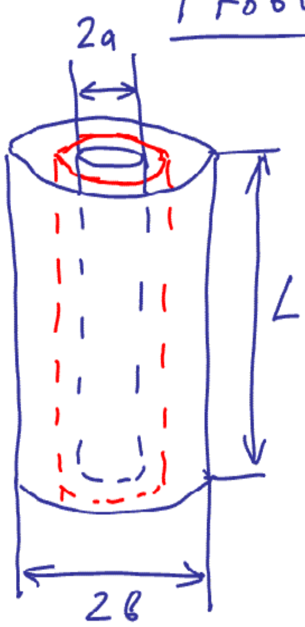
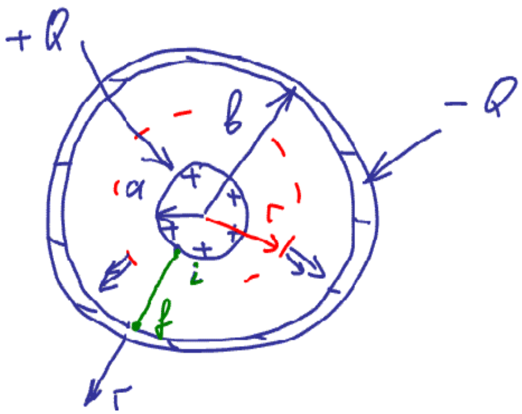
Problem 1



Derive an expression for the capacitance of the cylindrical capacitor in terms of a, b, L and ε_0



First step: Assume Q

Second step: Calculating $E(r)$ using Gauss's Law

$$\Phi_e = \oint \bar{E}\cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0} \leftarrow \text{Gauss's Law}$$

$$\bar{E} \parallel d\bar{A} \Rightarrow \bar{E}\cdot d\bar{A} = E\cdot dA$$

$$\oint E\cdot dA = E\oint dA = E\cdot 2\pi r\cdot L$$

$$q_{enc} = +Q$$

$$E\cdot 2\pi r\cdot L = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{1}{2\pi\varepsilon_0}\,\frac{Q}{L\cdot r}$$

Third step: Finding $\Delta V = -\int_i^f \bar{E}\cdot d\bar{s}$

The path $i \to f$ is indicated on the sketch, it is selected to be along radial direction

$$\measuredangle\, \bar{E}\cdot d\bar{s}, \qquad E > 0, \qquad ds = dr > 0$$

$$\bar{E} \parallel d\bar{s} \Longrightarrow \bar{E}\cdot d\bar{s} = \underline{\underline{E(r)\cdot dr}}$$

$$\Delta V = V_f - V_i < 0$$

$$|\Delta V| = |V_f - V_i| = +\int_i^f \bar{E}\cdot d\bar{s} = \int_i^f E(r)\cdot dr =$$

$$= \int_a^b E(r)\cdot dr = \int_a^b \frac{1}{2\pi\varepsilon_0}\frac{Q}{L\cdot r}\,dr =$$

$$= \frac{Q}{2\pi\varepsilon_0\cdot L}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0\cdot L}\,\ln r\Big|_a^b =$$

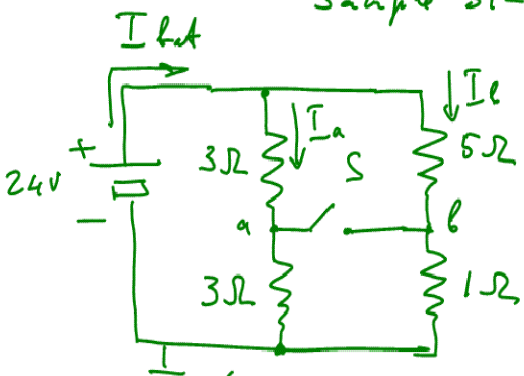
$$= \frac{Q}{2\pi\varepsilon_0\cdot L}(\ln b - \ln a) = \underline{\frac{Q}{2\pi\varepsilon_0\cdot L}\ln\frac{b}{a}}$$

Last step: Calculate C using the definition of capacitance $C = \frac{Q}{\Delta V}$

$$C = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon_0\cdot L}\ln\frac{b}{a}} = \underline{\underline{\frac{2\pi\varepsilon_0\cdot L}{\ln\frac{b}{a}}}}$$

Problem 2

Sample 31-63



What is I_{tot} and V_{ab} when the switch is:

a) Open-?

b) Closed-?

