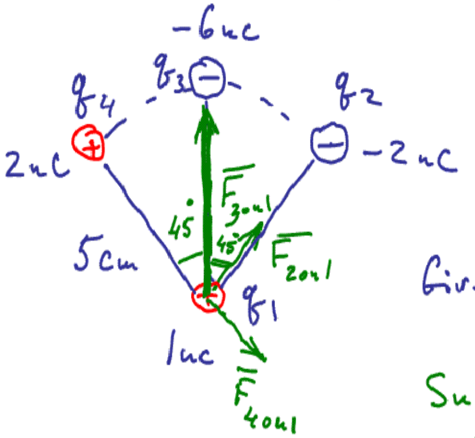
Review for Exam I

Problem 25-47



What is the force $\overline{F}$ on the 1nC charge at the bottom?

Give your answer in a component form

Summing the forces using unit-vector notation $(\hat{i}, \hat{j}, \hat{k})$

$$\overline{F}_{2on1} = \left(\frac{k \cdot |q_1| \cdot |q_2|}{r^2}, \text{ towards } q_2 \right) =$$

$$= \left(\frac{(9\times10^9)(1\times10^{-9})(2\times10^{-9})}{(5\times10^{-2})^2}, \text{ towards } q_2 \right) =$$

$$= (0.72\times10^{-5}\,N, \text{ towards } q_2) =$$

$$(0.72\times10^{-5}\,N)\cdot(\cos 45°\cdot\hat{i} + \sin 45°\cdot\hat{j})$$

$$\overline{F}_{4on1} = \left(\frac{k\cdot|q_1|\cdot|q_4|}{r^2}, \text{ away from } q_4 \right) =$$

$$(0.72\times10^{-5}\,N)\cdot(\cos 45°\cdot\hat{i} - \sin 45°\cdot\hat{j})$$

$$\overline{F}_{3on1} = \left(\frac{k\cdot|q_1|\cdot|q_3|}{r^2}, \text{ towards } q_3 \right) =$$

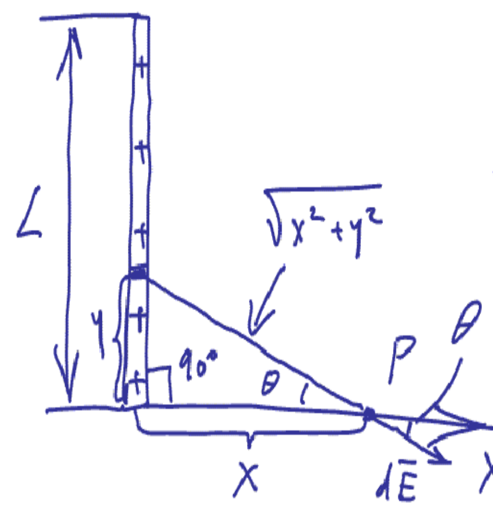
$$(2.16\times10^{-5}\,N, \text{ towards } q_3) = 2.16\times10^{-5}\cdot\hat{j}\ [N]$$

$$\overline{F}_{net\,on\,1} = \overline{F}_{2on1} + \overline{F}_{4on1} + \overline{F}_{3on1} =$$

$$= (0.72\times10^{-5})\cdot(2\cdot\cos 45°)\cdot\hat{i} + 2.16\times10^{-5}\cdot\hat{j} =$$

$$= \underline{(1.02\times10^{-5}\cdot\hat{i} + 2.16\times10^{-5}\,\hat{j})\ [N].}$$

Problem 26-45



Given L, Q

Find $\overline{E}(x)$,

where x - distance from the end of the rod

Give your answer in a component form

Steps:

1. Divide the rod into a series of little segments
2. Apply Coulomb Law for each segment Δy

3. $E_x = \int_{rod} dE_x$, $\quad E_y = \int_{rod} dE_y$

4. $\bar{E} = E_x \cdot \hat{i} + E_y \cdot \hat{j}$

$$d\bar{E} = |d\bar{E}| \cdot (\cos\theta \cdot \hat{i} - \sin\theta \cdot \hat{j})$$

$$|d\bar{E}| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x^2+y^2)}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$dq = \lambda \cdot dy = \frac{Q}{L}\cdot dy$$

$$\boxed{\lambda = \frac{Q}{L}}$$

$$d\bar{E} = \frac{Q/L}{4\pi\varepsilon_0}\left(\frac{x\cdot dy}{(x^2+y^2)^{3/2}}\cdot\hat{i} - \frac{y\cdot dy}{(x^2+y^2)^{3/2}}\cdot\hat{j}\right)$$

$$dE_x = \frac{Q/L}{4\pi\varepsilon_0}\frac{x\cdot dy}{(x^2+y^2)^{3/2}}$$

$$dE_y = -\frac{Q/L}{4\pi\varepsilon_0}\frac{y\cdot dy}{(x^2+y^2)^{3/2}}$$

$$E_x = \int_{rod} dE_x = \frac{Q/L}{4\pi\varepsilon_0}\int_0^L \frac{x\cdot dy}{(x^2+y^2)^{3/2}} =$$

$$= \frac{Q/L}{4\pi\varepsilon_0}\left(\frac{\cancel{x}\cdot y}{x^{\cancel{2}}(y^2+x^2)^{1/2}}\Bigg|_0^L\right) = \frac{Q/L}{4\pi\varepsilon_0}\frac{L}{x\sqrt{L^2+x^2}}$$

