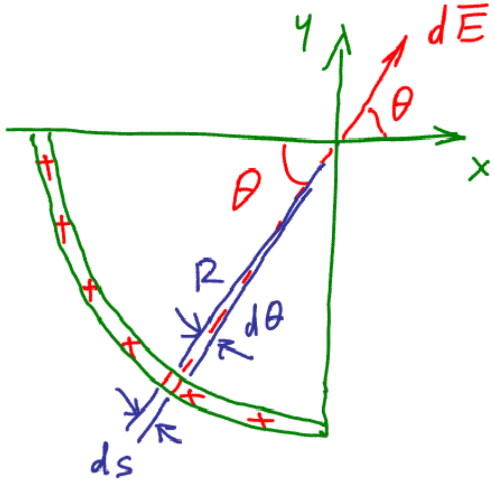
Review for Final

Preparation for Problems 1&2

Sample 26–49



A plastic rod with linear density λ is bent into the quarter circle.

1. Find $\bar{E}(x=0, y=0)$
2. Find $V(x=0, y=0)$
   a) Find expressions for dE_x and dE_y
   b) Find components $E_x = \int dE_x$, $E_y = \int dE_y$
   c) $\bar{E}$ in a component form

$\bar{E} = E_x \cdot \bar{i} + E_y \cdot \bar{j}$

Steps.

1. Use Coulomb Law to find $|d\bar{E}|$
2. Express components dE_x and dE_y
3. Integrate components: $E_x = \int dE_x$, $E_y = \int dE_y$
4. Express total El. field in a component form:

$$\bar{E} = E_x \cdot \bar{i} + E_y \cdot \bar{j}$$

$$|d\bar{E}| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{\cancel{R}\cdot d\theta}{R^{\cancel{2}}} = \underline{\underline{\frac{\lambda}{4\pi\varepsilon_0}\frac{1}{R}d\theta}}$$

$dq = \lambda \cdot ds$, $ds = R \cdot d\theta$

$$dE_x = |d\bar{E}| \cdot \cos\theta$$
$$dE_y = |d\bar{E}| \cdot \sin\theta$$

$$E_x = \int dE_x = \int \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{R}\cos\theta \cdot d\theta = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}\int_0^{\frac{\pi}{2}} \cos\theta \cdot d\theta =$$
$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot R}\sin\theta\Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}(1-0) = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}.$$

$(\sin\theta)' = \cos\theta$

$$E_y = \int dE_y = \int \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{R}\cdot\sin\theta \cdot d\theta = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}\int_0^{\frac{\pi}{2}} \sin\theta \cdot d\theta,$$

$(-\cos\theta)' = \sin\theta$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot R}(-\cos\theta)\Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}(0-(-1)) =$$
$$= \underline{\underline{\frac{\lambda}{4\pi\varepsilon_0 \cdot R}}}.$$

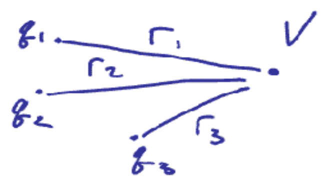
$$E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}.$$

$$\underline{\underline{\bar{E} = \frac{\lambda}{4\pi\varepsilon_0 \cdot R}(\bar{i} + \bar{j})}}$$

For point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$ — scalar, not a vector

Superposition principle:



$$V = \sum_i V_i$$

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

$$V = \int dV.$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R}$$

$$dq = \lambda\cdot ds = \lambda\cdot R\cdot d\theta$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cdot\cancel{R}\cdot d\theta}{\cancel{R}}$$

$$V = \int dV = \frac{\lambda\cdot\cancel{R}}{4\pi\varepsilon_0\cdot\cancel{R}}\int_0^{\frac{\pi}{2}} d\theta = \frac{\lambda}{4\pi\varepsilon_0}\,\theta\Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{\lambda}{4\cancel{\pi}\varepsilon_0}\frac{\cancel{\pi}}{2} = \boxed{\frac{\lambda}{8\varepsilon_0}} = \frac{\cancel{2}Q}{\pi R}\frac{1}{\cancel{8}_{4}\varepsilon_0} = \boxed{\frac{Q}{4\pi\varepsilon_0 R}}$$

