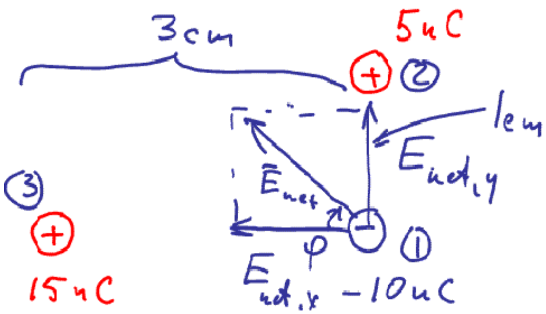
Problem 1: 25.41



Magnitude and direction $F_{net,1}$ - ?

$E_y > 0$

$E_x < 0$

Find F_y, F_x

Find $|\vec F_{net}| = \sqrt{(F_{net,x})^2 + (F_{net,y})^2}$

$tg\varphi = \frac{F_{net,y}}{|F_{net,x}|}$, $\varphi = arctg \frac{F_{net,y}}{|F_{net,x}|}$.

$$F_{net,y} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q_2}{r_{12}^2} = \frac{1}{4\cdot 3.1415\cdot 8.85\cdot 10^{-12}}\frac{10\cdot 10^{-9}\cdot 5\cdot 10^{-9}}{(10^{-2})^2} =$$

$$= 4.5\cdot 10^{-3} N$$

$$|F_{net,x}| = \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q_3}{r_{13}^2} = \frac{1}{4\cdot 3.1415\cdot 8.85\cdot 10^{-12}}\frac{10\cdot 10^{-9}\cdot 15\cdot 10^{-9}}{(3\cdot 10^{-2})^2} =$$

$$= 1.5\cdot 10^{-3} N$$

Component form:

$$\vec F_{net} = -|F_{net,x}|\cdot\hat i + F_{net,y}\cdot\hat j$$

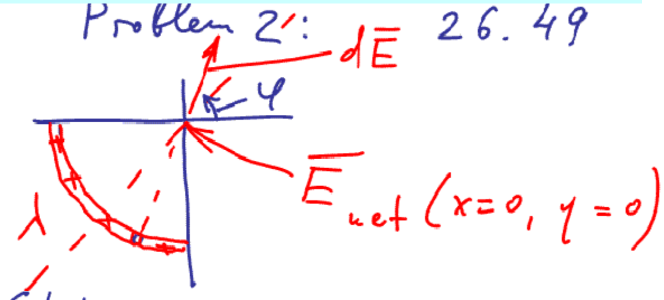
$$|\vec F_{net}| = \sqrt{F_{net,x}^2 + F_{net,y}^2} = 4.74\cdot 10^{-3} N$$

$$tg\varphi = \frac{E_y}{|E_x|} = \frac{4.5\cdot 10^{-3}}{1.5\cdot 10^{-3}} = 3$$

$\varphi = 71.6°$ – above $(-x)$ axis

Problem 2: 26.49

Problem 2: 26.49



Steps:

1. Divide into ds - infinitly small segments
2. Express $|d\vec E|$ – magnitude of the field from ds
3. Express components: $dE_x = |d\vec E|\cdot\cos\varphi$
   $dE_y = |d\vec E|\cdot\sin\varphi$
4. $E_{net,x} = \int_{\text{One end}}^{\text{Opposite end}} dE_x$
   $E_{net,y} = \int_{\text{One end}}^{\text{Opposite end}} dE_y$
5. $|E_{net}| = \sqrt{(E_{net,x})^2 + (E_{net,y})^2}$

6. $\varphi = \arctan \frac{E_{net,y}}{E_{net,x}} = 45°?$

$$|dE| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot \cancel{r} \cdot d\varphi}{r^{\cancel{2}}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{d\varphi}{r}$$

$$\boxed{dq = \lambda \cdot ds, \quad ds = r \cdot d\varphi, \quad d\varphi - \text{in rad}}$$

