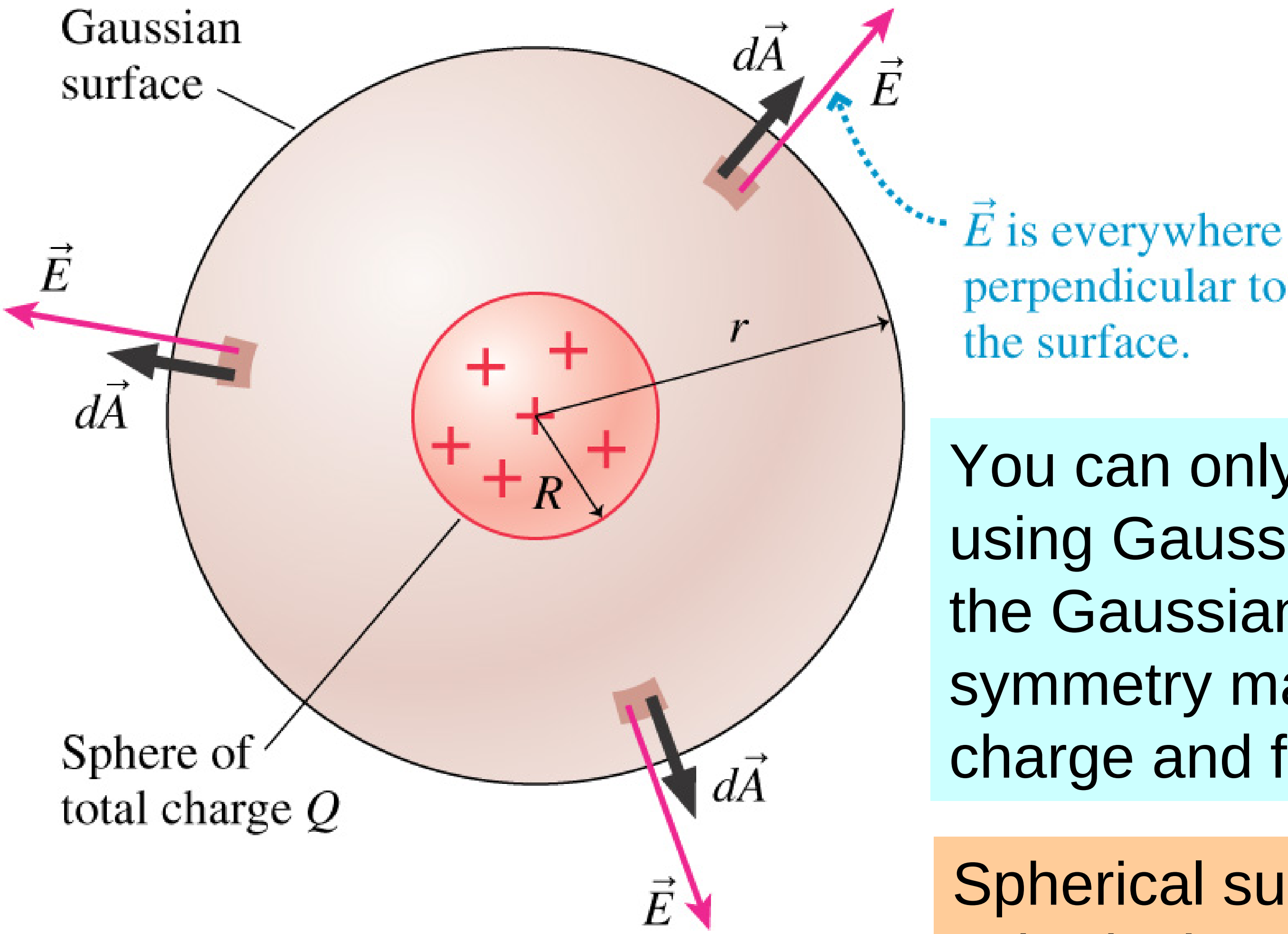
Lecture 6: Chapter 27, September 8 2005

The electric field outside a sphere of charge



You can only benefit from using Gauss's law if you select the Gaussian surface with the symmetry matching that for the charge and field distribution.

