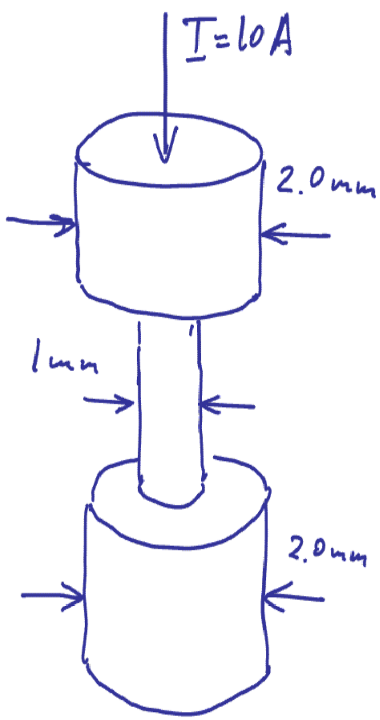
Review for Exam 2

PHYS 2102 Fall 2009

Problem 1

Preparation
- Problem 3 from Solution for Exam 2 in Fall 2008
- 31-57 — you can solve it yourself as a preparation
- 31-60 — see below

31-60



An Al wire consists of the three segments shown in figure. The current in the top segment is 10A. For each of these three segments, find:

a) Current I

b) Current density J

c) Electric field E

d) Drift velocity

e) Mean time between collisions

f) Electron current i.

(a) Current is conserved $I_{top} = I_{mid} = I_{bot} = 10A$

(b) $J = \frac{I}{A} = \frac{I}{\pi R^2} \Rightarrow$

$$J_{top} = J_{bot} = \frac{10\,A}{\pi(0.001m)^2} = 3.18\times10^{6}\,A/m^2$$

$$J_{mid} = \frac{10\,A}{\pi(0.0005m)^2} = 1.27\times10^{7}\,A/m^2$$

(c) $J = \sigma E \Rightarrow E = \frac{J}{\sigma}$.

$$E_{top} = E_{bot} = \frac{J_{top}}{\sigma} = \frac{3.18\times10^{6}\,\frac{A}{m^2}}{3.5\times10^{7}\,\Omega^{-1}m^{-1}} = 0.091\,\frac{V}{m}$$

$$E_{mid} = \frac{J_{mid}}{\sigma} = \frac{1.27\times10^{7}\,A/m^2}{3.5\times10^{7}\,\Omega^{-1}m^{-1}} = 0.364\,\frac{V}{m}$$

(d) $J = en v_d \Rightarrow v_d = J/ne$

$$(v_d)_{top} = (v_d)_{bot} = \frac{J_{top}}{ne} = \frac{3.18\times10^{6}\,A/m^2}{(6.0\times10^{28}\,m^{-3})(1.6\times10^{-19}C)} = 3.31\times10^{-4}\,\frac{m}{s}$$

$$(v_d)_{mid} = \frac{J_{mid}}{ne} = \frac{1.27\times10^{7}\,A/m^2}{(6.0\times10^{28}\,m^{-3})(1.6\times10^{-19}C)} = 1.33\times10^{-3}\,\frac{m}{s}$$

(e) $\tau = \frac{m v_d}{eE} \Rightarrow$

$$\tau_{top} = \tau_{bot} = \frac{m(v_d)_{top}}{eE_{top}} = \frac{(9.11\times10^{-31}kg)(3.31\times10^{-4}\frac{m}{s})}{(1.6\times10^{-19}C)(0.091\,V/m)} = 2.07\times10^{-14}s$$

$$\tau_{mid} = \frac{m(v_d)_{mid}}{eE_{mid}} = \frac{(9.11\times10^{-31}kg)(1.33\times10^{-3}\frac{m}{s})}{(1.6\times10^{-19}C)(0.364\,V/m)} = 2.07\times10^{-14}s$$

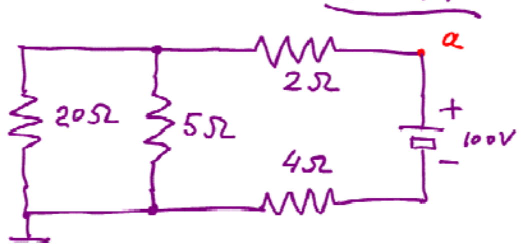
The mean times between the collisions are the same in all three segments because they are determined by thermal motion of electrons which is only weakly perturbed by the electric field.

(f) — We will not consider electron current. We consider only current I.

