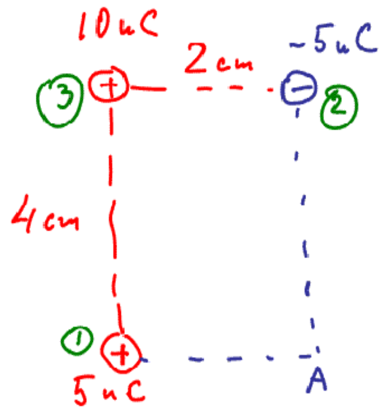
The focused preparation for Final Exam is provided through solving problems in two sources:

1) Review for Exam in Spring 2007: Final. You can get this review at my website.

2) A few additional problems solved below. Both sources have equal importance. In order to provide better preparation you can find analogous problems at the end of each Chapter.

Problem 29-29



a) What is the electric potential at point A?

b) What is the potential energy of a proton at point A?

Let $q_1 = +5\,\mu C$, $q_2 = -5\,\mu C$, $q_3 = 10\,nC$. Also, $r_1 = 2\,cm$, $r_2 = 4\,cm$, and $r_3 = \sqrt{(2cm)^2 + (4cm^2)} = 4.47\,cm$

Potential is a scalar, not a vector, so the net potential is simply the sum of the potentials of each of the charges

$$V_A = V_1 + V_2 + V_3 = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} =$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right) =$$

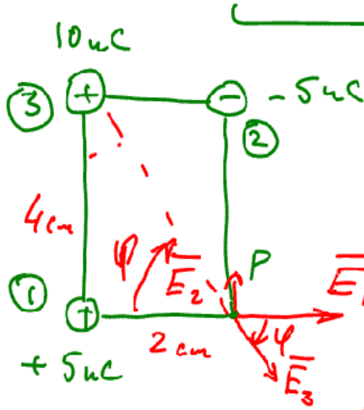
$$= (9\times10^9)\cdot\left(\frac{5\times10^{-9}}{0.02} + \frac{-5\times10^{-9}}{0.04} + \frac{10\times10^{-9}}{0.0447}\right) = \underline{\underline{3140\,V}}$$

b) The potential energy of a proton at point A is

$$U_{proton} = q_{proton}\cdot V_A = eV_A =$$

$$= (1.6\times10^{-19})\cdot(3140) = \underline{\underline{5.02\times10^{-16}}}\ [J]$$

Problem 26-31



$\vec E(P)$ - ?
a/ Comp. form
b/ Magnitude and direction

$V(P)$ - ?

$$\vec E_{net} = \vec E_1(P) + \vec E_2(P) + \vec E_3(P)$$

$$V_{net}(P) = V_1(P) + V_2(P) + V_3(P)$$

$$|\vec E_1| = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{K} \frac{q_1}{r_1^2} = \frac{9\times10^9(5\times10^{-9})}{(0.02)^2} = 112,500 \frac{N}{C}$$

$$\vec E_1 = 112,500\cdot\hat i \left[\frac{N}{C}\right]$$

$$|\vec E_2| = 28,120 \frac{N}{C}$$

$$\vec E_2 = 28,120\cdot\hat j \frac{N}{C}$$

$$|\vec E_3| = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3^2} = \frac{(9\times10^9)\cdot(10\cdot10^{-9})}{(0.02)^2+(0.04)^2} = 45,000 \frac{N}{C}$$

$$\tan\varphi = \frac{4}{2} \Rightarrow \varphi = \tan^{-1}\left(\frac{4}{2}\right) = 63.43^\circ$$ - with x-axis

$$\vec E_3 = |\vec E_3|\cdot\cos\varphi\cdot\hat i - |\vec E_3|\cdot\sin\varphi\cdot\hat j =$$

$$(20,130\,\hat i - 40,250\,\hat j)\ \frac{N}{C}$$

$$\vec E_{net} = \vec E_1 + \vec E_2 + \vec E_3 = (132,600\,\hat i - 12,130\cdot\hat j)\frac{N}{C}$$

$$|\vec E_{net}| = \sqrt{E_{x,net}^2 + E_{y,net}^2} = \sqrt{(132,600)^2 + (-12,130)^2}$$

$$= 133,200 \frac{N}{C}$$

Direction of $\vec E_{net}$ is given by angle θ

$$\theta = \tan^{-1}\left(\frac{E_x}{E_y}\right) = 5.23^\circ$$

$E_{net} = 133,200 \frac{N}{C}$, $\theta = 5.23^\circ$ - below the x-axis

$V(P)$ - ?

$$V(P) = V_1(P) + V_2(P) + V_3(P)$$

$$\boxed{V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

$$V(P) = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Pay attention to the sign of q_i