Table integral: $\boxed{\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}}$

$$E_y = \int_{rod} dE_y = \frac{-Q/L}{4\pi\varepsilon_0}\int_0^L \frac{y\cdot dy}{(x^2+y^2)^{3/2}} =$$

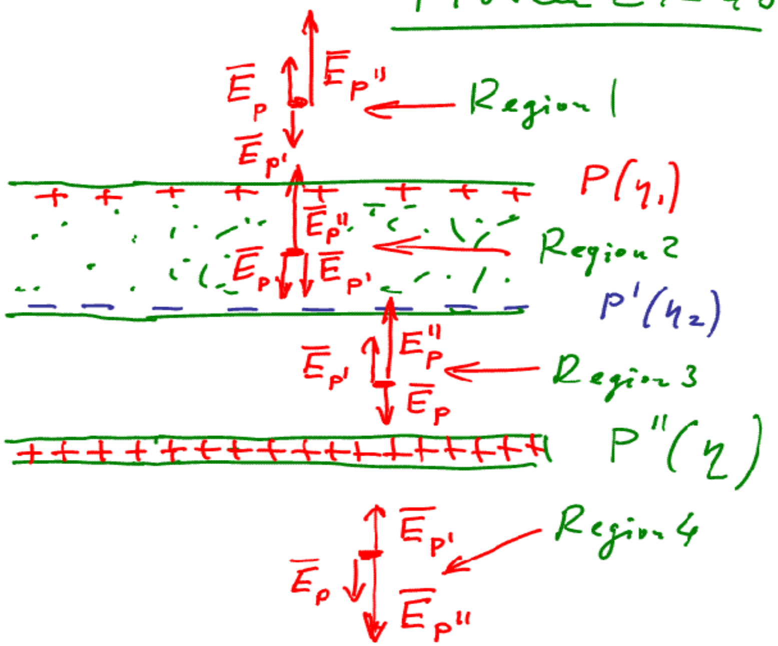
$$= \frac{Q/L}{4\pi\varepsilon_0}\frac{1}{(y^2+x^2)^{1/2}}\Bigg|_0^L = \frac{Q/L}{4\pi\varepsilon_0}\left[\frac{1}{(L^2+x^2)^{1/2}} - \frac{1}{x}\right] =$$

Table integral: $\boxed{\int\frac{x\cdot dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}}$

$$= \frac{Q/L}{4\pi\varepsilon_0\cdot x}\left[\frac{x}{\sqrt{x^2+L^2}} - 1\right]$$

$$\bar{E} = \frac{Q/L}{4\pi\varepsilon_0\cdot x}\left[\frac{L}{\sqrt{x^2+L^2}}\cdot\hat{i} + \left(\frac{x}{\sqrt{x^2+L^2}}-1\right)\cdot\hat{j}\right]$$

Problem 27-48



E_1 - ?

E_2 - ?

E_3 - ?

E_4 - ?

The plane of charge (P'') polarizes the conductor in such a way that the face of the conductor adjacent to the plane of charge is negatively charged ($P'(\eta_2)$). This makes the other face of the conductor positively charged ($P(\eta_1)$).

Since the metal is electrically neutral, we have $\eta_2 = -\eta_1$.

Model: It becomes to be a problem with three infinite planes of charge - $P(\eta_1)$, $P'(\eta_2)$ and $P''(\eta)$

Additional condition: El. field inside conductor is zero (Region 2)

$$\bar{E}_p + \bar{E}_{p'} + \bar{E}_{p''} = 0$$

Using general expression for an el. field due to a plane of charge $\left(E = \frac{\eta}{2\varepsilon_0}\right)$, we have for Region 2:

$$-\frac{\eta_1}{2\varepsilon_0}\cdot\hat{j} + \frac{\eta_2}{2\varepsilon_0}\hat{j} + \frac{\eta_3}{2\varepsilon_0}\hat{j} = 0$$

$$-\eta_1 + \eta_2 + \eta = 0$$

On the other hand we know that $\eta_2 = -\eta_1 \Rightarrow$

$$-2\eta_1 + \eta = 0 \Rightarrow \eta_1 = \frac{1}{2}\eta \quad \text{and} \quad \eta_2 = -\frac{1}{2}\eta$$

Now we know densities in signs of the charges for each plane. Let us find the fields region by region.