Problem 2

Preparation
- Problem 32-56 — Solved in Review for Exam 2 In Spring 2007 (problem 2)
- Problem 32-62 — You can solve it yourself as a preparation
- Problem 32-64 — see below

32-64



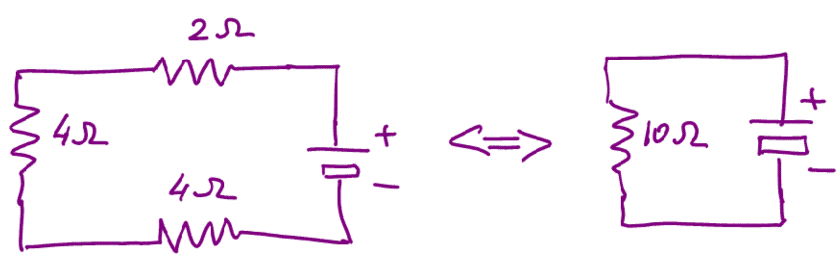
Find:

a) I through 2Ω resistor

b) The power dissipated by the 20Ω resistor

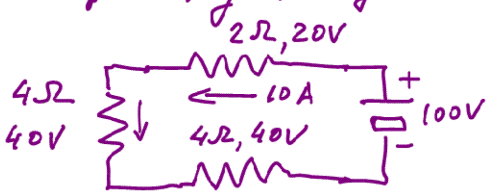
c) The potential at point a

$$\frac{1}{R} = \frac{1}{20} + \frac{1}{5} = \frac{1+4}{20} = \frac{5}{20} = \frac{1}{4} \Rightarrow R = 4\Omega$$



$$I = \frac{100V}{10\Omega} = 10\,A$$

Now reconstruct the original circuit step by step using the fact that $I = 10\,A$



$\Delta V_2 = (10A)\cdot(2\Omega) = 20V$

$\Delta V_4(\text{left}) = (10A)\cdot(4\Omega) = 40V$

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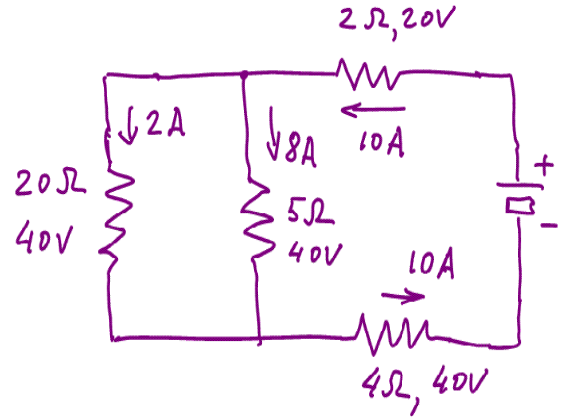


Two resistors, 20Ω and 5Ω, must have the same potential difference $\Delta V = \Delta V_4(\text{left}) = 40V$

$$I_5 = \frac{40V}{5\Omega} = 8A$$

$$I_{20} = \frac{40V}{20\Omega} = 2A$$

(b) The power dissipated by the 20Ω resistor is $I_{20}^2(20\Omega) = (2)^2(20) = 80W$

(c) To find $V(a)$ let us start at the left-bottom corner with $V=0$ and move clockwise.

Two rules:

1. If you move with the current through the resistor, the potential is reduced by $-IR$.
2. If you traverse the battery from minus to plus, the potential is increased by $\mathcal{E}$.

$$0V - (10A \times 2\Omega) + 100V = \underline{\underline{60V}} = V_a$$

Alternatively, you can move counterclockwise:

$$0V + (10A \times 4\Omega) + (10A \times 2\Omega) = \underline{\underline{60V}} = V_a$$

Same result.

Problem 3

Problem 2 in Solution for Exam 2 in Fall 2008

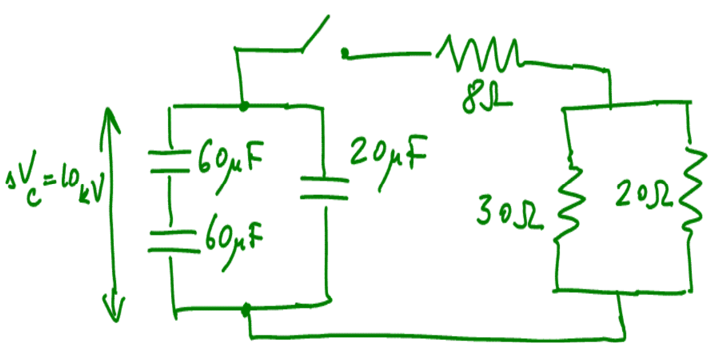
Problem 4

Preparation

32-72 – You can solve it yourself as a preparation

32-76 – see below

32-76



The capacitors are charged and the switch is closed at $t = 0\,s$.

At what time has the current in the 8Ω resistor decays to a half the value it had immediately after the switch was closed?

