Lecture 8: Chapter 27, September 20 2005

The Coulomb law has shape analogous to the gravitational law ⇒ we can introduce electric potential energy:

Note Title 9/20/2005

$$\Delta U = -W(i \to f) = -\int_{s_i}^{s_f} \bar{F}_s \cdot d\bar{s}$$

$$U_f - U_i = -\int_{s_i}^{s_f} \bar{F}_s \cdot d\bar{s}$$

Let us request $\underline{U_f = 0}$ for $s_f \to \infty$

$$U_i = \int_{s_i}^{s_f} \bar{F}_s \cdot d\bar{s} \equiv U$$

U – can be considered as an alternative way of describing el. field (instead of $\bar{E}$)

$\cdot\ U(x, y, z)$

$\oplus$

Q

The main advantage of $U(x,y,z)$ compared to $\bar{E}(x,y,z)$:

$U(x,y,z)$ — Scalar $\qquad$ $\bar{E}(x,y,z)$ — Vector

The disadvantage of $U(x,y,z)$ is the fact that it depends on both

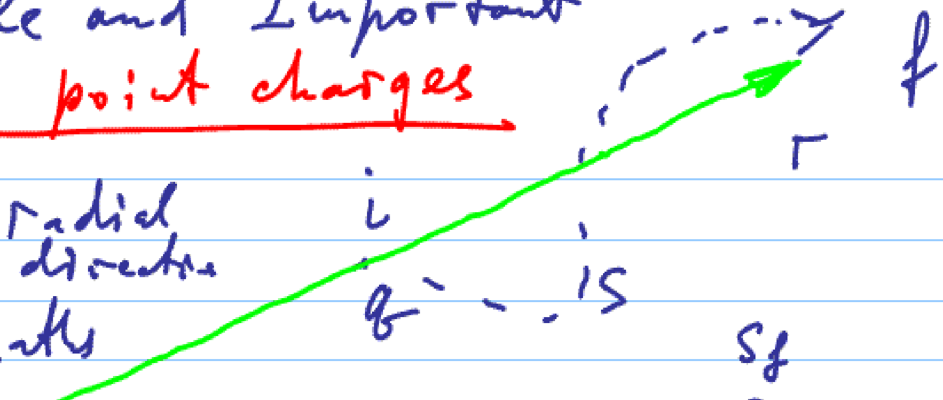
Q — source charge

q — probe charge

Most simple and Important Case: Two point charges

r – straight, radial direction

s - random paths



$$U_f - U_i = -\int_{s_i}^{s_f} \bar{F}_s \cdot d\bar{s} =$$

• Since W does not depend on the path why don't we move the charge q along straight line (r – direction)

$$= -\int_{r_i}^{r_f} F \cdot dr = -\int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot q}{r^2} \cdot dr =$$

$$= -\frac{Q \cdot q}{4\pi\varepsilon_0} \int_{r_i}^{r_f} \frac{dr}{r^2} = \frac{Q \cdot q}{4\pi\varepsilon_0} \left. \frac{1}{r} \right|_{r_i}^{r_f} =$$

$$= \frac{Q \cdot q}{4\pi\varepsilon_0} \left(\frac{1}{r_f} - \frac{1}{r_i} \right) = U_f - U_i$$

$$U_f = \frac{Q \cdot q}{4\pi\varepsilon_0} \frac{1}{r_f} \xrightarrow[r_f \to \infty]{} 0 \quad \text{— good!}$$

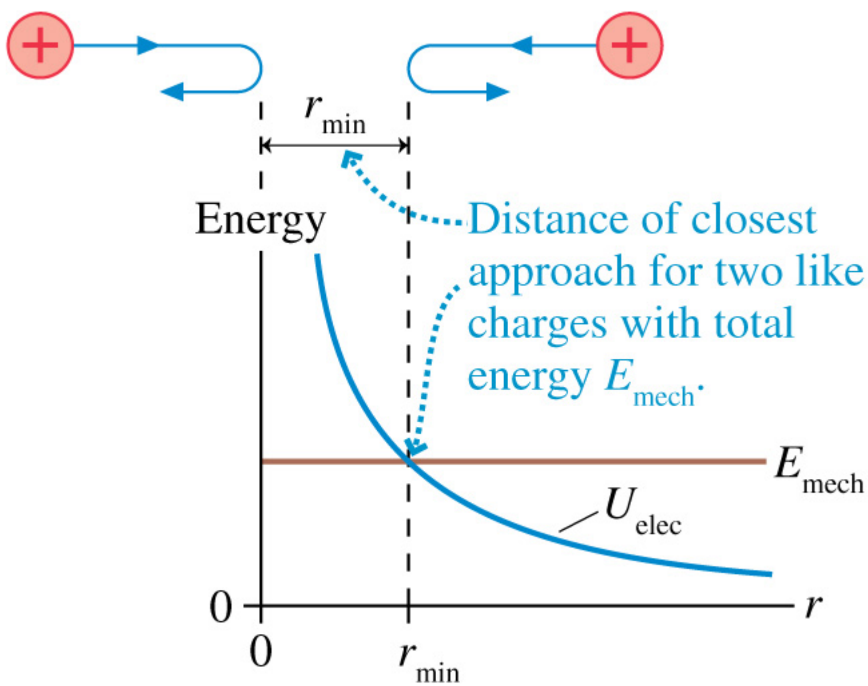
$$U_i = \frac{Q \cdot q}{4\pi\varepsilon_0} \frac{1}{r_i}$$