$$dE_x = \frac{\lambda}{4\pi\varepsilon_0}\frac{\cos\varphi}{r} \cdot d\varphi$$

$$dE_y = \frac{\lambda}{4\pi\varepsilon_0}\frac{\sin\varphi}{r} \cdot d\varphi$$

$$E_{net,x} = \int_{\text{One end}}^{\text{Opposite end}} dE_x = \frac{\lambda}{4\pi\varepsilon_0 \cdot r}\int_0^{\frac{\pi}{2}} \cos\varphi \cdot d\varphi =$$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot r} \sin\varphi \Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 \cdot r}(1-0) = \frac{\lambda}{4\pi\varepsilon_0 \cdot r}$$

$$E_{net,y} = \int_{\text{One end}}^{\text{Opposite end}} dE_y = \frac{\lambda}{4\pi\varepsilon_0 \cdot r}\int_0^{\frac{\pi}{2}} \sin\varphi \cdot d\varphi =$$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot r}(-\cos\varphi)\Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 \cdot r}(0-(-1)) =$$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot r}.$$

$$\bar{E}(0,0) = E_{net,x} \cdot \bar{i} + E_{net,y} \cdot \bar{j} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot r}(\bar{i} + \bar{j})$$

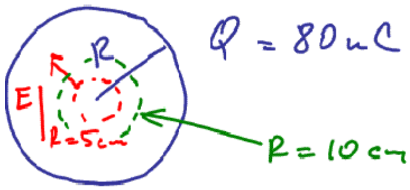
If we are given Q (total charge)

$$Q = \ell \cdot \lambda, \text{ where } \ell = \frac{1}{4} 2\pi r = \frac{\pi r}{2}$$

$$Q = \frac{\pi r}{2} \cdot \lambda \Rightarrow \lambda = \frac{2Q}{\pi \cdot r}$$

$$\bar{E}(0,0) = \frac{2Q}{\pi \cdot r \cdot 4\pi\varepsilon_0 \cdot r}(\bar{i} + \bar{j}) = \frac{Q}{2\pi^2\varepsilon_0 r^2}(\bar{i} + \bar{j})$$

Problem 3: 27.37



$R = 20\,cm$

$Q = 80\,nC$

a) ρ (density) – ?

b) Q_{enc} by sphere with $R = 5cm$, $10cm$, $20cm$

c) $E(R=5cm)$ – ?

$E(R=10cm)$ – ?

$E(R=20cm)$ – ?

$$\rho = \frac{Q}{V} \text{ (Uniform case) } = \frac{Q}{\frac{4}{3}\pi R^3}.$$

$$Q_{enc}(R=5cm) = \rho \cdot V(R=5cm) =$$

$$= \frac{Q}{\frac{4}{3}\pi R^3\Big|_{R=20cm}} \cdot \frac{4}{3}\pi R^3\Big|_{R=5cm} = \frac{Q \cdot 5^3}{20^3} =$$

$$= \frac{Q \cdot 125}{8000} = Q \cdot 0{,}0156 \cong \underline{\underline{1{,}25\,nC}}$$

$$Q_{enc}(R=10cm) = \rho \cdot V(R=10cm) =$$

$$= \frac{Q}{\frac{4}{3}\pi R^3\Big|_{R=20cm}} \cdot \frac{4}{3}\pi R^3\Big|_{R=10cm} = \frac{Q \cdot 1000}{8000} = \underline{\underline{10\,nC}}$$

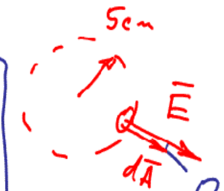
$$Q_{enc}(R=20cm) = 80\,nC$$

$E(R=5cm)$ – ?

Gauss's Law! $\quad \Phi_e = \dfrac{Q_{enc}}{\varepsilon_0}$

$$\Phi_e = \oint \bar{E} \cdot d\bar{A} =$$

$$\boxed{\begin{aligned} \bar{E} \cdot d\bar{A} &= |\bar{E}| \cdot |d\bar{A}| \cdot \cos\theta = \\ &= |\bar{E}| \cdot |d\bar{A}| = E(r) \cdot dA \end{aligned}}$$