Capacitors in series

1. $\frac{1}{C} = \frac{1}{60} + \frac{1}{60} = \frac{2}{60} = \frac{1}{30} \Rightarrow C = 30\,\mu F$

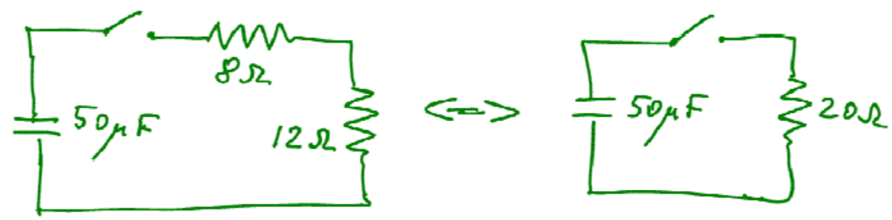
Capacitors in parallel

$30\,\mu F + 20\,\mu F = 50\,\mu F$

Resistors in parallel:

$\frac{1}{R} = \frac{1}{30} + \frac{1}{20} = \frac{2+3}{60} = \frac{5}{60} = \frac{1}{12} \Rightarrow R = 12\,\Omega$

Equivalent circuit



Discharge process:

$Q_c = Q_0 \cdot e^{-\frac{t}{\tau}}$, where $\tau = R_{eq} \cdot C_{eq}$

$\tau = R_{eq} \cdot C_{eq} = (20\,\Omega) \cdot (50\,\mu F) = 10^3 \times 10^{-6}\,s = 10^{-3}\,s = \underline{\underline{1\,ms}}$

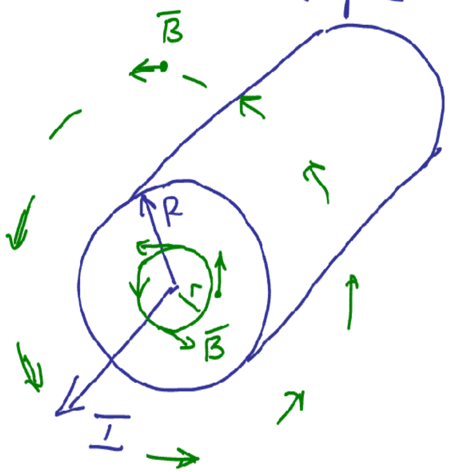
$I = I_0 \cdot e^{-\frac{t}{\tau}}$, $I(t_1) = I_0/2$.

$I_0/2 = I_0 \cdot e^{-\frac{t_1}{\tau}} \Rightarrow \ln 0.5 = -\frac{t_1}{\tau} \Rightarrow \underline{\underline{t_1 = 0.69\,ms}}$

Problem 5

Ampere's Law
Example 33.8

> Additionally, you can study Problem 5 in Review for Spring 2008. However this is too advanced level.



A wire of radius R carries current I.
Find $\bar{B}$ inside at distance $r<R$ from the axis.
The current density is uniform.

$j = \frac{I}{A_0}$, $\quad \bar{B}(r) - ?$ $\quad r<R$

Ampere's Law:

$$\oint \bar{B}\cdot d\bar{s} = \mu_0 \cdot I_{through}, \quad I_{through} - ?$$

$$j = \frac{I}{A_0} = \frac{I}{\pi R^2}.$$

$$I_{through} = j \times A_{green\ circle} = j \times \pi r^2 =$$

$$= \frac{I}{\pi R^2}\pi r^2 = I\left(\frac{r}{R}\right)^2.$$

$$\bar{B}\cdot d\bar{s} = |\bar{B}|\cdot|d\bar{s}|\cdot\cos\varphi = B\cdot ds.$$

$$\varphi = \measuredangle\, \bar{B}, d\bar{s}, \quad \varphi = 0^\circ \Rightarrow \cos\varphi = 1$$

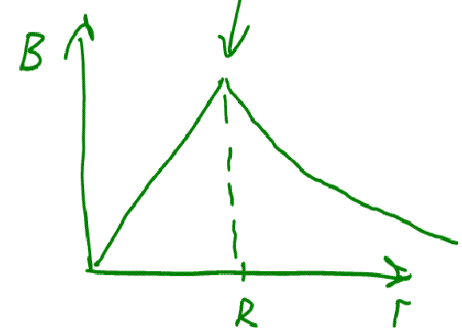
$$\oint \bar{B}\cdot d\bar{s} = \oint B\cdot ds = B\oint ds = B\cdot 2\pi r$$