Problem 26-48

Charge Q is uniformly distributed along L

$\vec{E}$ - ?

$$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2} =$$

$$\boxed{dq = \lambda\cdot ds = \underbrace{\frac{Q}{L}}_{\lambda}\cdot\underbrace{R\cdot d\varphi}_{ds}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\frac{\cancel{R}\cdot d\varphi}{R^{\cancel{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{LR}\cdot d\varphi.$$

$$dE_x = |d\vec{E}|\cdot\sin\varphi$$

$$E_x = \int_{1}^{2} dE_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L\cdot R}\int_0^{\pi}\sin\varphi\, d\varphi$$

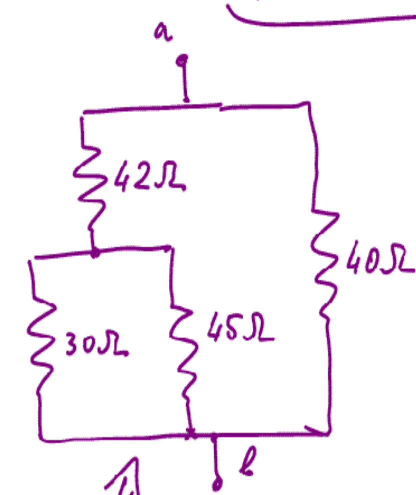
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(-\cos\varphi)}{L\cdot R}\Big|_0^{\pi} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{LR}\left(\underbrace{-(-1)+1}_{2}\right) =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2Q}{L\cdot R}.$$

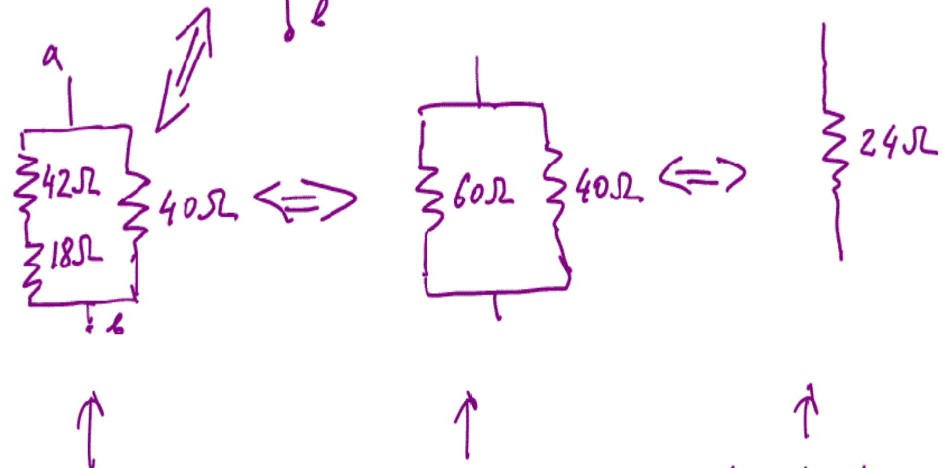
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{L\cdot R}\right)\cdot\hat{i}$$

Since $R = \frac{L}{\pi}$ => $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\pi Q}{L^2}\right)\hat{i} = 1.7\times10^{5}$ [N/C]

Problem 31-31



What is the equivalent resistance between points "a" and "b" in Figure?



$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{45}$ =>

$R_{eq} = 18\Omega$

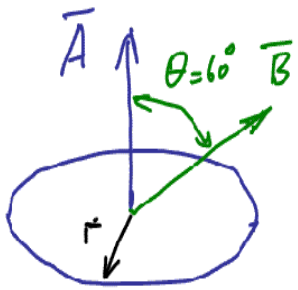
$R_{eq_2} = 18 + 42 = 60\Omega$

$\frac{1}{R_{eq_3}} = \frac{1}{60} + \frac{1}{40}$ =>

$R_{eq_3} = 24\Omega$

The equivalent resistance of circuit is 24Ω

Problem 33-28



A 100-turn, 2-cm-diam. coil is at rest in a horizontal plane. A uniform magnetic field 60° away from vertical increases from 0.5 T to 1.50 T in 0.6 s. What is $\mathcal{E}_{ind}$ in the coil?

