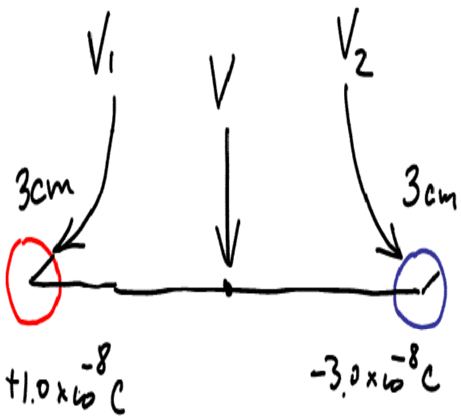
Review for Exam 1

Problem 1

(Source is not the Night's Text)



Two metal spheres, each of radius 3.0 cm, have a center-to-center separation of 2.0 m. One has charge of $+1.0\times10^{-8}$ C; the other has a charge of -3.0×10^{-8} C. Assume that the separation is large enough relative to the size of spheres to permit us to consider the charge on each to be uniformly distributed (the spheres do not affect each other). With $V=0$ at infinity, calculate:

a) The potential at the point halfway between their centers

b) The potential of each sphere.

(a) The electric potential is a sum of contributions of individual spheres. Let q_1 be the charge on the left sphere and q_2 be the charge on the other, and d be their separation.

The point halfway is separated by the distance $d/2 = 1.0$ m from the center of each sphere.

$$V = k\frac{q_1}{(d/2)} + k\frac{q_2}{(d/2)} = k\frac{2(q_1+q_2)}{d} =$$

$$= 9\times10^{9}\,\frac{2(1.0\times10^{-8} - 3.0\times10^{-8})}{2} = -1.8\times10^{2}\,V$$

(b) The distance from the center of one sphere to the surface of the other is $(d-R)$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is:

$$V_1 = k\frac{q_1}{R} + k\frac{q_2}{d-R} = k\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] =$$

$$= 9\times10^{9}\left[\frac{1.0\times10^{-8}}{0.03} - \frac{3.0\times10^{-8}}{2.0-0.03}\right] = \underline{\underline{2.9\times10^{3}\ V}}$$

$$V_2 = k\frac{q_1}{d-R} + k\frac{q_2}{R} = k\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] =$$

$$9\times10^{9}\left[\frac{1.0\times10^{-8}}{2.0-0.03} - \frac{3.0\times10^{-8}}{0.03}\right] = \underline{\underline{-8.9\times10^{3}\ V}}$$

Problem 2

(Source is not the Night's Text)

A particle of charge q is fixed at point P, and a second particle of mass m and the same charge q is initially held a distance r_1 from P. The second particle is then released. Determine its speed when it is a distance r_2 from P. Let $q = 3.1\,\mu C$, $m = 20\,mg$, $r_1 = 0.9\,mm$, and $r_2 = 2.5\,mm$.

The main concept of this problem is energy conservation: $E_i = E_f$.

$E_i = U_i + K_i$, where $K_i = 0$ since $v_i = 0$

$$U_i = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{r_1} > 0$$

$E_f = U_f + K_f$, where $K_f = \dfrac{m v_f^2}{2}$

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{r_2}$$

Conservation of energy ($E_i = E_f$) yields

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_2} + \frac{m v_f^2}{2}$$

$$v_f = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{K \cdot 2 \cdot q^2}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} =$$

$$\boxed{\text{we used here } \frac{1}{4\pi\varepsilon_0} = K, \text{ where } K = 9 \times 10^9\, N \cdot m^2/C^2}$$

$$= \sqrt{\frac{(9\times10^9)\times 2\times(3.1\times10^{-6})^2}{20\times10^{-6}}\left(\frac{1}{0.9\times10^{-3}} - \frac{1}{2.5\times10^{-3}}\right)} =$$

$$= 2.5 \times 10^3\, \frac{m}{s}$$

Problem 3

The sample is Problem 29-70 from Night

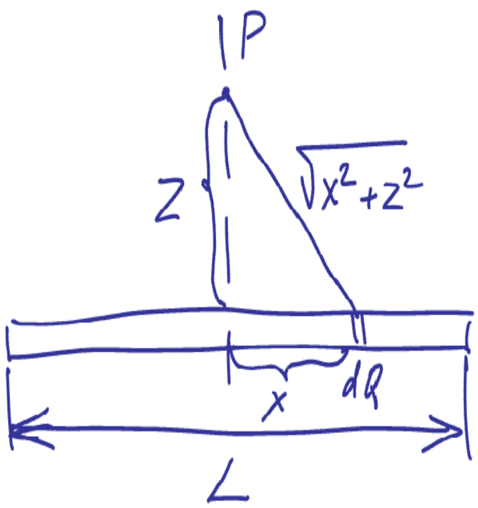


Figure shows a thin rod of length L and charge Q. Find V a distance z away from the center of the rod on the line that bisects the rod

