Review for Final

Problem 1 (From a different text)

A point charge $+q$ is placed at the center of an electrically neutral, spherical conductive shell with inner radius a and outer radius b.

What charge appears on

(a) the inner surface of the shell

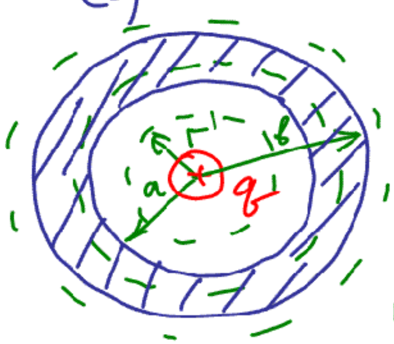
(b) the outer surface?

What is the net electric field at a distance r from the center of the shell if:

(c) $r < a$

(d) $a < r < b$

(e) $r > b$



Gauss's Law:

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0}$$

$r' < a$

$$\oint \bar{E}\cdot d\bar{A} = \oint E(r')\cdot dA = E(r')\oint dA = E(r')\cdot 4\pi r'^2$$

$$E(r')\cdot 4\pi r'^2 = \frac{q_{enc}}{\varepsilon_0}, \quad q_{enc} = q$$

(c) $\boxed{E(r') = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'^2}}$ – Coulomb Law

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$a < r' < b \Rightarrow E = 0$ inside the metal

$$\oint \bar{E}\cdot d\bar{A} = 0 \Rightarrow q_{enc} = 0$$

$$q_{enc} = q(r=0) + q_{inner} = 0$$

(a) $\boxed{q_{inner} = -q}$

$r' > b$

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0}.$$

$$q_{enc} = \underbrace{q(r=0) + q_{inner}}_{0} + q_{outer} = q_{outer}$$

$$\oint \bar{E}\cdot d\bar{A} = E(r')\cdot 4\pi r'^2$$

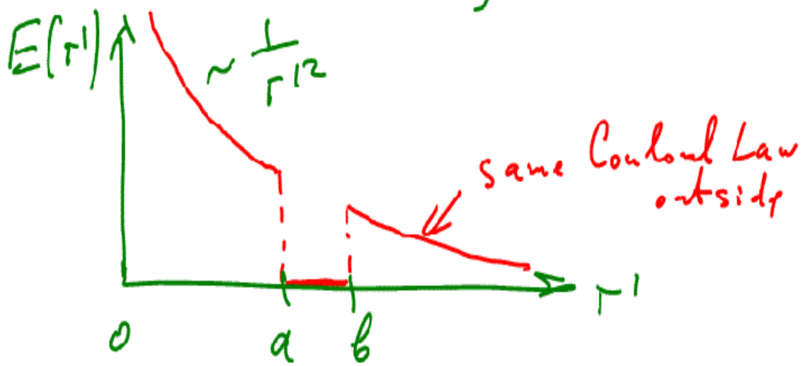
$$E(r')\cdot 4\pi r'^2 = \frac{q_{outer}}{\varepsilon_0}$$

$q_{inner} = -q_{outer}$ – consequence of 0 total charge on metal sphere

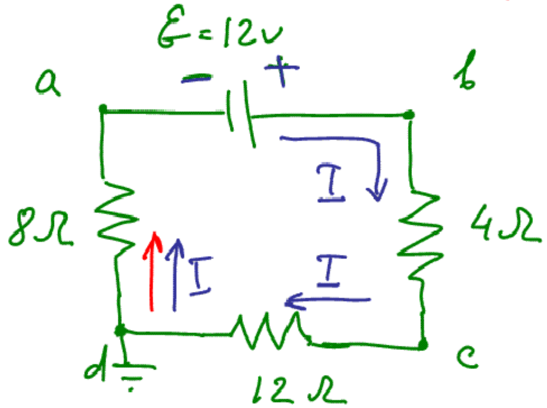
(b) $q_{outer} = q(r=0)$

(e) $E(r') \cdot 4\pi r'^2 = \frac{q}{\varepsilon_0}$

$$\boxed{E(r') = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'^2}}$$ – Same formula!



Problem 2 (Knight 31-58)



d – has $V_d = 0$

V_a – ?