In region 1: $\bar{E}_p = \frac{\eta}{4\varepsilon_0}\hat{j}$, $\bar{E}_{p'} = -\frac{\eta}{4\varepsilon_0}\hat{j}$, $E_{p''} = \frac{\eta}{2\varepsilon_0}\hat{j}$

The net el. field is $\bar{E}_{net} = \bar{E}_p + \bar{E}_{p'} + \bar{E}_{p''} = \underline{\left(\frac{\eta}{2\varepsilon_0}\right)\cdot\hat{j}}$

In region 2: $\underline{\bar{E}_{net} \equiv 0}$ - see above

In region 3: $\bar{E}_p = -\frac{\eta}{4\varepsilon_0}\hat{j}$, $\bar{E}_{p'} = \frac{\eta}{4\varepsilon_0}\hat{j}$, $\bar{E}_{p''} = \frac{\eta}{2\varepsilon_0}\cdot\hat{j}$

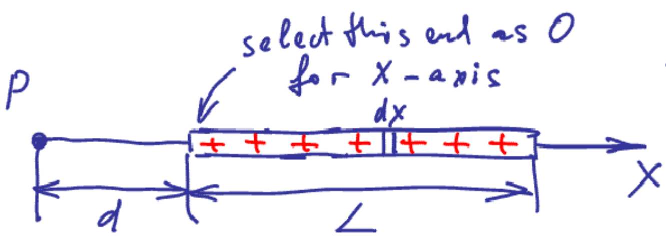
The net el. field is $\underline{\bar{E}_{net} = \left(\frac{\eta}{2\varepsilon_0}\right)\cdot\hat{j}}$

In region 4: $\bar{E}_p = -\frac{\eta}{4\varepsilon_0}\cdot\hat{j}$, $\bar{E}_{p'} = \frac{\eta}{4\varepsilon_0}\hat{j}$, $\bar{E}_{p''} = -\frac{\eta}{2\varepsilon_0}\hat{j}$

The net el. field: $\underline{\bar{E}_{net} = -\left(\frac{\eta}{2\varepsilon_0}\right)\cdot\hat{j}}$

In addition study Gauss's law through examples 27.4, 27.5 and 27.6

Problem 4 (Not from the Night's book)



The plastic rod has length L and a nonuniform linear charge density $\lambda = cX$, where c – positive constant. Find the electric potential $V(P)$ at point P on the axis, at distance d from one end.

Consider infinitly small segment of the rod located between x and $(x+dx)$.

It contains charge $dq = \lambda \cdot dx$, but $\lambda = cx \Rightarrow$

$$dq = c \cdot x \cdot dx$$

A small potential created by this element at distance d:

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0} \frac{c \cdot x \cdot dx}{d+x}$$

The total potential of the rod:

$$V(P) = \int_{rod} dV = \frac{c}{4\pi\varepsilon_0} \int_0^L \frac{x \cdot dx}{d+x} =$$

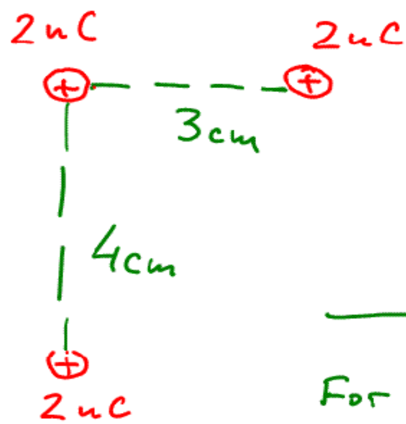
$$\boxed{\int \frac{x \cdot dx}{d+x} = x - d \cdot \ln(x+d)}$$ Standard integral

$$= \frac{c}{4\pi\varepsilon_0} \left[x - d \cdot \ln(x+d) \right] \Big|_0^L =$$

$$= \frac{c}{4\pi\varepsilon_0} \left[L - d \cdot \ln(L+d) + d \cdot \ln d \right] =$$

$$= \frac{c}{4\pi\varepsilon_0} \left[L - d \cdot \ln\left(\frac{L+d}{d}\right) \right]$$

Problem 29-5



What is the electric potential energy of the group of charges in Figure?

For a system of charges, the potential energy is the sum of the potential energies due to all mutual interactions:

$$U_{elec} = \sum_{i,j} \frac{K q_i q_j}{r_{ij}} = U_{12} + U_{13} + U_{23} =$$

$$= (9 \times 10^{9}\, N.m/C^2)\cdot(2 \times 10^{-9}\, C)\cdot(2 \times 10^{-9}\, C) \times$$

$$\times \left[\frac{1}{0.03m} + \frac{1}{0.04m} + \frac{1}{\sqrt{(0.03m)^2 + (0.04m)^2}}\right] =$$

$$= 1.2 \times 10^{-6}\, J + 0.9 \times 10^{-6}\, J + 0.72 \times 10^{-6}\, J = \underline{\underline{2.82 \times 10^{-6}\, J}}$$

Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$