$$U = \frac{Q \cdot q}{4\pi\varepsilon_0} \frac{1}{r} \quad \text{— Potential energy}$$

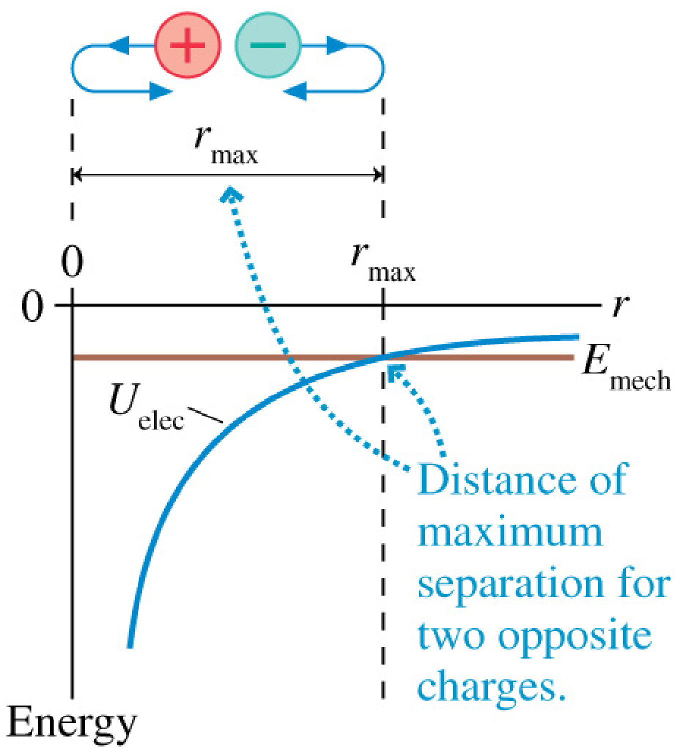
1. If the sign of charges Q and q is the same: $U(r) > 0$

2. If the sign of charges Q and q is opposite: $U(r) < 0$

(a) Like charges



(b) Opposite charges



What if we divide $U(r)$ by q
as we previously did for $\bar{E}(x,y,z) = \frac{\bar{F}(x,y,z)}{q}$

$$\frac{U_f}{q} - \frac{U_i}{q} = -\frac{1}{q}\int_{s_i}^{s_f} \bar{F}_f \cdot d\bar{s}$$

Define new quantity, electric potential,

$$\begin{cases} V_f = \dfrac{U_f}{q} \\ V_i = \dfrac{U_i}{q} \end{cases} \Rightarrow \boxed{V = \frac{U}{q}}$$

$$\boxed{V_f - V_i = -\int_{s_i}^{s_f} \frac{\bar{F}_f}{q} \cdot d\bar{s} = -\int_{s_i}^{s_f} \bar{E} \cdot d\bar{s}}$$

Let us apply this definition for a point charge

$$U = \frac{1}{4\pi \varepsilon_0} \frac{Q \cdot q}{r}, \qquad V = \frac{U}{q} \Rightarrow$$

$$V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r}$$

Since it has been obtained for <u>point</u> charges let us use dV instead of V and dq insted of Q:

$$dV = \frac{1}{4\pi \varepsilon_0} \frac{dq}{r}.$$

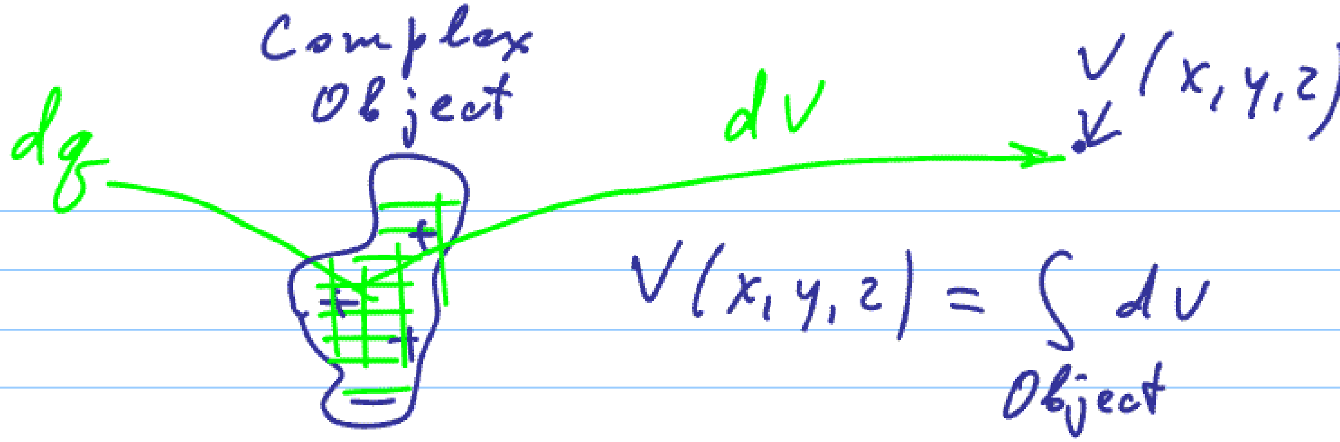
What is the use of $V(x, y, z)$?

- Provides a description of electric field in energy-related terms

$$U = V(x, y, z) \cdot q$$

- It is much easier to work with complex charge distributions since $V(x, y, z)$ is a scalar not a vector as $\overline{E}(x, y, z)$

- Superposition principle applies to electric potential

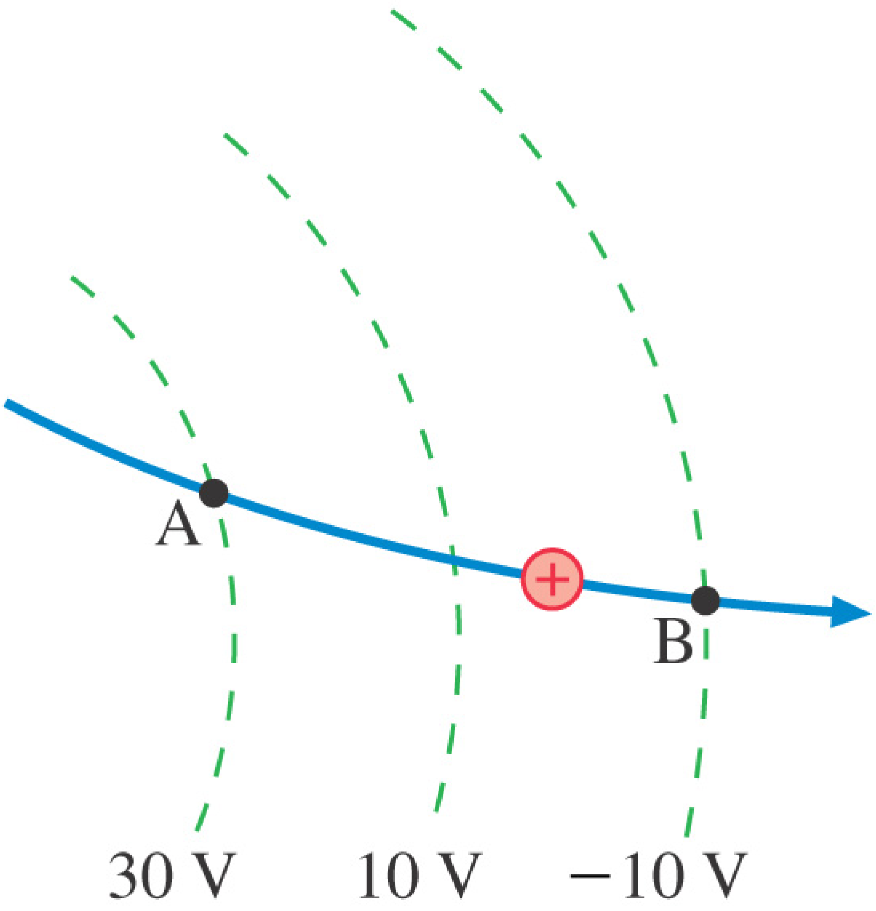


$$V(x,y,z) = \int_{\text{Object}} dV$$

- We can always reconstruct el. field $\vec{E}$ if we know $V(x,y,z)$:

$$\begin{cases} E_x = -\dfrac{\partial V}{\partial x} \\ E_y = -\dfrac{\partial V}{\partial y} \\ E_z = -\dfrac{\partial V}{\partial z} \end{cases}$$

A couple of problems…



Problem 44

A proton’s speed as it passes point A is 50,000 m/s. It follows the trajectory shown in figure. What is the proton’s speed at point B?