V_b – ?

V_c – ?

V_d – ?

Loop Rule:

$\cancel{V_d} - I\cdot 8 + 12 - I\cdot 4 - I\cdot 12 = \cancel{V_d}$

$12 = 24\cdot I \Rightarrow I = 0.5\,A$

$V_d = 0$

$V_a = V_d - I\cdot 8 = 0 - 0.5\cdot 8 = -4V$

$V_b = V_a + \mathcal{E} = -4V + 12V = 8V$

$V_c = V_b - I\cdot 4 = 8 - 0.5\cdot 4 = 6V$

$V_d = V_c - I\cdot 12 = 6 - 0.5\cdot 12 = 0V$

In addition study problems from Chapter 31: 67–70

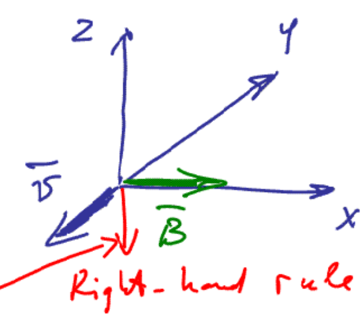
Problem 3 (Knight 32-31)

An electron moves in the magnetic field $\bar{B} = 0.5\cdot\hat{i}\,[T]$ with a speed of $1.0\times 10^7\,\frac{m}{s}$ in the direction shown in Figures (Two). For each Figure, what is magnetic force $\bar{F}$ on electron?

Give your answer in a component form

$\bar{F}_B$ – ?

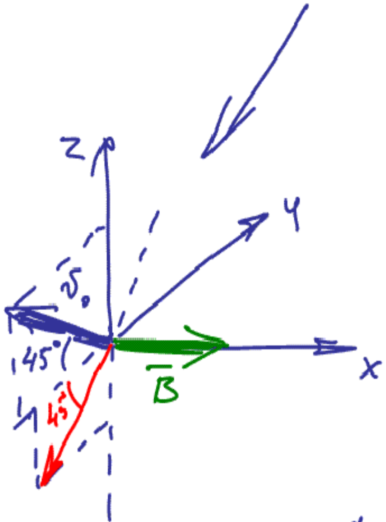
$$\boxed{\bar{F}_B = q\,\bar{v}\times\bar{B}}$$



Right-hand rule gives direction of $\bar{v}\times\bar{B}$ along z. However, since $q<0$ the direction of $\bar{F}_B$ – is opposite: $\bar{F}_B = -q v B\cdot\hat{k}$

Part 2

Let us use more rigorous way to solve:



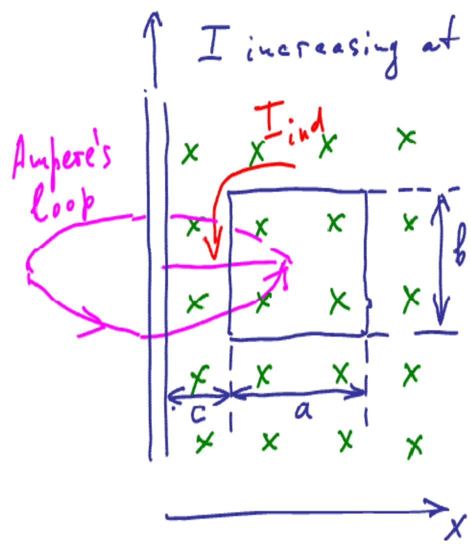
$$\bar{F}_B = -q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$= -q \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \left(-\frac{v_0}{\sqrt{2}}\right) & \left(\frac{v_0}{\sqrt{2}}\right) \\ B & 0 & 0 \end{vmatrix} =$$

$$q\left[-\hat{i}\begin{vmatrix} \left(-\frac{v_0}{\sqrt{2}}\right) & \left(\frac{v_0}{\sqrt{2}}\right) \\ 0 & 0 \end{vmatrix} + \hat{j}\begin{vmatrix} 0 & \left(\frac{v_0}{\sqrt{2}}\right) \\ B & 0 \end{vmatrix} - \hat{k}\begin{vmatrix} 0 & \left(-\frac{v_0}{\sqrt{2}}\right) \\ B & 0 \end{vmatrix}\right] = \left[0 - B\frac{v_0}{\sqrt{2}}\cdot\hat{j} - B\frac{v_0}{\sqrt{2}}\hat{k}\right]\cdot q = \frac{q v_0}{\sqrt{2}} B\left(-\hat{j} - \hat{k}\right) =$$

$$= \frac{|e| \cdot v_0 \cdot B}{\sqrt{2}}\left(-\hat{j} - \hat{k}\right).$$

Problem 4 (33.31 from Knight)



A 2.0 cm x 2.0 cm square loop of wire with $R = 0.01\,\Omega$ is parallel to a long straight wire.