(a) – Open S

$$I_{tot} = \frac{24V}{3\Omega} = 8A$$

Branch a: $24V = I_a(3\Omega + 3\Omega) \Rightarrow I_a = \frac{24V}{6\Omega} = 4A$

Branch b: $24V = I_b(5\Omega + 1\Omega) \Rightarrow I_b = \frac{24V}{6\Omega} = 4A$

$$V_a = \underbrace{V_{initial}}_{=0} + I_a \cdot (3\Omega) = 4A \cdot 3\Omega = 12V$$

$$V_b = 0 + I_b \cdot (1\Omega) = 4A \cdot 1\Omega = 4V$$

$$V_{ab} = V_a - V_b = 12V - 4V = 8V$$

Let us start alternatively from the top part of the circuit: (V_{top})

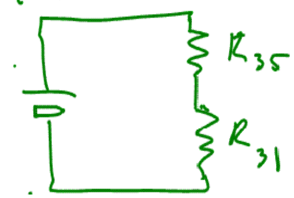
$$V_a = V_{top} - I_a \cdot (3\Omega) = V_{top} - 12V$$

$$V_b = V_{top} - I_b \cdot (5\Omega) = V_{top} - 20V$$

$$V_{ab} = V_a - V_b = \cancel{V_{top}} - 12V - \cancel{V_{top}} + 20V = 8V$$

So, it does not matter where we start, the result is the same!

(b) – S-closed

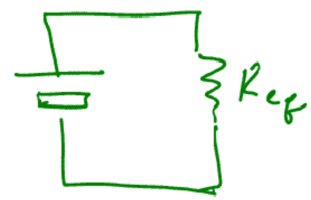


$$\frac{1}{R_{35}} = \frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$$

$$R_{35} = \frac{15}{8}\Omega$$

$$\frac{1}{R_{31}} = \frac{1}{3} + \frac{1}{1} = \frac{1+3}{3} = \frac{4}{3}$$

$$R_{31} = \frac{3}{4}\Omega$$



$$R_{eq} = R_{35} + R_{31} = \frac{15}{8} + \frac{3}{4} = \frac{15+6}{8} = \frac{21}{8}\Omega$$

$$I_{tot} = \frac{24V}{\frac{21}{8}\Omega} = 9.143\ A$$

$$V_{ab} = 0$$

Problem 3

A couple of problems 30-45 and 29-48

Problem 30-45

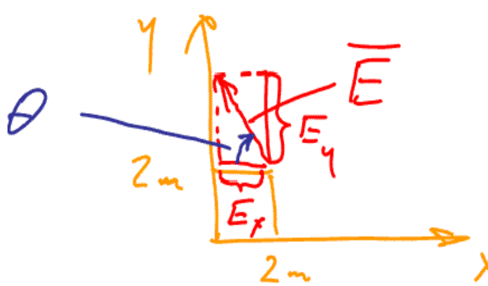
Given $V = (150x^2 - 200y^2)$ [V],

where x and y - in meters

What is the strength and direction of the el. field at point ($x = 2.0$ m, $y = 2.0$ m)?

$$E_x = -\frac{\partial V}{\partial x} = -2x \cdot 150 = -300x = -600\,\frac{V}{m}$$

$$E_y = -\frac{\partial V}{\partial y} = +2y \cdot 200 = 400y = 800\,\frac{V}{m}$$

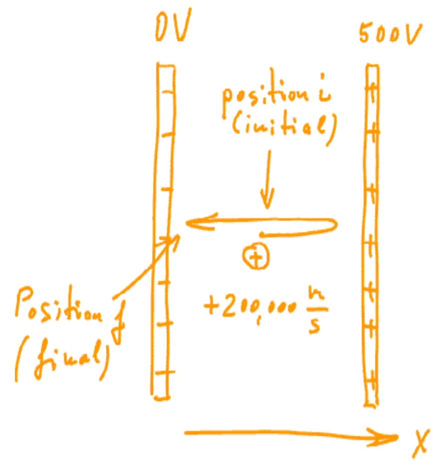


$$|\bar{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{(-600)^2 + (800)^2} = 1000\,\frac{V}{m}$$

$$\tan\theta = \frac{E_y}{|E_x|} = \frac{800}{600} = \frac{4}{3} \Rightarrow \theta = 53.1^\circ$$

a above the $-x$-axis

Problem 29-48



A proton is fired with $v = 2 \cdot 10^5\,\frac{m}{s}$ from the mid point of a capacitor towards the + plate.

a) Show that v is insufficient to reach + plate

b) What is the proton velocity (v_f) as it collides with the negative plate?