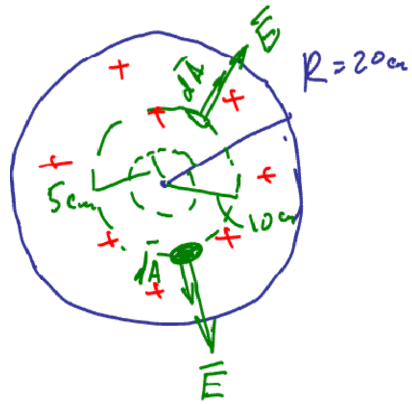
If you are given Q

$$\lambda = \frac{Q}{\underbrace{\pi R/2}_{\text{length of the } \frac{1}{4} \text{ ring}}} = \frac{2Q}{\pi R}$$

$2\pi R$, $\frac{2\pi R}{4} = \frac{\pi R}{2}$

Preparation for Problem 3

Sample problem 27-37



A 20cm-radius ball is uniformly charged to 80 μC.

a) what is the ball charge density, $\rho\left(\frac{C}{m^3}\right)$ - ?

b) How much charge is enclosed by spheres with radii 5, 10 and 20cm?

c) What is the el. field strength at points 5, 10, 20 cm from the center.

Step by step:

1. Calculate density ρ
2. Use three Gaussian spheres ($r=5, 10, 20$ cm)
3. Calculate Q_{enc} ($r=5, 10, 20$ cm)
4. Use Gauss's Law and calculate $E(r=5, 10, 20\,cm)$

$$\rho = \frac{Q}{V}, \quad V = \frac{4}{3}\pi R^3 \Rightarrow \rho = \frac{Q\cdot 3}{4\pi R^3} =$$

$$= \frac{80 \cdot 10^{-9} C \cdot 3}{4 \cdot \pi \, (0.2)^3} = 2{,}387 \times 10^{-6} \frac{C}{m^3}$$

$$Q\Big|_{r=5cm} = \rho \cdot V\Big|_{r=5cm} = \rho \, \frac{4}{3}\pi r^3\Big|_{5cm} = \underline{\underline{1{,}25\,\mu C}}$$

$$Q\Big|_{r=10cm} = \rho \cdot V\Big|_{r=10cm} = \underline{\underline{10\,\mu C}}$$

$$Q\Big|_{r=20cm} = \underline{\underline{80\,\mu C}}$$

Gauss's Law:

$$\Phi_e = \frac{Q_{enc}}{\varepsilon_0}, \quad \Phi_e = \oint_G \bar{E} \cdot d\bar{A} =$$

$$\boxed{\begin{array}{l} \bar{E} \cdot d\bar{A} = |\bar{E}| \cdot |d\bar{A}| \cdot \cos\varphi = |\bar{E}| \cdot |d\bar{A}| = E \cdot dA. \\ \quad \varphi = 0 \Rightarrow \cos\varphi = 1 \end{array}}$$

$$= \oint_G E \cdot dA = \oint_G E(r) \cdot dA = E(r) \cdot \oint_G dA =$$

$$E(r) \cdot 4\pi r^2.$$

$$E(r) \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}.$$

$$E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q_{enc}}{r^2}.$$

$$E\Big|_{r=5} = \frac{Q_5}{4\pi\varepsilon_0 \cdot r^2\Big|_{5cm}} = \frac{1.25 \cdot 10^{-9} C}{8.85 \cdot 10^{-12} \cdot 4\pi \cdot (0.05m)^2} =$$

$$= \underline{\underline{4500 \frac{N}{C}}}$$

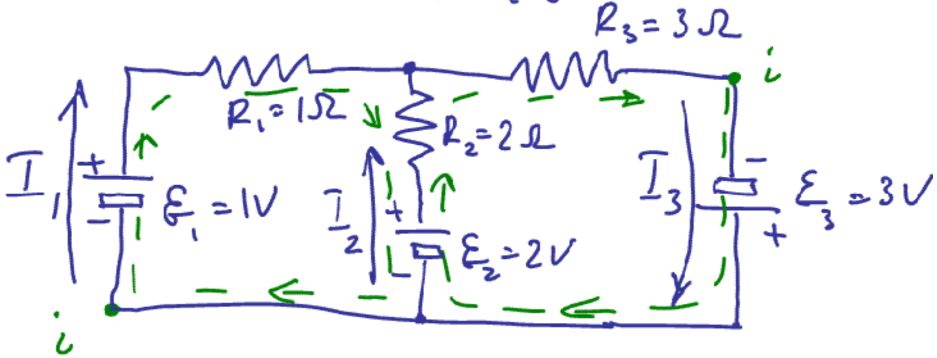
$$E\Big|_{r=10} = \underline{\underline{9000 \frac{N}{C}}}$$

$$E\Big|_{r=20cm} = 18000 \frac{N}{C}$$

For Problem 4

Solve any of the Problems from 31-67 up to 31-71

For Problem 5



Select direction for the currents

Select directions for loops: cw or ccw

cw - for the left loop

cw - for the right loop

Three unknowns: I_1, I_2, I_3

We need three Eq.