I_{wire} increasing at 100 A/s

What is the current in the loop?

$\bar{B}$ in the sketch (x) is due to I_{wire}.

Since $I_{wire} \uparrow \Rightarrow$ flux $\Phi \uparrow \Rightarrow \mathcal{E}$ and I_{ind}

Magnitude: $I_{ind} = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right|$

We used Faraday's Law: $\mathcal{E} = \left|\frac{d\Phi}{dt}\right|$

Direction of I_{ind} is counter clockwise due to Lenz's Law.

Continue calculation of the magnitude....

$$\Phi = \int_{Loop} \bar{B} \cdot d\bar{A} \qquad (1)$$

B – magn. field created by long straight wire

B can be found using Ampere's Law:

$$\oint \bar{B}\cdot d\bar{s} = \mu_0 \cdot I_{wire}$$

Line integral

$$\boxed{\bar{B} \parallel d\bar{s} \Rightarrow \cos\varphi = 1 \Rightarrow \bar{B}\cdot d\bar{s} = B(x)\cdot ds}$$

$$\oint \bar{B}\cdot d\bar{s} = B(x)\cdot \oint ds = B(x)\cdot 2\pi x$$

$$B(x)\cdot 2\pi x = \mu_0 \cdot I_{wire}$$

$$\boxed{B(x) = \frac{\mu_0}{2\pi}\frac{I_{wire}}{x}} \Leftarrow$$ Let us substitute it in (1)

$dA = b\cdot dx$ _ Let us substitute it in (1)

$$\Phi = \oint_{Loop} \bar{B}\cdot d\bar{A} = \int_{Loop} \frac{\mu_0}{2\pi}\frac{I_{wire}}{x} b\cdot dx =$$

$$\boxed{\text{Since } \bar{B} \parallel d\bar{A} \Rightarrow \bar{B}\cdot d\bar{A} = B\cdot dA}$$

$$= \frac{\mu_0 \cdot b\cdot I_{wire}}{2\pi}\int_c^{c+a}\frac{dx}{x} = \frac{\mu_0\cdot b\cdot I_{wire}}{2\pi}\ln x\Big|_c^{c+a} =$$

$$= \frac{\mu_0\cdot b\cdot I_{wire}}{2\pi}\ln\left(\frac{c+a}{a}\right)$$

$$I_{ind} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\frac{\mu_0\cdot b}{2\pi}\ln\left(\frac{c+a}{a}\right)\frac{dI}{dt} =$$

$$\frac{(4\pi\times 10^{-7}\,T\cdot m/A)\cdot(0.02\,m)}{(0.01\,\Omega)\cdot 2\pi}\ln\left(\frac{0.03}{0.01}\right)\left(100\,\frac{A}{s}\right) = \underline{\underline{43.9\,\mu A}}$$

Problem 5 (From a different text)

Radiation pressure from the Sun reaching Earth (just outside the atmosphere) has an intensity of $1.4\ kW/m^2$.

(a) Assuming that Earth (and atmosphere) behaves like a flat disk perpendicular to the Sun's rays and that all the incident energy is absorbed, calculate the force on Earth due to radiation pressure

(b) Compare it with the force due to the Sun's gravitational attraction.

(a) $$F_r = p_r\cdot A = p_r\cdot(\pi R_e^2) = \frac{I}{c}(\pi\cdot R_e^2) =$$

$p_r = \frac{I}{c}$ _ true in the case of total absorption, see 34.7 from Knight

R_e (radius of Earth) $= 6.37\times 10^6\ m$

$$= \frac{\pi\cdot(1.4\times 10^3\,W/m^2)\cdot(6.37\times 10^6\,m)^2}{3\cdot 10^8\,m/c} = \underline{\underline{6.0\times 10^8\,N}}$$

The force produced by the Sun's radiation pressure is significant. Can it be however compared with the gravitational force?