A little charge dQ with the size dx exerts a little potential dV at P:

$$dV = k\frac{dQ}{r} = k\frac{\lambda \cdot dx}{\sqrt{x^2+z^2}}.$$

$$V(P) = \int dV = k\lambda \int_{-\frac{L}{2}}^{L/2} \frac{dx}{\sqrt{x^2+z^2}} =$$

$$\boxed{\text{The standard integral: } \int \frac{dx}{\sqrt{x^2+z^2}} = \ln\left(\sqrt{x^2+z^2}+x\right)}$$

$$= k\lambda\left[\ln\left(\sqrt{\left(\tfrac{L}{2}\right)^2+z^2}+\tfrac{L}{2}\right) - \right.$$

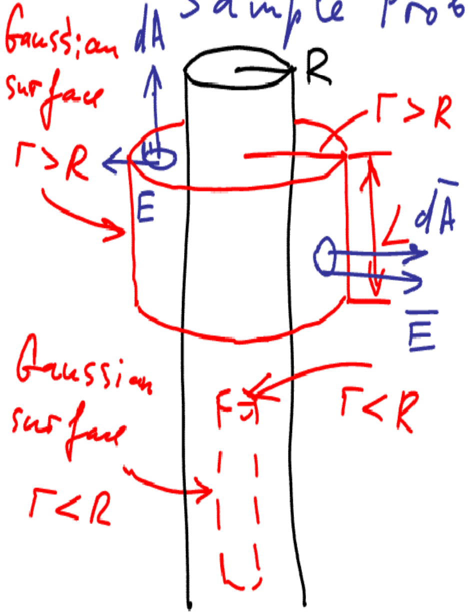
$$\left. - \ln\left(\sqrt{\left(\tfrac{L}{2}\right)^2+z^2}-\tfrac{L}{2}\right)\right] =$$

$$= k\lambda\left[\ln\left(\frac{\sqrt{\left(\frac{L}{2}\right)^2+z^2}+\frac{L}{2}}{\sqrt{\left(\frac{L}{2}\right)^2+z^2}-\frac{L}{2}}\right)\right] =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\ln\left(\frac{\sqrt{\left(\frac{L}{2}\right)^2+z^2}+\frac{L}{2}}{\sqrt{\left(\frac{L}{2}\right)^2+z^2}-\frac{L}{2}}\right)$$

Problem 4

Sample Problem 28-52 from Night



A very long, uniformly charged cylinder has radius R and linear charge density λ.

Find $\bar{E}$:

(a) Outside the cylinder $r \geq R$

(b) Inside the cylinder $r < R$

(c) Show that your answers to part "a" and "b" match at $r=R$

The black cylinder is represented by a continuous material charged with the uniform density $\rho = \frac{Q}{V}\left[\frac{C}{m^3}\right]$. However, in this problem we are given a linear density $\lambda = \frac{Q}{L}\left[\frac{C}{m}\right]$.

(a) For $r \geq R$: Gauss' Law $\rightarrow \oint \bar{E} \cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0}$

$$\oint \bar{E} \cdot d\bar{A} = \oint_{Bases} \bar{E} \cdot d\bar{A} + \oint_{Sidewall} \bar{E} \cdot d\bar{A}$$

$\oint_{Bases} \bar{E} \cdot d\bar{A} = 0$ since $\bar{E} \perp d\bar{A}$ as shown in the Figure: $\bar{E} \cdot d\bar{A} = E \cdot dA \cdot \cos 90° = 0$

$$\oint_{Sidewall} \bar{E} \cdot d\bar{A} = \int_{Sidewall} E \cdot dA = E \cdot \oint_{Sidewall} dA = \underline{\underline{E \cdot 2\pi r \cdot L}}$$

Along sidewall surface $\bar{E} \parallel d\bar{A} \Rightarrow$

$$\bar{E} \cdot d\bar{A} = E \cdot dA \cdot \cos 0° = E \cdot dA$$

In addition E is a const for given r.

$$q_{enc} = \lambda \cdot L \Rightarrow$$

$$E \cdot 2\pi \cdot r \cdot L = \frac{\lambda L}{\varepsilon_0} \quad \text{— Gauss' Law}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}, \qquad \underline{\underline{\bar{E} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \cdot \hat{r}}}$$

(b) For $r < R$ Let us use same approach, but for a small red dashed cylinder:

$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot A = \frac{q_{enc}}{\varepsilon_0}.$$ How to calculate q_{enc}?

$\rho = \frac{Q}{V}$ Let us select a part of the real (black cylinder with height L)

For such cylinder $\left.\begin{array}{l} Q = \lambda \cdot L \\ V = \pi R^2 \cdot L \end{array}\right\} \Rightarrow$

$$\rho = \frac{\lambda \cdot L}{\pi \cdot R^2 \cdot L} = \frac{\lambda}{\pi \cdot R^2}$$

$q_{enc} = \rho \cdot V_r$, where V_r is the volume of the small red cylinder

$$V_r = \pi r^2 \cdot L$$

$$q_{enc} = \frac{\lambda}{\pi \cdot R^2} \cdot \pi \cdot r^2 \cdot L = \lambda \cdot L \left(\frac{r}{R}\right)^2$$

Back to Gauss' Law for a small red cylinder

$$E \cdot 2\pi r \cdot L = \frac{\lambda \cdot L \left(\frac{r}{R}\right)^2}{\varepsilon_0} \Rightarrow$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{r}{R^2} \quad \text{or} \quad \bar{E} = \frac{1}{2\pi\varepsilon_0} \frac{r}{R^2} \cdot \hat{r}$$

(c) Easy to check that at $r = R$ both solutions give $E = \frac{1}{2\pi\varepsilon_0} \frac{1}{R}$.