2 Loop rules

Junction rule

Left loop:

$$1 - I_1 \cdot 1 + I_2 \cdot 2 - 2 = 0 \quad (1)$$

Right loop:

$$3 + 2 - I_2 \cdot 2 - I_3 \cdot 3 = 0 \quad (2)$$

Junction rule: $I_1 + I_2 = I_3 \quad (3)$

Exclude unknowns one by one: Subst. (3) into (2) to get rid of I_3

$$5 - 2I_2 - 3(I_1 + I_2) = 0$$

$$5 - 2I_2 - 3I_1 - 3I_2 = 0$$

$$5 - 5I_2 - 3I_1 = 0 \text{ — solve jointly with (1)}$$

$$1 - I_1 + 2I_2 - 2 = 0 \Rightarrow I_1 = -1 + 2I_2 \Leftarrow \text{to exclude } I_1.$$

$$5 - 5I_2 - 3(-1 + 2I_2) = 0$$

$$5 - 5I_2 + 3 - 6I_2 = 0$$

$$8 = 11 I_2 \Rightarrow I_2 = \frac{8}{11}\,A$$

$$I_1 = -1 + 2I_2 = -1 + \frac{16}{11} = \frac{-11+16}{11} = \frac{5}{11}\,A$$

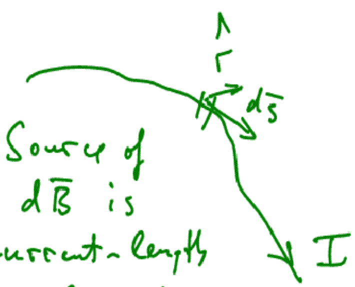
$$I_3 = I_1 + I_2 = \frac{8}{11} + \frac{5}{11} = \frac{13}{11}\,A$$

For Problem 6

$dq\ \hat{r}$ $\cdot O$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

O - observation



Source of $d\vec{B}$ is current-length element: $I \cdot d\vec{s}$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$|\hat{r}| = 1$$

Sample Problem 32-50

$\bar{B}(0) - ?$

$$\boxed{d\bar{B} = \frac{\mu_0}{4\pi}\frac{I\cdot d\bar{s}\times\hat{r}}{r^2}}$$

$$\bar{B}(0) = \bar{B}_1(0) + \bar{B}_2(0) + \bar{B}_3(0)$$

$$d\bar{s} \parallel \hat{r} \Rightarrow \varphi = 0 \Rightarrow |d\bar{s}\times\hat{r}| = |d\bar{s}|\cdot|\hat{r}|\cdot\sin\varphi = 0$$

$$B_1 = \int_1 dB_1 = 0, \quad B_3 = \int_3 dB_3 = 0$$

Part 2: $|d\bar{s}\times\hat{r}| = |d\bar{s}|\cdot|\hat{r}|\cdot\sin\varphi =$

$= ds\cdot 1\cdot 1 = ds$

$$B_2 = \int_2 dB_2, \quad dB_2 = \frac{\mu_0}{4\pi}\frac{I\cdot ds}{R^2} =$$

$$\boxed{ds = R\,d\theta}$$

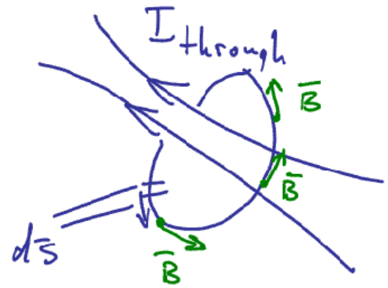
$$\frac{\mu_0}{4\pi}\frac{I\cdot \cancel{R}\,d\theta}{R^{\cancel{2}}} = \frac{\mu_0}{4\pi}\frac{I}{R}\,d\theta$$

$$B_2 = \int_2 dB_2 = \frac{\mu_0}{4\pi}\frac{I}{R}\int_0^{\theta} d\theta =$$

$$= \frac{\mu_0}{4\pi}\frac{I}{R}\,\theta$$

Sample for Problem 7

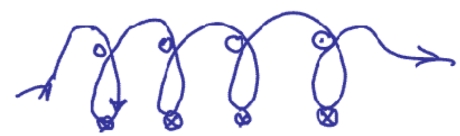
Ampere's Law:

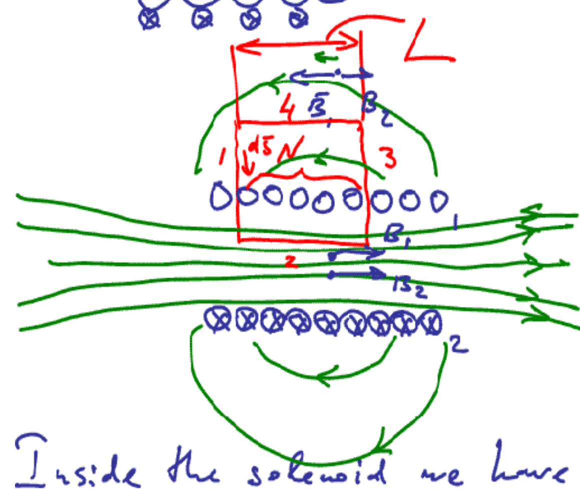


$$\oint \bar{B}\cdot d\bar{s} = \mu_0 I_{through}$$

$$\mu_0 = 1.26\times 10^{-6}\ T\,m/A$$

Finding the field ($\bar{B}$) of a solenoid





$B_2 \lesssim B_1$

$B_{net} = B_1 - B_2 \ll B_1$

There is no strong $\bar{B}$ outside the solenoid

Inside the solenoid we have very strong

$\bar{B}_{net} = \bar{B}_1 + \bar{B}_2$

$I_{through} = N\cdot I$

$$\oint \bar{B}\cdot d\bar{s} = \mu_0 \cdot I_{through} = \mu_0 \cdot N \cdot I$$

$$\oint \bar{B}\cdot d\bar{s} = \int_1 \bar{B}\cdot d\bar{s} + \int_2 \bar{B}\cdot d\bar{s} + \int_3 \bar{B}\cdot d\bar{s} + \int_4 \bar{B}\cdot d\bar{s}$$

~ 0

Side 1: $\bar{B}\cdot d\bar{s} = |\bar{B}|\cdot|d\bar{s}|\cdot \cos\varphi = 0$

$\bar{B} \perp d\bar{s} \Rightarrow \varphi = 90^\circ \Rightarrow \cos\varphi = 0$

$$\int_1 \bar{B}\cdot d\bar{s} = 0$$

Side 4: $\int_4 \bar{B}\cdot d\bar{s} \sim 0 \quad B\downarrow$ as $r\uparrow$

Side 2: $\int_2 \bar{B}\cdot d\bar{s} = \int_2 B\cdot ds = B\int_2 ds = B\cdot L$

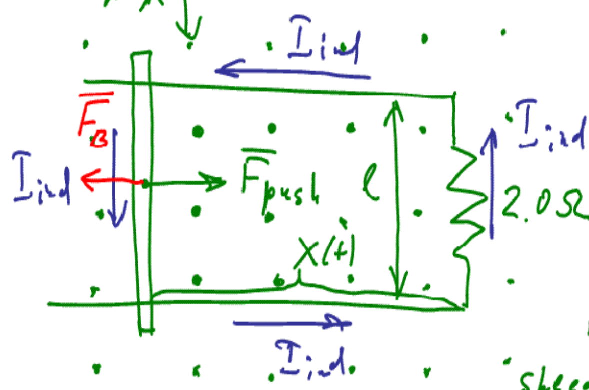
$$\boxed{\begin{array}{l} \bar{B} \parallel d\bar{s} \Rightarrow \varphi = 0 \Rightarrow \cos\varphi = 1 \\ \bar{B}\cdot d\bar{s} = |\bar{B}|\cdot|d\bar{s}|\cdot\cos\varphi = B\cdot ds \end{array}}$$

$$BL = \mu_0 \cdot N \cdot I \Rightarrow B = \mu_0 \frac{N}{L} I = \underline{\underline{\mu_0 \cdot n \cdot I}}$$

$$n = \frac{N}{L}$$

Sample for Problems 8 and 9

$\bar{B}$ - out of the page Problem 33-44



10 cm wide zero resistance slide wire is pushed towards 2Ω resistor at steady speed $0.5\,\frac{m}{s}$, $B = 0.5\,T$

a) F_{push} (force required for v = const) - ?

b) Power supplied to wire (mechanical) - ?

c) $I_{induced}$ - ?

d) Power dissipated in the resistor - ?

