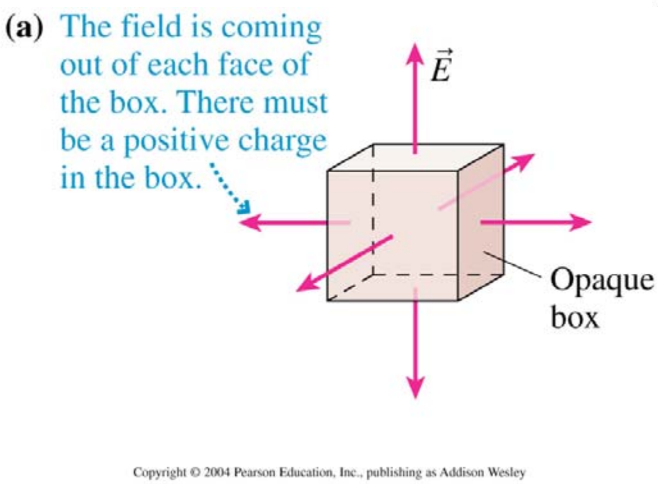
What symmetry can provide and what it can't

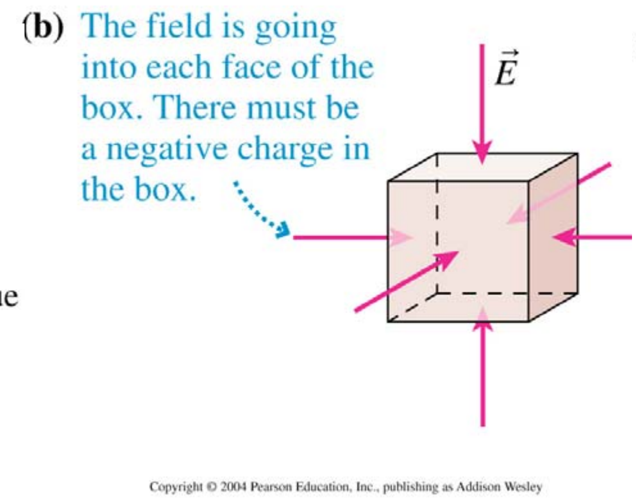
- Can provide ***shape*** of *E*
- Can't provide ***strength*** and ***distance dependence*** $E(r)$
- This is the place for Gauss's Law: knowing the shape to find $E(r)$

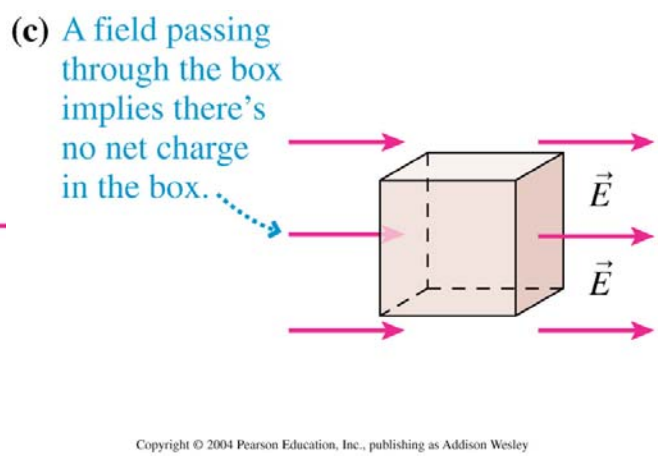
$$\Phi_e = \oint_{surface} \vec{E} d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

What is the flux Φ_e?

Can you figure out what is in the box?