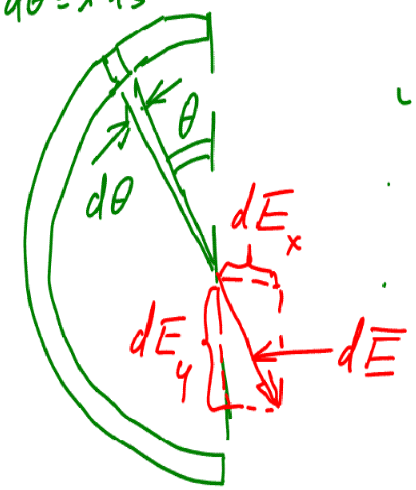
Another important geometry is spherical.. Study Problem 28-38"

Also, study potential of the charged sphere at page 898

Problem 5

Sample Problem 27-46

$d\theta = \lambda \cdot ds$



Charge Q is uniformly distributed along a thin, flexible rod of length L. The rod is then bent into a semicircle

Find $\bar{E}$ at the center P

Hint: $ds = R \cdot d\theta$

$$dE = k\frac{dQ}{R^2} = k\frac{\lambda \cdot ds}{R^2} = k\frac{\lambda \cdot R \cdot d\theta}{R^2} = k\frac{\lambda}{R}d\theta$$

$$\begin{cases} dE_x = (dE) \cdot \sin\theta \\ dE_y = (dE) \cdot \cos\theta \end{cases}$$

$$E_x = \int dE_x$$

$E_y = \int dE_y = 0$ – from symmetry

For each element $d\bar{s}$ we can find another, symmetrically located element which will produce dE_y with the same magnitude, but opposite sign.

That is why $\int dE_y = 0$.

Let us consider Semiring

$$E_x = \int dE_x = \frac{k\lambda}{R}\int_0^{\pi} \sin\theta \cdot d\theta =$$

$$= \frac{k\lambda}{R}(-\cos\theta)\Big|_0^{\pi} = \frac{k\lambda}{R}[1-(-1)] = \frac{2k\lambda}{R} =$$

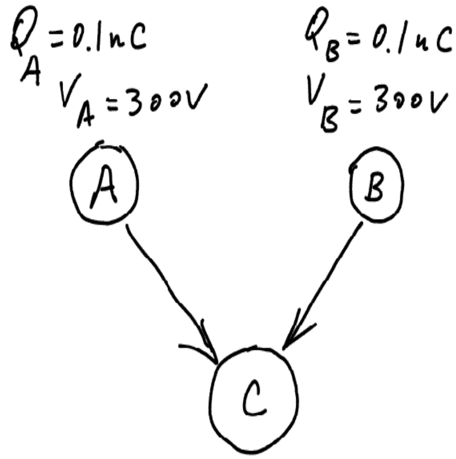
$$\boxed{k = \frac{1}{4\pi\varepsilon_0},\quad \lambda = \frac{Q}{L},\quad L = \pi \cdot R}$$

$$= \frac{2 \cdot Q \cdot \pi}{4\pi\varepsilon_0 \cdot L \cdot L} = \frac{1}{2\varepsilon_0}\frac{Q}{L^2}.$$

$$\bar{E} = \frac{1}{2\varepsilon_0}\frac{Q}{L^2} \cdot \hat{i}$$

Problem 6

Sample Problem 29-62



Two spherical droplets of mercury each have a charge of 0.1nC and a potential of 300V at the surface. The two droplets merge to form a single droplet. What is the potential at the surface of the new droplet?

Two main laws:

1. Charge conservation

$$Q_A + Q_B = Q_C = 0.2nC$$

2. Volume concervation

$$\left[Vol_C = \frac{4}{3}\pi R_c^3\right] = \left[Vol\,A + Vol\,B = \frac{4}{3}\pi R_A^3 + \frac{4}{3}\pi R_B^3\right]$$

$$R_c^3 = R_A^3 + R_B^3$$

$$R_c = \left(R_A^3 + R_B^3\right)^{1/3}$$

Another concept we need to use is a surface potential of a sphere:

$$V = k\frac{Q}{R} \Rightarrow V_A = k\frac{Q_A}{R_A} \Rightarrow$$

$$R_A = k\frac{Q_A}{V_A} = \frac{(9\times10^{9})\times(0.1\times10^{-9})}{300} = 3\times10^{-3}\,m$$

$$R_A = R_B$$

From the Eq.

$$R_c = \left(R_A^3 + R_B^3\right)^{1/3} = 3.8\times10^{-3}\,m$$

$$V_c = k\frac{Q_c}{R_c} = \frac{(9\times10^{9})\times(0.2\times10^{-9})}{3.8\times10^{-3}} = \underline{\underline{470V}}$$