$\theta = 0, \; \cos\theta = 1$

$$= \oint E(r) \cdot dA = E(r) \oint_{\text{Red sphere}} dA = E(r) \cdot 4\pi r^2$$

$$E(r) \cdot 4\pi r^2 = \frac{Q_{enc}(r=5cm)}{\varepsilon_0}.$$

$$E(r) = \frac{Q_{enc}(r=5cm)}{4\pi r^2\big|_{r=5cm} \cdot \varepsilon_0} = \frac{1{,}25 \cdot 10^{-9}}{4 \cdot 3{,}1415 \cdot (5 \cdot 10^{-2})^2 \cdot 8{,}85 \cdot 10^{-12}}$$

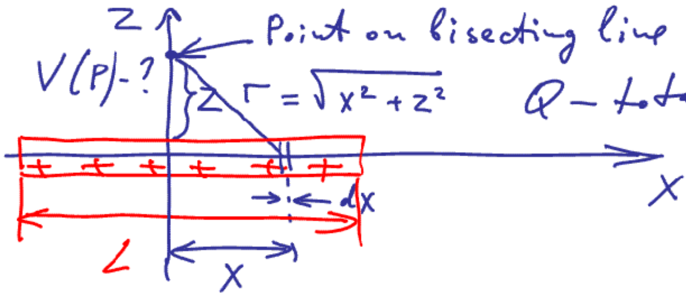
$$= 4.5 \cdot 10^3 \frac{N}{C}$$

For $r = 10cm$:

$$E(r=10cm) = \frac{Q_{enc}(r=10cm)}{4\pi r^2\big|_{r=10cm} \cdot \varepsilon_0} = \underline{\underline{9.0 \cdot 10^3}} \frac{N}{C}$$

For $r = 20cm$: $\quad E(r=20cm) = \underline{\underline{18 \cdot 10^3}} \dfrac{N}{C}$

Problem 4: 29.69



Q – total charge given

$$\lambda = \frac{Q}{L}, \qquad dq = \lambda \cdot dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{\sqrt{x^2+z^2}}.$$

$V = 2 \cdot V_{1/2}$, $V_{1/2}$ – Potential created by a half of rod from 0 to $\frac{L}{2}$

$$V_{1/2} = \int_0^{L/2} dV = \int_0^{L/2} \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{\sqrt{x^2+z^2}} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\sqrt{x^2+z^2}} =$$

$$\boxed{\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)}$$

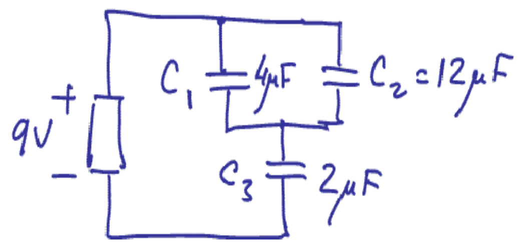
$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(x+\sqrt{x^2+z^2}\right)\Big|_0^{L/2} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(\frac{L}{2}+\sqrt{\left(\frac{L}{2}\right)^2+z^2}\right) - \ln z\right] =$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L}{2z}+\sqrt{\left(\frac{L}{2z}\right)^2+1}\right)$$

$$\underbrace{V = 2\cdot V_{1/2} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{L}{2z}+\sqrt{\left(\frac{L}{2z}\right)^2+1}\right).}$$

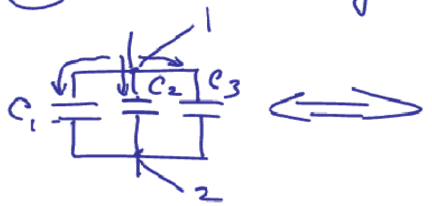
Problem 5: 30.64

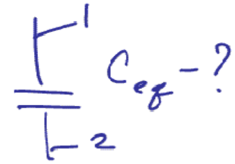


Q_i – ? $\quad i = 1,2,3$

V_i – ?