Spherical surface matches the spherical symmetry of this problem

Gauss's Law:

$$\Phi_e = \frac{q_{enc}}{\varepsilon_0}, \qquad \Phi_e = \oint \bar{E} \cdot d\bar{A}$$

$$\bar{E} \cdot d\bar{A} = \underbrace{E}_{\text{Magnitude}} \cdot \underbrace{dA}_{\text{Magnitude}} \cdot \underbrace{\cos\theta}_{\boxed{1 \text{ since } \theta = 0}} = E \cdot dA$$

$$\Phi_e = \oint E \cdot dA \underbrace{=}_{E(r) - \text{Const}} E(r) \cdot \oint dA = E(r) \cdot A =$$

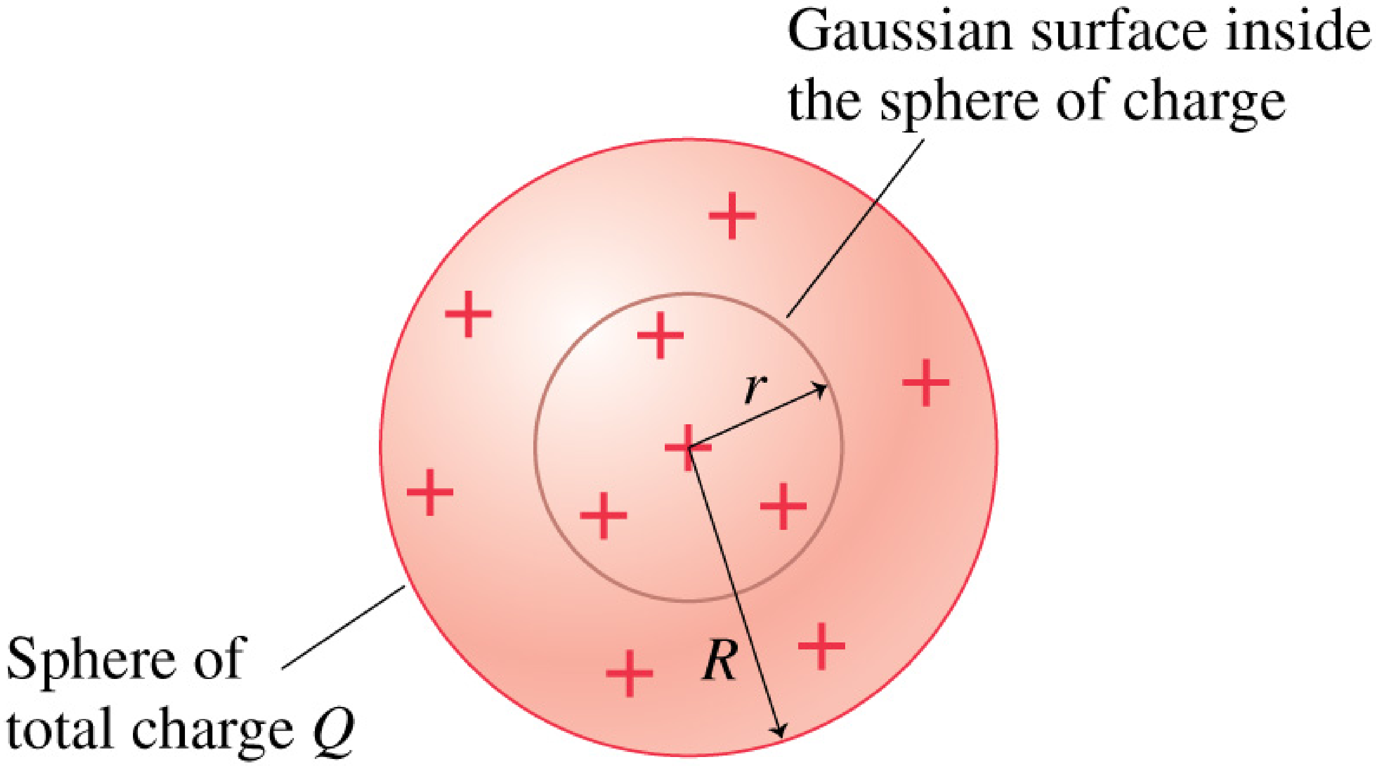
$A = 4\pi r^2$ — Surface of a sphere

$$= 4\pi r^2 \cdot E(r).$$