(b) $F_{grav} = G \frac{m_s \cdot m_e}{d^2} =$

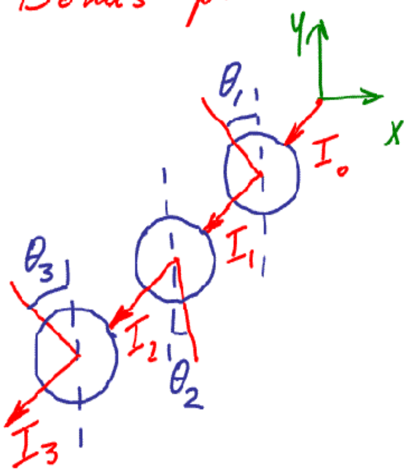
m_s (mass of Sun) $= 2 \cdot 10^{30}$ kg
m_e (mass of Earth) $= 5{,}98 \cdot 10^{24}$ kg
d (distance from Sun to Earth) $= 1{,}5 \cdot 10^{11}$ m

$$= \frac{(6.67 \cdot 10^{-11}\ N \cdot m^2/kg^2) \cdot (2 \cdot 10^{30}\ kg) \cdot (5{,}98 \cdot 10^{24}\ kg)}{(1.5 \cdot 10^{11}\ m)^2} =$$

$$= \underline{\underline{3.6 \times 10^{22}\ N}}$$

The gravitational force is 14 orders of magnitude larger.

Bonus problem (From a different text)



Unitially unpolarized light is sent through three polarizing sheets whose polarization directions make angles:

$\theta_1 = 40^\circ$, $\theta_2 = 20^\circ$ and $\theta_3 = 40^\circ$

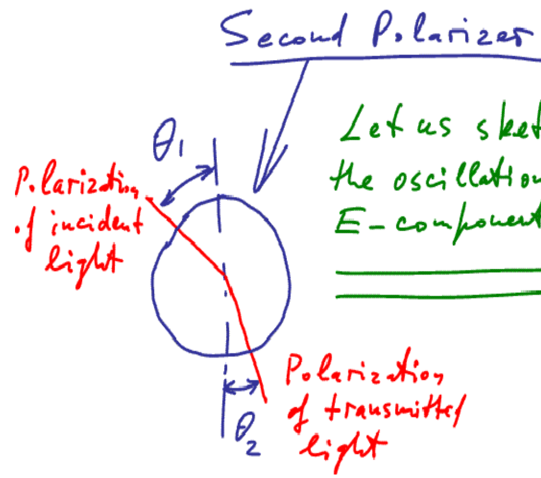
with the direction of the y-axis.

What percentage of the light's intial intensity (I_0) is transmittey by the system?

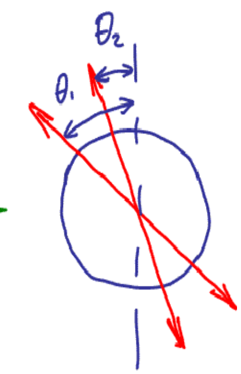
First Malus's Law:

$I_1 = \frac{1}{2} I_0$

Second Malus's Law: $I_2 = I_1 \cdot \cos^2\theta$
What is θ?



Let us sketch the oscillations of E-component of wave



θ – is the angle between the directions of oscillations of E-component in the incident and transmitted waves.

It is seen that $\theta = \theta_1 - \theta_2 = 40^\circ - 20^\circ = 20^\circ$

Show using similar arguments that:

$I_3 = I_2 \cos^2\theta$, where $\theta = \theta_3 - \theta_2 = 40^\circ - 20^\circ = 20^\circ$

$$I_3 = I_2 \cdot \cos^2(20^\circ) = I_1 \cdot \cos^4(20^\circ) = \frac{I_0}{2} \cos^4(20^\circ)$$

$$\underline{\underline{\frac{I_3}{I_0} = \frac{\cos^4(20^\circ)}{2}}}$$