① Connection of capacitors in parallel





Read end of Chapter 3.

$$Q_{eq} = Q_1 + Q_2 + Q_3$$

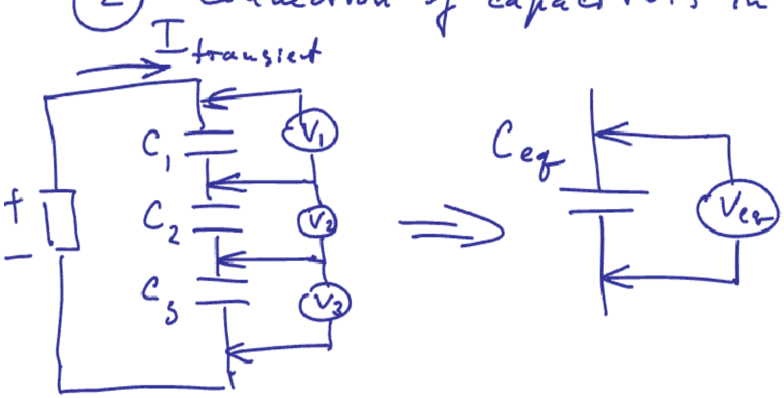
$$V_1 = V_2 = V_3 = V_{eq}$$

$$C_{eq} = \frac{Q_{eq}}{V_{eq}} = \frac{Q_1 + Q_2 + Q_3}{V_{eq}} = \frac{Q_1}{V_{eq}} + \frac{Q_2}{V_{eq}} + \frac{Q_3}{V_{eq}} =$$

$$= \frac{Q_1}{V_1} + \frac{Q_2}{V_2} + \frac{Q_3}{V_3} = \underbrace{C_1 + C_2 + C_3}$$

Rule of connecting C's in parallel.

② Connection of capacitors in series



$$V_{eq} = V_1 + V_2 + V_3$$

$$Q_{eq} = Q_1 = Q_2 = Q_3$$

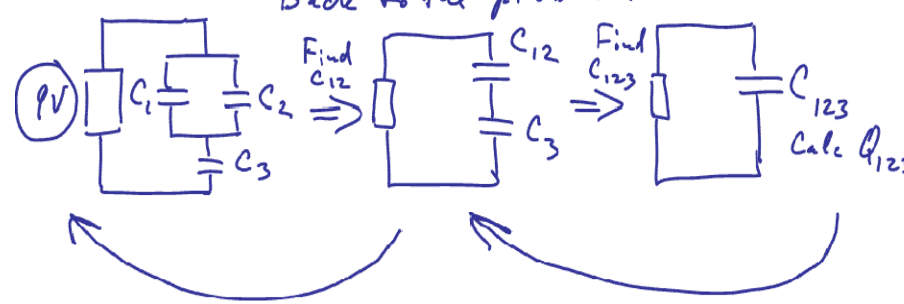
$$\frac{1}{C_{eq}} = \frac{V_{eq}}{Q_{eq}} = \frac{V_1 + V_2 + V_3}{Q_{eq}} = \frac{V_1}{Q_{eq}} + \frac{V_2}{Q_{eq}} + \frac{V_3}{Q_{eq}} =$$

$$= \frac{V_1}{Q_1} + \frac{V_2}{Q_2} + \frac{V_3}{Q_3} = \underbrace{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Rule of connecting C's in series

$$\boxed{\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}}$$

Back to the problem



1. $C_{12} = C_1 + C_2 = 4 + 12 = 16\mu F$

2. $\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{16} + \frac{1}{2} = \frac{1+8}{16} = \frac{9}{16}$

   $C_{123} = \frac{16}{9}\mu F$

3. $V_{123} = V = 9V$, $C_{123} = \frac{Q_{123}}{V} \Rightarrow$

$$Q_{123} = C_{123} \cdot V = \frac{16}{\cancel{9}}(\mu F) \cdot \cancel{9}(V) = 16\mu C$$

Now we can go in the backward direction

$\frac{16}{9}\cdot 10^{-6}(F)\cdot 9(V) = 16\cdot 10^{-6}(C) = 16\mu C$

4. $Q_{12} = Q_3 = Q_{123} = 16\mu C$

$$\left.\begin{aligned} V_{12} &= \frac{Q_{12}}{C_{12}} = \frac{16\mu C}{16\mu F} = 1V \\ V_3 &= \frac{Q_3}{C_3} = \frac{16\mu C}{2\mu F} = 8V \end{aligned}\right\} V_{12}+V_3 = 9V \quad \text{OK}$$

5. $Q_1 = C_1\cdot V_1 = 4\mu F\cdot 1V = 4\mu C,\ V_1 = V_{12} = 1V$

$Q_2 = C_2\cdot V_2 = 12\mu F\cdot 1V = 12\mu C,\ V_2 = V_{12} = 1V$