Mechanical energy is conserved if the forces are conservative

$$E_A = E_B, \quad \begin{cases} E_A = K_A + U_A \\ E_B = K_B + U_B \end{cases}$$

$$K_A + U_A = K_B + U_B$$

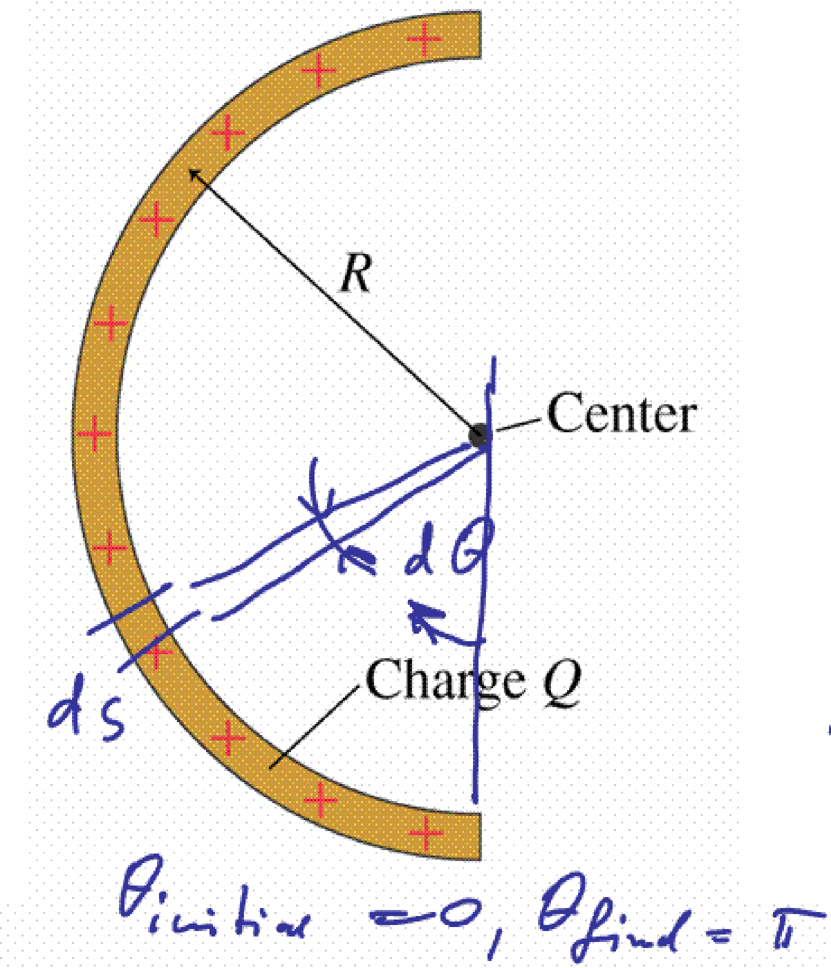
$$\frac{m v_A^2}{2} + U_A = \frac{m v_B^2}{2} + U_B \qquad \begin{cases} K_A = \dfrac{m v_A^2}{2} \\ K_B = \dfrac{m v_B^2}{2} \end{cases}$$

$$v_B^2 = \frac{2}{m}\left[\frac{m v_A^2}{2} + U_A - U_B\right]$$

$$U_A - U_B - ? \qquad U_A - U_B = (V_A - V_B) \cdot e$$

positive for proton

To finish it substitute the numbers



Problem 71

Figure shows a thin rod with charge Q that has been bent into a simicircle of radius *R*. Find an expression for the electric potential at the center.

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}.$$

$$dq = \lambda \cdot ds = \lambda \cdot R \cdot d\theta$$

$$V = \int_{\text{Semicircle}} dV = \int_0^{\pi} \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cdot \cancel{R} \cdot d\theta}{\cancel{R}} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^{\pi} d\theta = \frac{\lambda \cdot \cancel{\pi}}{4\cancel{\pi}\,\varepsilon_0} = \frac{Q}{\pi \cdot R \cdot 4\varepsilon_0} =$$

$$\boxed{\lambda = \frac{Q}{\pi \cdot R}} \qquad = \frac{Q}{4\pi\varepsilon_0 \cdot R}$$

End of Lecture 8
Reading: Finish Chapter 29
Review for Quiz 4
Home Work 4