Lenz's Law (direction of I_{ind}):

Flux of the $B_{ext}\downarrow$ since $A\downarrow \Rightarrow$

B_{ind} should be out of the page to resist changes. $\Rightarrow$ I_{ind} — $\underline{\underline{CCW}}$

Faraday's Law (magnitude of $\mathcal{E} \Rightarrow I_{ind}$)

$$\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = Bl\left|\frac{dx}{dt}\right| = Bl v$$

$$\Phi_B = B\cdot A = B\cdot \ell\cdot x$$

$$I_{ind} = \frac{\mathcal{E}}{R} = \frac{B\cdot \ell\cdot v}{R}.$$

$$\bar F = I\cdot \bar L\times \bar B;\quad |\bar F| = I\cdot|\bar L|\cdot|\bar B|\cdot \sin\varphi = I\cdot L\cdot B,\quad \varphi = 90°$$

$|\bar F_B| = I_{ind}\cdot \ell\cdot B$ = — directed leftward

$$= \frac{B^2\cdot \ell^2\cdot v}{R},\qquad |\bar F_{push}| = |\bar F_B|$$

$$F_{push} = \frac{B^2\cdot \ell^2\cdot v_{term}}{R}$$

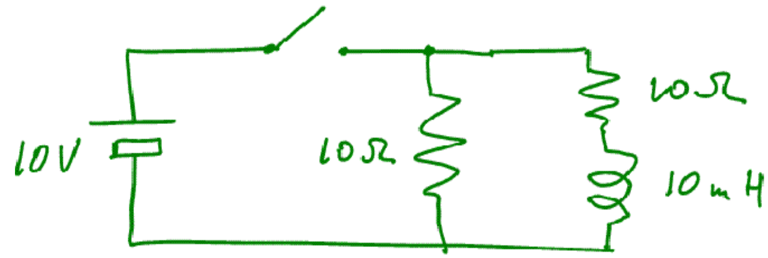
$$v_{term} = \frac{F_{push}\cdot R}{B^2\cdot \ell^2}$$

$$F_{push} = \frac{B^2\cdot \ell^2\cdot v_{term}}{R} = \frac{(0.5)^2\cdot(0.1)^2\cdot(0.5)}{2} = 6.25\times 10^{-4}\,N$$

$$P_{mech} = F_{push}\cdot v = (6.25\times 10^{-4}\,N)\cdot(0.5\,\tfrac{m}{s}) = 3.13\times 10^{-4}\,W$$

$$P_{elect} = I_{ind}^2\cdot R = \frac{B^2\cdot \ell^2\cdot v^2}{R^2}\cdot R = \frac{B^2\cdot \ell^2\cdot v^2}{R}$$ — same

Problems 33-72, 33-73



a) I through the battery immed. after closing the switch

b) I through the battery after long time with the switch closed

a) $\Delta V_L = -L\dfrac{dI}{dt}$ — for an inductor (1)

Immediately after closing the switch the inductor accomodates all the voltage available to it.

In this problem it is 10V from the battery.

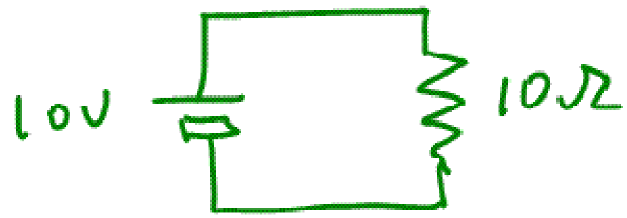
At this stage the current (I) through the inductor is extremely small (practically zero), but

$\dfrac{dI}{dt}$ — is not small. It is just enough

to create $\Delta V_L = -L\dfrac{dI}{dt} = 10V$

It is easy to see that at this initial stage of the transient process the inductor can be modeled as a "break" – extremely high R.

The equivalent circuit:



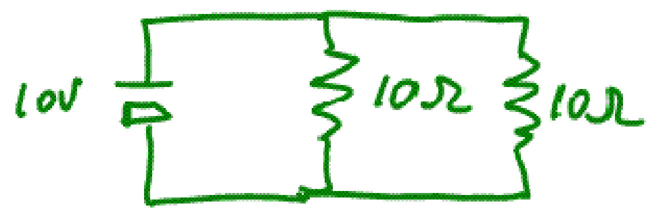
$\Rightarrow I_{battery} = \frac{10V}{10\Omega} = 1A$

b) After long time $\frac{dI}{dt} \sim 0 \Rightarrow$

$$\Delta V_L = -L\frac{dI}{dt} \sim 0$$

This means that the inductor behaves as a wire with zero R or as a "short"

The equivalent circuit:



$$I_{battery} = \frac{10V}{5\Omega} = 2A$$