The flux for a single loop:

$$\Phi = \bar{A} \cdot \bar{B} = B \cdot A \cdot \cos\theta$$

Total flux $\Phi_{total} = N \cdot \Phi = NBA \cdot \cos\theta$

According to Faraday's Law:

$$\mathcal{E} = \left| \frac{d\Phi_{total}}{dt} \right| = NA \cdot \cos\theta \cdot \left| \frac{dB}{dt} \right| = N\pi r^2 \cos\theta \left| \frac{\Delta B}{\Delta t} \right| =$$

$$= 100 \cdot \pi \cdot (0.01)^2 \cdot (\cos 60°) \left| \frac{1.50 - 0.5}{0.6} \right| =$$

$$= 2.62 \times 10^{-2}\ V = \underline{\underline{26.2\ mV}}$$

Problem 33-61

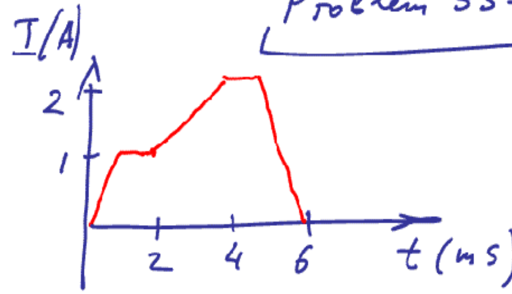


Figure shows the current through a 10 mH inductor. Draw a graph showing the potential difference ΔV_L across the inductor for these 6 ms.

Question is about ΔV_L.

The main self-inductance formula:

$$\Delta V_L = -L \frac{dI}{dt}$$

$$\Delta V_L = -L \frac{\Delta I}{\Delta t} \Rightarrow$$

For the first interval 0 ms to 1 ms:

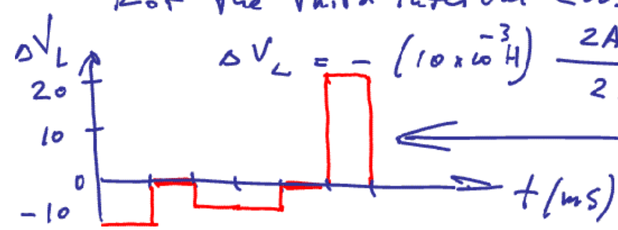
$$\Delta V_L = -(10 \times 10^{-3}\ H) \frac{1A - 0A}{(1 - 0) \times 10^{-3}\ s} = -10V$$

For the second interval 1 ms to 2 ms:

$$\Delta V_L = 0 \text{ since } \Delta I = 0$$

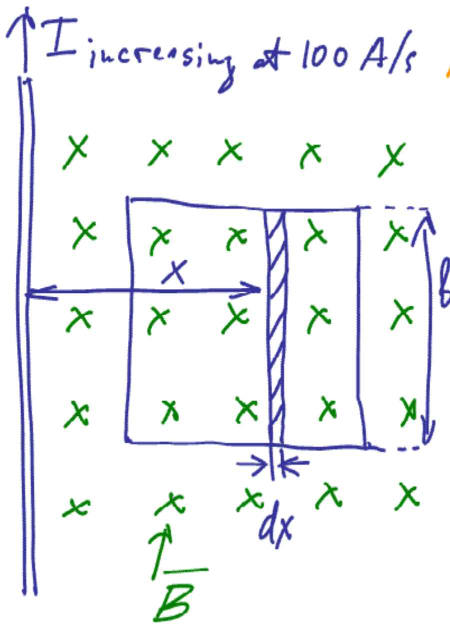
For the third interval 2 ms to 4 ms:

$$\Delta V_L = -(10 \times 10^{-3}\ H) \frac{2A - 1A}{2 \times 10^{-3}\ s} = -5V$$



Finally

Problem 33-31



A 2.0 cm x 2.0 cm square loop of wire with resistance 0.0010 Ω is parallel to a long straight wire. The near edge of the loop is 1.0 cm from the wire. The current in the wire is increasing at the rate 100 A/s. What is the current in the loop?

Faraday's Law: $\mathcal{E}_{loop} = \left| \frac{d\Phi}{dt} \right|$

$$I_{loop} = \frac{\mathcal{E}_{loop}}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right|$$

We need to find Φ.

This is a separate problem considered in detail in Example 33.5.

You should study this Example 33.5

It is shown there that:

$$\Phi = \frac{\mu_0 \cdot I \cdot b}{2\pi} \ln\left(\frac{c+a}{c}\right),$$

where c - is the distance of the near edge from the wire ($c = 1\,cm$ in our case)

In our case $b = a = 2\,cm$

$$I_{loop} = \frac{1}{R} \frac{\mu_0 \cdot b}{2\pi} \ln\left(\frac{c+a}{c}\right) \frac{dI}{dt}$$

$$= \frac{(4\pi \times 10^{-7}) \cdot (0.02)}{(0.01) \cdot 2\pi} \ln\left(\frac{0.03}{0.01}\right) (100\,A/s) = 43.9\,\mu A$$