$$4\pi r^2 \cdot E(r) = \frac{Q}{\varepsilon_0} \Rightarrow E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}.$$

- Same field as it would be if the charge Q were a *point* one

The electric field inside a sphere of charge



Gauss's Law:

$$\Phi_e = \frac{q_{enc}}{\varepsilon_0}, \qquad \Phi_e = \oint \bar{E}(r) \cdot d\bar{A}$$

Simplifying Property 1:

$$\bar{E}(r) \parallel d\bar{A} \Rightarrow \boxed{\theta = 0} \Rightarrow \cos\theta = 1 \Rightarrow$$

$$\bar{E}(r) \cdot d\bar{A} = E(r) \cdot dA \cdot \cos\theta = E(r) \cdot dA$$

$$\Phi_e = \oint E(r) \cdot dA = E(r) \oint dA = E(r) \cdot 4\pi r^2$$

Simplifying Property 2:

$$\boxed{E(r) = \text{Const} - \text{Magnitude}}$$

Difference with the previous problem is about q_{enc}.

$$V(R) \longrightarrow Q$$

$$V_{Gaussian}(r) \longrightarrow q_{enc}$$

$$V(R) = \frac{4}{3}\pi R^3$$

$$V_{Gaussian}(r) = \frac{4}{3}\pi r^3$$

$$\boxed{Q \sim V}$$

$$\frac{V(R)}{V_{Gaussian}(r)} = \frac{Q}{q_{enc}} = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{R^3}{r^3}.$$

$$q_{enc} = Q\frac{r^3}{R^3}$$

r – radius of the Gaussian sphere

R – radius of the sphere

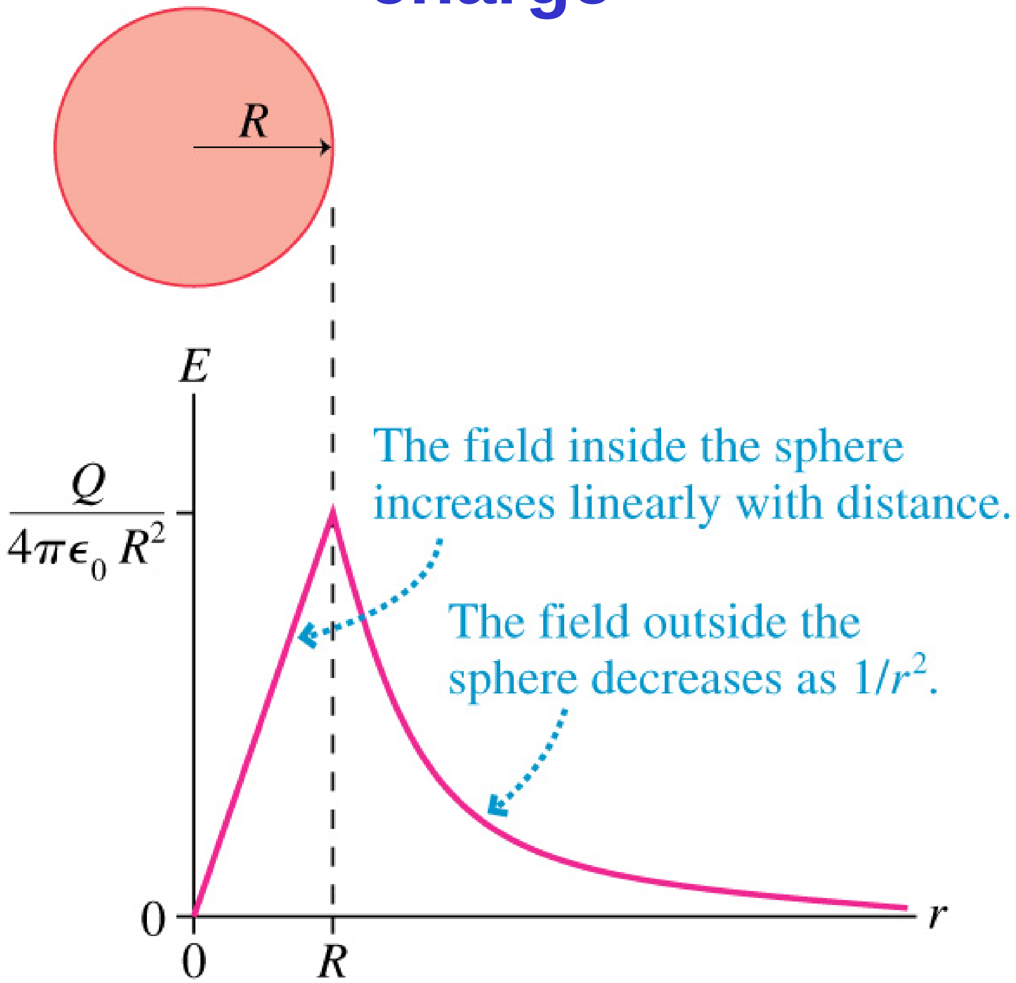
$$\Phi_e = \frac{q_{enc}}{\varepsilon_0}.$$