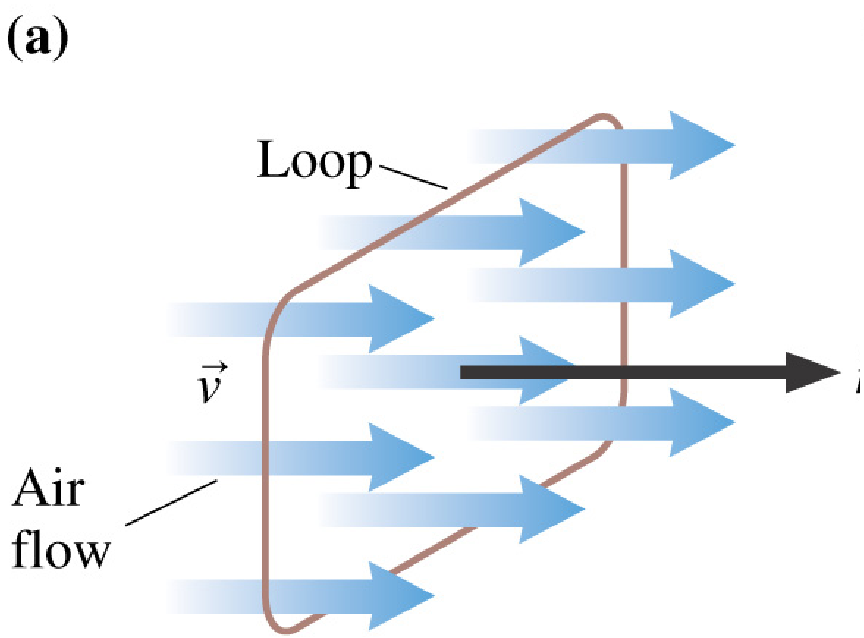




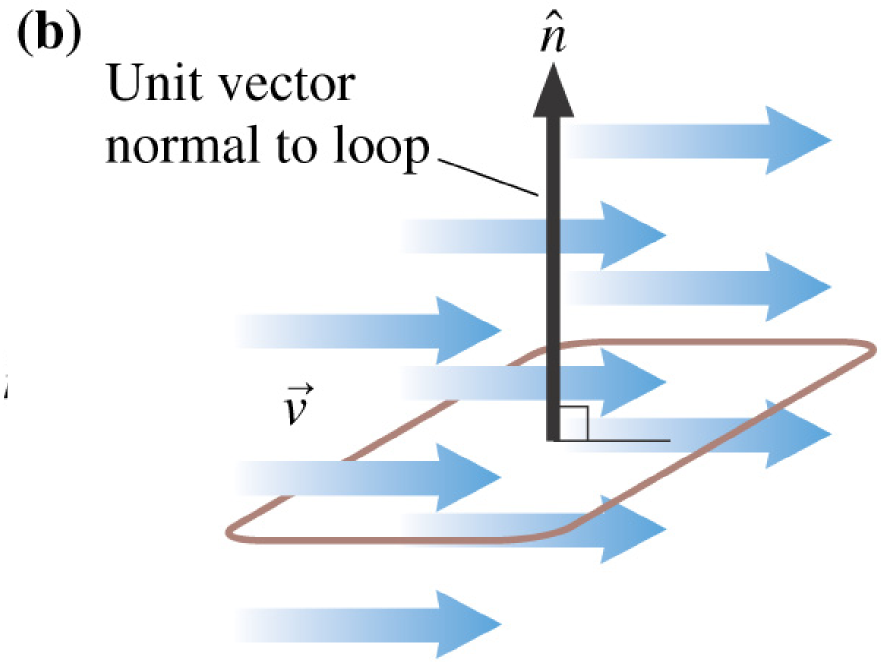


- Is there any charge inside the box?
- Is it positive or negative?

The Basic Definition of Flux



The air flowing through the loop is maximum when $\theta = 0°$.



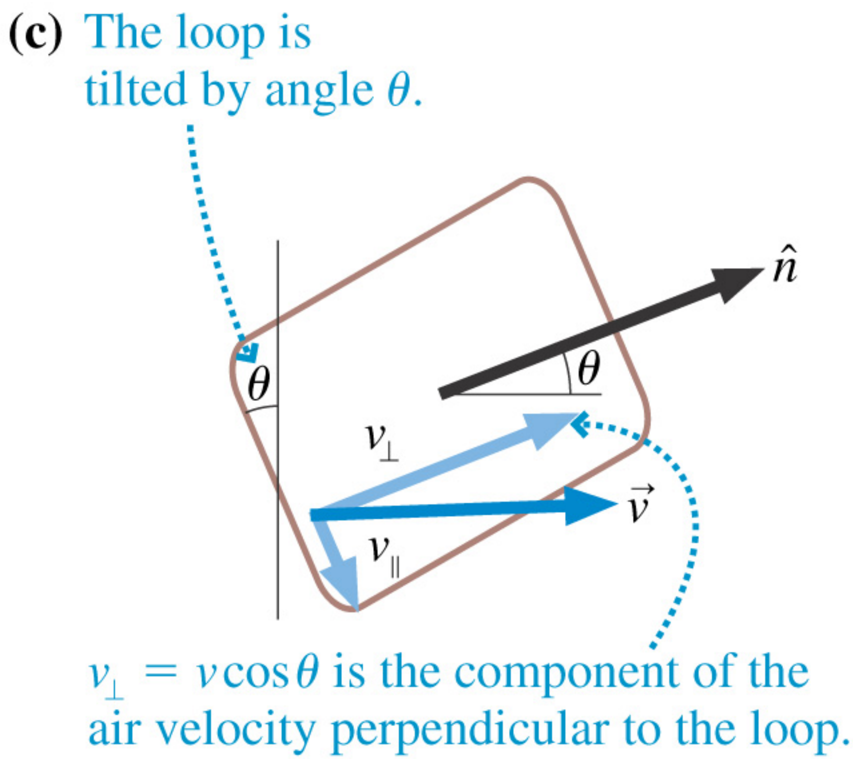
No air flows through the loop when $\theta = 90°$.