Green loop

$$B\cdot 2\pi r = \mu_0\cdot I\left(\frac{r}{R}\right)^2$$

$$B = \frac{\mu_0 I r}{2\pi\cdot R^2}$$

$$B = \frac{\mu_0 I}{2\pi\cdot R}$$



$r > R$ – outside the wire

$$\oint \bar{B}\cdot d\bar{s} = \mu_0\cdot I_{through}, \quad I_{through} = I$$

$$\bar{B}\cdot d\bar{s} = |\bar{B}|\cdot|d\bar{s}|\cdot\cos\varphi = B\cdot ds$$

$$\oint B\cdot ds = B\oint ds = B\cdot 2\pi r$$

$$B\cdot 2\pi r = \mu_0\cdot I$$

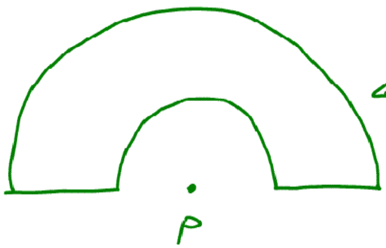
$$B = \frac{\mu_0 I}{2\pi r}.$$

At the surface of the wire $r = R$

Problem 6

33-46 Solved as Problem 3
in Review for Fall 2006

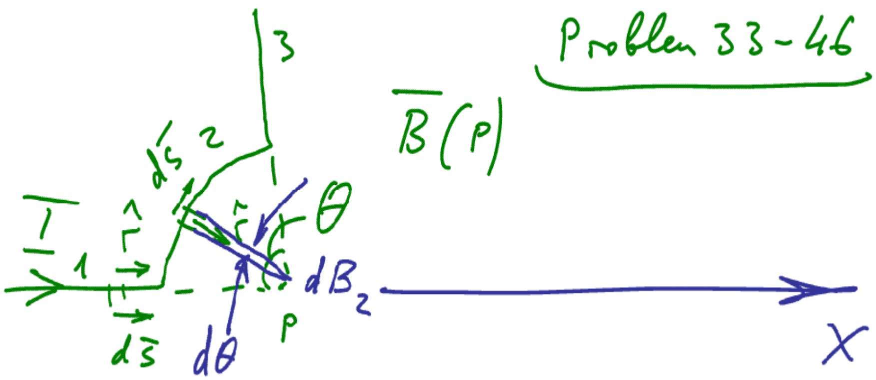
33-53 Solved as Problem 6
in Solution for Exam 2 Fall 2008



Take a look
a this

Problem 33-46

$\overline{B}(P)$



$$\overline{B}(P) = \overline{B}_1(P) + \overline{B}_2(P) + \overline{B}_3(P).$$

$$\overline{B}_1(P) = 0$$

Biot-Savart Law:

$$d\overline{B} = \frac{\mu_0}{4\pi} \frac{(I \cdot d\bar{s}) \times \hat{r}}{r^2},$$

$$d\bar{s} \parallel \hat{r}$$

$$d\bar{s} \times \hat{r} = 0$$

Each $dB_1 = 0 \Rightarrow B_1 = \int dB_1 = 0$

$$|d\bar{s} \times \hat{r}| = |d\bar{s}| \cdot |\hat{r}| \cdot \sin\theta.$$

$$d\bar{s} \cdot \hat{r} = |d\bar{s}| \cdot |\hat{r}| \cdot \cos\theta$$

$$B_3 = B_1 = 0.$$

$$\boxed{\sin\theta = \sin 90° = 1}$$

$$\boxed{|\hat{r}| = 1}$$

$$dB_2 = \frac{\mu_0}{4\pi} \frac{(I \cdot d\bar{s}) \times \hat{r}}{r^2}$$

$$|d\bar{s} \times \hat{r}| = |d\bar{s}| \cdot |\hat{r}| \cdot \sin\theta = \underline{ds}.$$

$$dB_2 = \frac{\mu_0}{4\pi} \frac{I \cdot ds}{r^2} =$$

$$\boxed{ds = r \cdot d\theta}$$

$$= \frac{\mu_0}{4\pi} \frac{I \cdot r}{r^2} d\theta = \frac{\mu_0}{4\pi} \frac{I}{r} d\theta$$

$$B_2(\text{in page}) = \int_{\text{Arc}} dB_2 = \int_0^{\theta} \frac{\mu_0}{4\pi} \frac{I}{r} d\theta =$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r} \theta \Big|_0^{\theta} = \frac{\mu_0}{4\pi} \frac{I}{r} \theta$$

You can measure the angle θ from $+X$-axis. It gives the same result.

$-\pi + \theta$

$$B_2 = \int_{\text{Arc}} dB_2 = \int_{-\pi}^{-\pi+\theta} \frac{\mu_0}{4\pi} \frac{I}{r} d\theta =$$

$$\frac{\mu_0}{4\pi} \frac{I}{r} \theta \Big|_{-\pi}^{-\pi+\theta} =$$

$$\frac{\mu_0}{4\pi} \frac{I}{r} (\cancel{-\pi} + \theta + \cancel{\pi}) = \underline{\underline{\frac{\mu_0}{4\pi} \frac{I}{r} \theta.}}$$