(a) Inside the capacitor $V(x) = E \cdot x \Rightarrow V_i = \frac{500V - 0V}{2} = 250\,V$

The energy required to reach + plate:

$$\Delta U = e\Delta V = 1.6 \cdot 10^{-19}\,C \cdot (250V) = 4 \times 10^{-17}\,J$$

The proton kinetic energy is:

$$K = \frac{1}{2} m v_i^2 = \frac{1}{2}(1.67 \cdot 10^{-27}\,kg) \cdot \left(2 \cdot 10^5\,\frac{m}{s}\right)^2 = 3.34 \times 10^{-17}\,J$$

$K < \Delta U \Rightarrow$ not enough energy to reach + plate

(b) Use energy conservation:

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + qV_i = \frac{1}{2} m v_f^2 + qV_f \Rightarrow$$

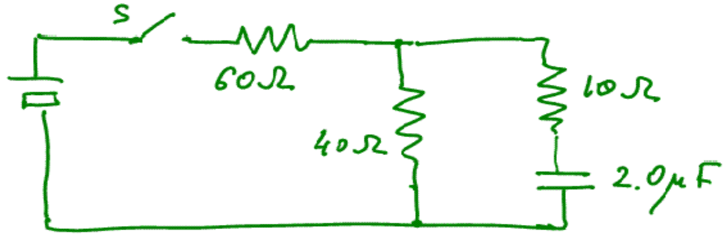
$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 + q(V_i - V_f)$$

$$v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_i - V_f)} = \sqrt{\left(2 \times 10^5\,\frac{m}{s}\right)^2 + \frac{2(1.6 \times 10^{-19}\,C)(250V - 0V)}{1.67 \times 10^{-27}\,kg}} = 2.96 \times 10^5\,\frac{m}{s}$$

Problem 4

Samples 31-77 and 30-31

Problem 31-77



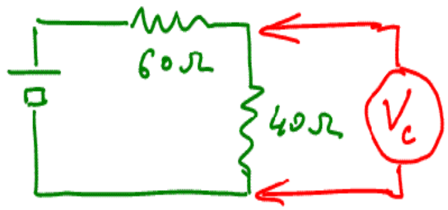
The switch S has been closed for a very long time

a) what is the charge on the capacitor?

b) The switch is open at $t=0s$. At what time has the charge on the capacitor decreased to 10% of its initial value?

(a) In steady state the capacitor is a break $\Rightarrow$

An equivalent circuit is a voltage divider:



Voltage available for the capacitor:

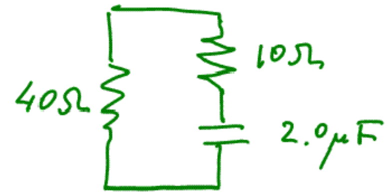
$$V_{40} = \frac{\mathcal{E}\cdot 40}{60+40} = 40V$$

In steady state I through 10Ω resistor $= 0 \Rightarrow$

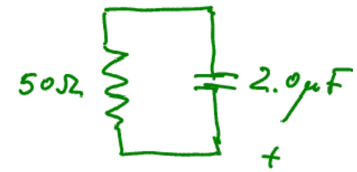
$$V_{10} = I_{10}\cdot 10 = 0 \Rightarrow V_c = V_{40} = 40V$$

Since $C = \frac{Q}{V_c} \Rightarrow Q = C\cdot V_c = (2\times 10^{-6}F)\cdot(40V) = \underline{\underline{80\mu C}}$

Reopening creates a new circuit:



$\Longleftrightarrow$



$$Q = Q_0\cdot e^{-\frac{t}{\tau}}$$

$Q_0 = 80\mu C$

$\tau = RC = 50\Omega\cdot 2\cdot 10^{-6}F = \underline{\underline{10^{-4}s}}$

$Q(t) = 0.1\cdot Q_0 \Rightarrow$

$0.1\cancel{Q_0} = \cancel{Q_0}\cdot e^{-\frac{t}{\tau}}$

$\ln 0.1 = \ln\left(e^{-\frac{t}{\tau}}\right) = -\frac{t}{\tau}$

$-2.3 = -\frac{t}{\tau} \Rightarrow t = 2.3\cdot\tau = 2.3\cdot 10^{-4}s = \underline{\underline{0.23ms}}$

Problem 30-31

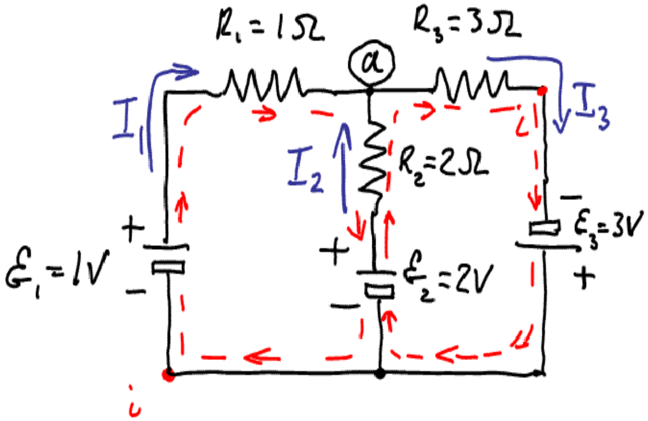
Capacitor 2 has half the capacitance and twice the potential difference as capacitor 1

$$\left(C_2 = \tfrac{1}{2}C_1\ ,\quad \Delta V_{C_2} = 2\Delta V_{C_1}\right)$$

What is the ratio of their potential energies $\frac{U_{C_1}}{U_{C_2}}$ -?

$$\frac{U_{C_1}}{U_{C_2}} = \frac{\frac{1}{2}C_1(\Delta V_{C_1})^2}{\frac{1}{2}C_2(\Delta V_{C_2})^2} = \frac{C_1(\Delta V_{C_1})^2}{\left(\frac{1}{2}C_1\right)\cdot 4(\Delta V_{C_1})^2} = \frac{1}{2}$$

Problem 5



Find the currents I_1, I_2 and I_3 in each resistor using Kirchoff's Law

Step 1: Assume directions for the currents I_1, I_2 and I_3 in all branches.
If your assumption is correct you will eventually find this current I_i with sign +

If you made a wrong assuption, you will find sign −. In any case you are safe to assume directions for the currents ARBITRARY.

Step 2: Assume directions (clockwise or counterclockwise) of making loops.
Generally, the directions are selected ARBITRARY
In this example we selected clockwise directions for both small loops starting from the point indicated "i"

Step 3: To find three unknowns I_1, I_2 and I_3 we need three independent equations.

Let us use two loop rules and one junction rule. They will give us a complete system of independent equations.

Left loop:

$$1 - \underbrace{I_1 \cdot 1} + \underbrace{I_2 \cdot 2} - 2 = 0 \quad (1)$$

we go through the resistor R_1 with the current. Hence, according to the "resistor" rule the sign is negative

we go through R_2 against the current I_2 ⟹ the sign is positive

Right loop:

$$3 + 2 - I_2 \cdot 2 - I_3 \cdot 3 = 0 \quad (2)$$

Junction rule at point a:

$$I_1 + I_2 = I_3 \quad (3)$$

substitute (3) in (2):

$$5 - I_2 \cdot 2 - (I_1 + I_2) \cdot 3 = 0$$

$$5 - I_2 \cdot 5 - I_1 \cdot 3 = 0$$

From (1): $I_1 = -1 + I_2 \cdot 2$
plug it into the previous eq.:

$$5 - I_2 \cdot 5 - (-1 + I_2 \cdot 2) \cdot 3 = 0$$

$$5 - I_2 \cdot 5 + 3 - 6 I_2 = 0$$

$$8 = 11 \cdot I_2 \Rightarrow I_2 = \frac{8}{11} A$$

$$I_1 = -1 + 2 I_2 = -1 + \frac{16}{11} = \frac{-11 + 16}{11} = \frac{5}{11} A$$

$$I_3 = I_1 + I_2 = \frac{8}{11} + \frac{5}{11} = \frac{8+5}{11} = \frac{13}{11} A$$