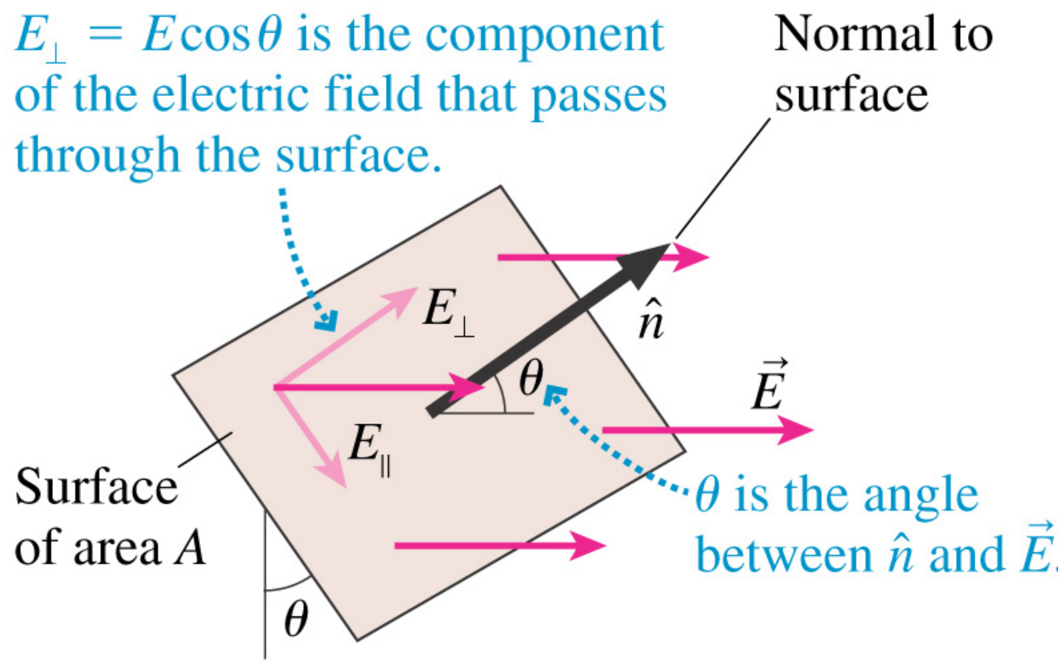
Only $v_{\perp} = v \cos\theta$ carries air through the loop

Flux: volume of air flowing through the loop each second (m^3/s):

$$\Phi = v_{\perp} A = v A \cos\theta$$

Electric Flux Φ_e





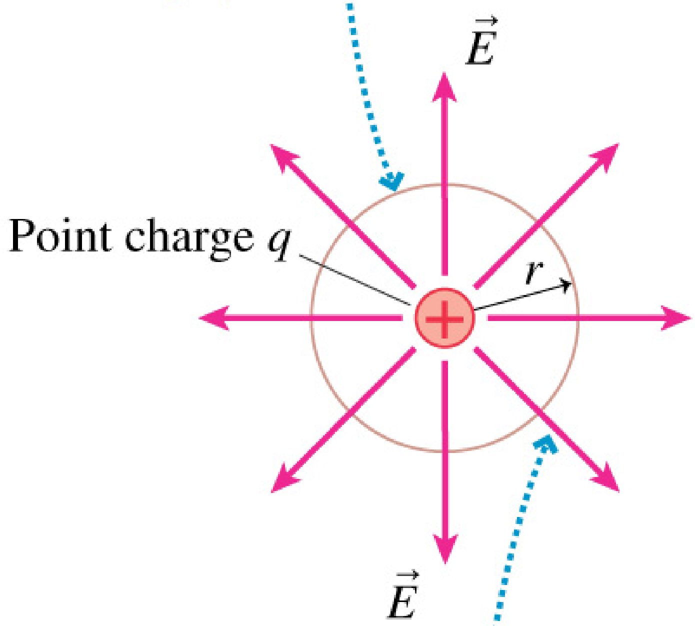
Definition of the ***electric flux***: $\Phi_e = E_\perp A = EA\cos\theta$
Let us define an ***area vector***: ***A*** = A***n***
Then it can be represented as a ***scalar (or dot) product***