$$E(r)\cdot 4\pi \cancel{r^2} = \frac{Q}{\varepsilon_0}\frac{r^{\cancel{3}}}{R^3}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}$$

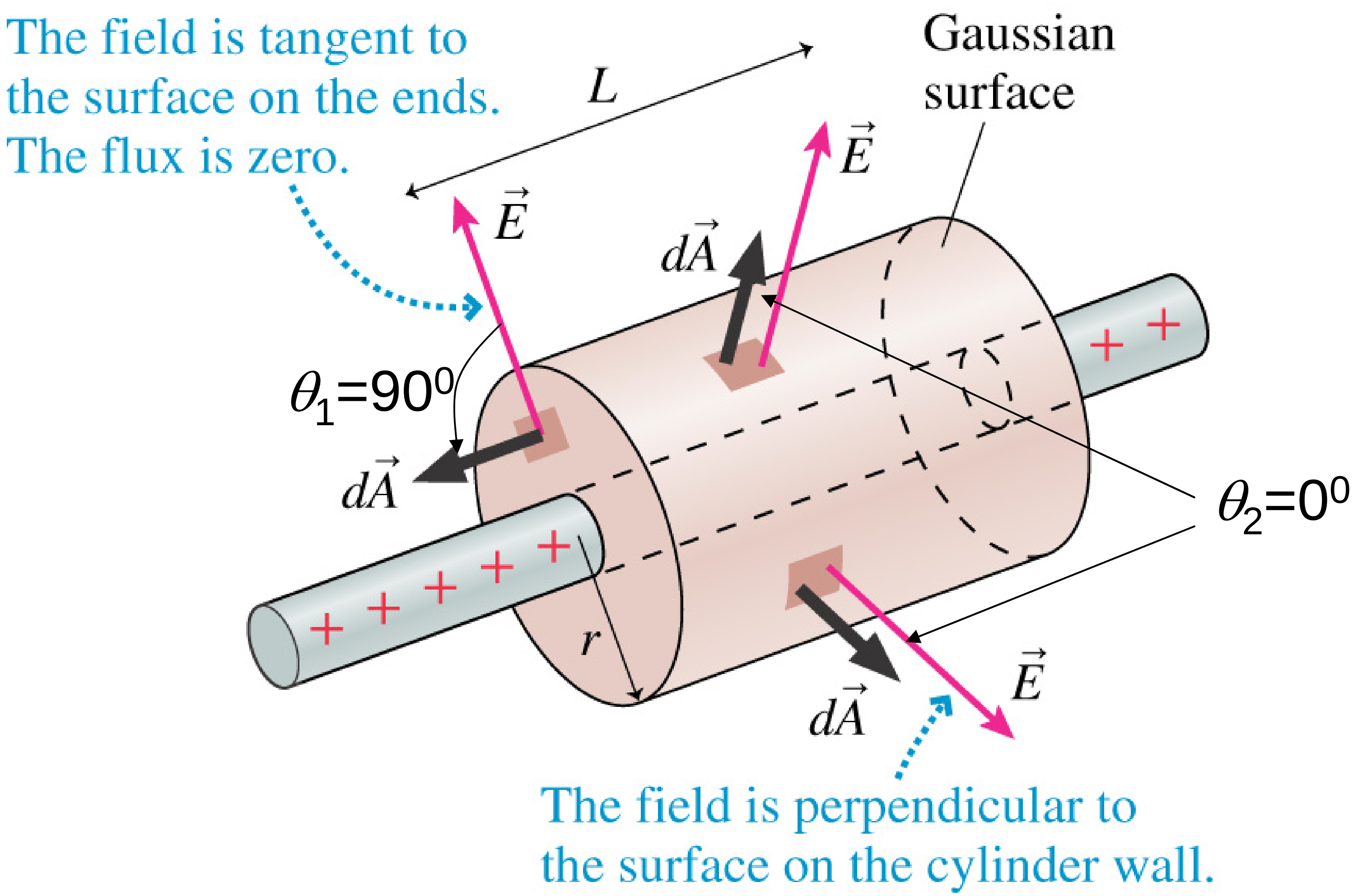
Inside the charged sphere $E(r) \sim r$

Solution for the field *inside* and *outside* a sphere of charge



• This field distribution (E(r)) inside the sphere is very different from that for the field outside the sphere. Checkpoint: Can you explain this using Gauss's law?

Now switch to the cylindrical geometry: The electric field of a long, charged wire



Gauss's Law:

$$\Phi_e = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi_e = \Phi_{bases} + \Phi_{sidewall}$$

$$\Phi_{bases} = \bar{E}(r)\cdot d\bar{A} = E(r)\cdot dA\cdot \cos\theta = $$

$$E(r)\cdot dA\cdot \cos 90^\circ = 0$$

Since $\theta = 0$ all along the sidewall surface

$$\Phi_e = \oint \bar{E}\cdot d\bar{A} = \oint E\cdot dA\cdot \underbrace{\cos\theta}_{=1} =$$