$$\Phi_e = \vec{E}\cdot\vec{A}$$

Electric flux of constant electric field

Gauss's Law is equivalent to Coulomb's Law

Cross section of a Gaussian sphere of radius r. This is a mathematical surface, not a physical surface.



The electric field is everywhere perpendicular to the surface *and* has the same magnitude at every point.

Consider flux:

$$\Phi_e = \oint_{surface} \vec{E} d\vec{A} = EA_{sphere}$$

Use Coulomb's law:

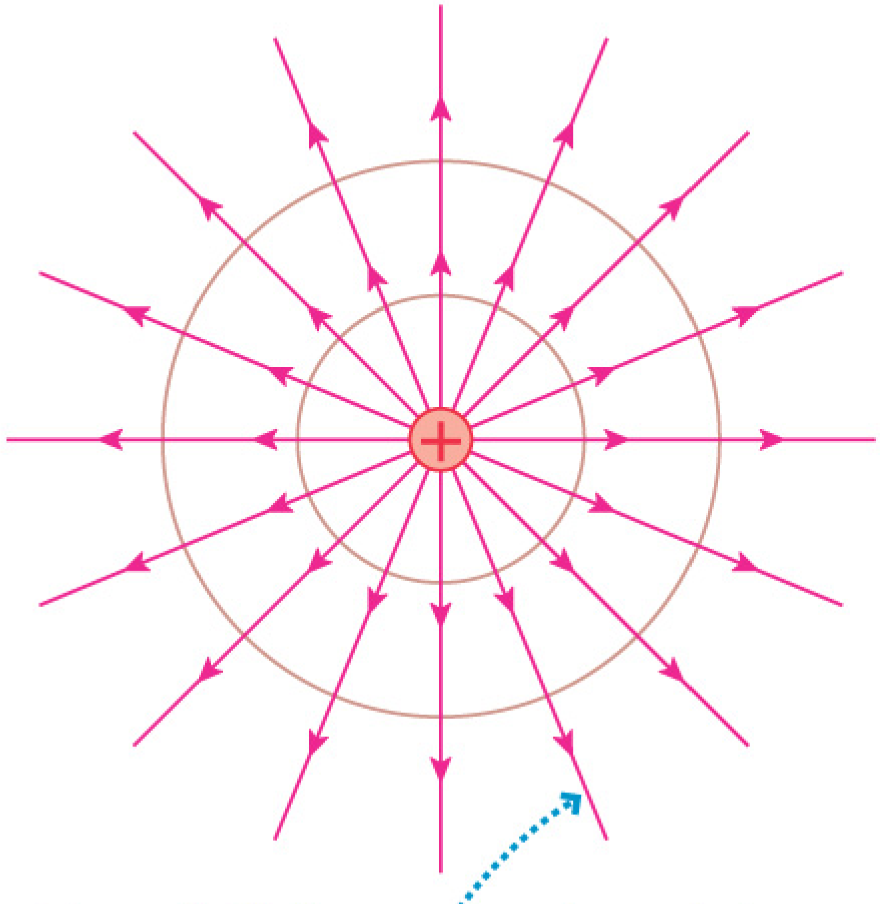
$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}$$

Obtain Gauss's law:

$$\Phi_e = \frac{q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

This however is not a proof of the Gauss's law

Flux is independent of the Surface Shape and Radius



Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

The flux through any closed surface surrounding a point charge q is

$$\Phi_e = \oint_{surface} \vec{E} d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

This is a consequence of the inverse-square force law

End of Lecture 5
Reading: Entire Chapter 27
Home Work 2 in Mastering Physics