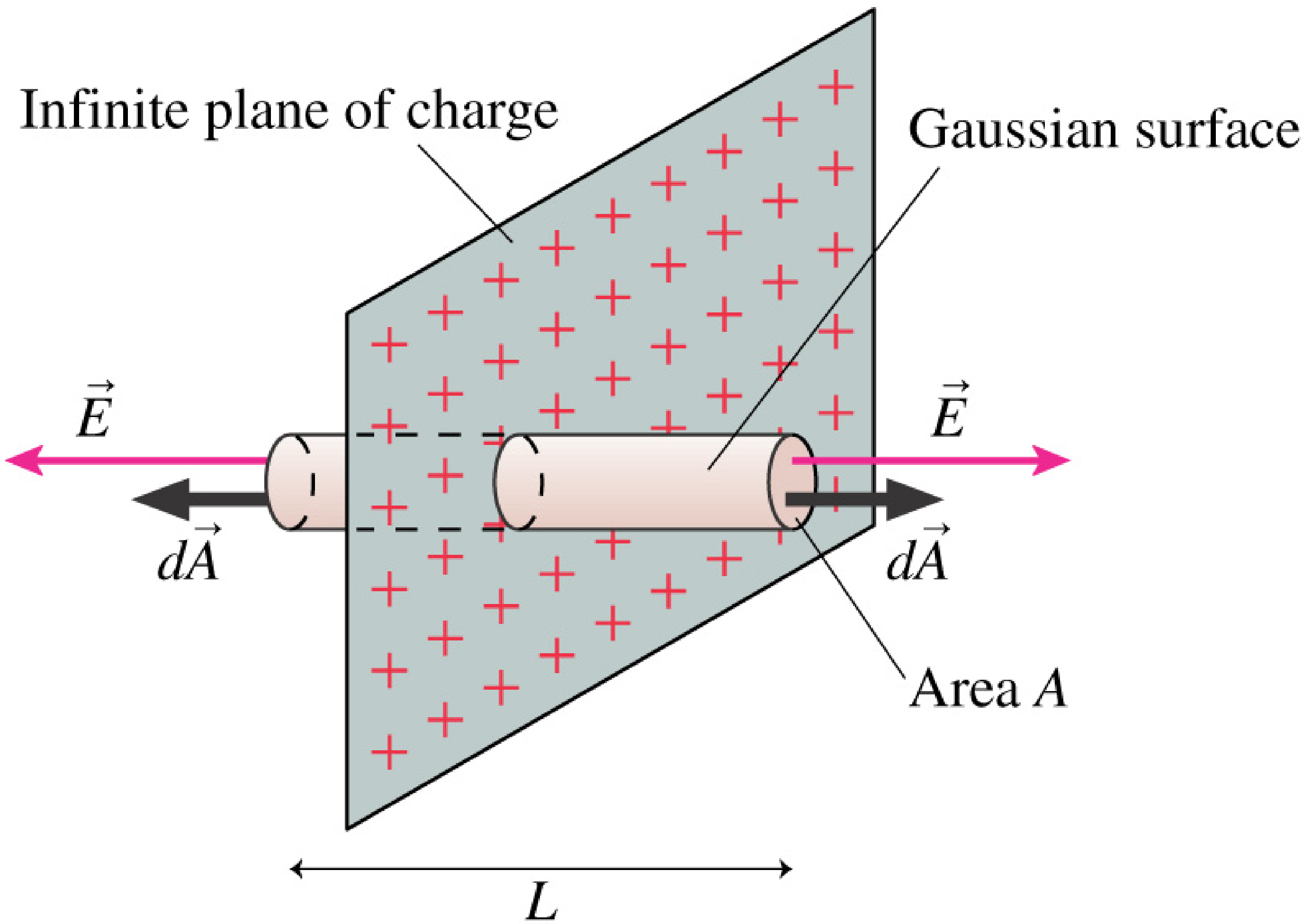
$$= \oint E\cdot dA = E(r)\oint dA = E(r)\cdot 2\pi r\cdot L$$

$$\Phi_e = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E(r)\cdot 2\pi r\cdot \cancel{L} = \frac{\lambda\cdot \cancel{L}}{\varepsilon_0}$$

$$E(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

This formula has been previously obtained (Chapter 26) in a more complex way.

One more geometry:
The electric field of a plane of charge



Gauss's Law:

$$\Phi_e = \Phi_{sidewall} + \Phi_{faces}$$

$$\Phi_{sidewall} = \int_{sidewall} \vec{E} \cdot d\vec{A} = \int E \cdot dA \cdot \underset{0}{\underset{\parallel}{\cos\theta}} = 0$$

since $\theta = 90°$

$$\Phi_{faces} = 2E \cdot A$$

$$\Phi_e = \frac{q_{enc}}{\varepsilon_0}, \quad q_{enc} = \eta A$$

$$2E\cancel{A} = \frac{\eta \cancel{A}}{\varepsilon_0} \Rightarrow E = \frac{\eta}{2\varepsilon_0}$$

Electric field created by the infinite plane does not depend on *r*.

End of Lecture 6
Reading: Entire Chapter 27
Review for Quiz 3
Home Work 3 in Mastering Physics