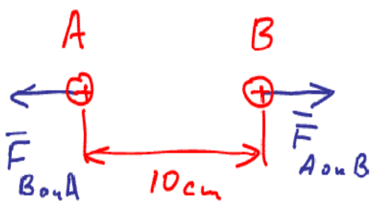
For problem 1

25-37



$m_A = 0.1\,kg$ $\quad m_B = 0.1\,kg$

$q_A = 2q_B$ $\quad q_B$

$\bar{F}_{A\,on\,B} = 0.45\,N$

Objects A and B are both + charged, $m_A = m_B = 100\,g$, $q_A = 2q_B$.

When A and B are placed 10 cm apart, B experiences an el. force $F_{A\,on\,B} = 0.45\,N$

a) $F_{B\,on\,A}$ - ?

b) q_A - ? and q_B - ?

c) If released what is a_A - ?

3rd Newton's Law:

a) $|\bar{F}_{B\,on\,A}| = |\bar{F}_{A\,on\,B}| = k\dfrac{q_A \cdot q_B}{r^2} = k\dfrac{2q_B^2}{r^2} = 0.45\,N$

b) $q_B^2 = \dfrac{(0.45)\cdot r^2}{2k}$

$q_B = r\sqrt{\dfrac{(0.45)}{2k}} = 0.1 \times \sqrt{\dfrac{0.45}{2\times 9\times 10^9}} =$

$= 5\times 10^{-7}\,C = 0.5\,\mu C$

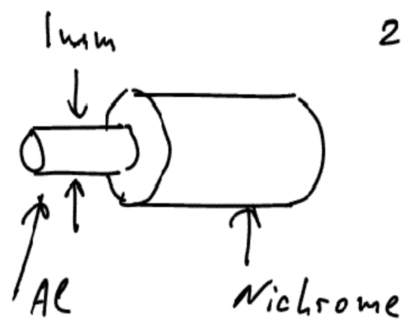
c) 2-nd Newton's Law

$a_A = \dfrac{F_{B\,on\,A}}{m_A} = \dfrac{0.45}{0.1} = 4.5\left[\dfrac{m}{s^2}\right].$

Problem 2

Current

28-50



What diameter should be the nichrome wire in order for electric field strength to be the same in both wires?

$j = \dfrac{I}{A}\left[\dfrac{A}{m^2}\right].$

$j = \sigma \cdot E,$

σ - conductivity, E - el. field strength.

1 - states for Al

2 - states for Nichrome

$I_1 = I_2.$

$I = j\cdot A = \sigma\cdot E\cdot A.$

$I_1 = \sigma_1 \cdot E_1 \cdot A_1, \quad I_2 = \sigma_2 \cdot E_2 \cdot A_2$

$$\boxed{\sigma_1 \cdot E_1 \cdot A_1 = \sigma_2 \cdot E_2 \cdot A_2}$$

Problem suggests that $E_1 = E_2$ - this is not a general law

$\sigma_1 \cdot A_1 = \sigma_2 \cdot A_2$

$$A_1 = \frac{\pi D_1^2}{4}, \quad A_2 = \frac{\pi D_2^2}{4}.$$

$$\sigma_1 \cdot \frac{\pi D_1^2}{4} = \sigma_2 \frac{\pi D_2^2}{4}$$

$$D_2^2 = D_1^2 \frac{\sigma_1}{\sigma_2}$$

$$D_2 = D_1 \cdot \sqrt{\frac{\sigma_1}{\sigma_2}} = 10^{-3} \times \sqrt{\frac{3.5 \times 10^7}{6.7 \times 10^5}} =$$

$$= 7.22 \times 10^{-3} =$$

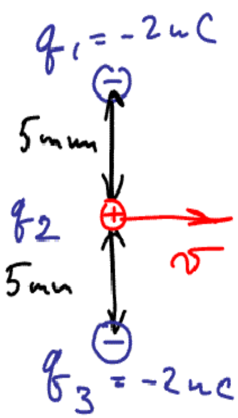
$$\underline{\underline{7.22\ mm}}$$

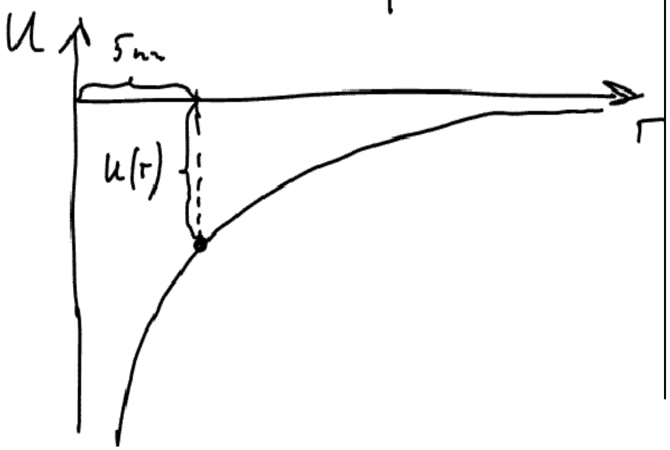
Problem 3

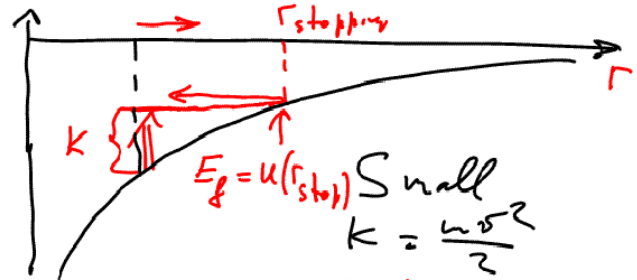
29-49

Calculate v_{escape}

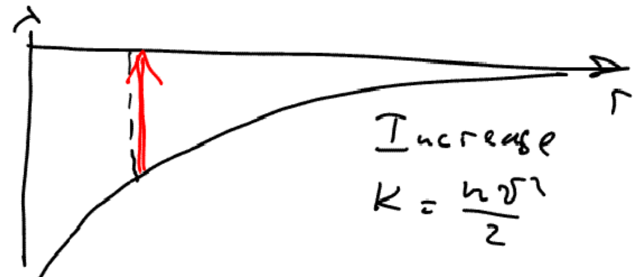
$$U(r) = K \frac{q_1 \cdot q_2}{r}$$







$$E_i = U(<0) + K$$



Definition of the escape velocity:
It is a minimal velocity with no return

$$U + K = 0 \Rightarrow K = -U$$

$$U_{12} = K \frac{q_1 \cdot q_2}{r}.$$

$$U(+\text{charge}) = 2 \times U_{12} = 2K \frac{q_1 \cdot q_2}{r}.$$

$$\left| 2K \frac{q_1 \cdot q_2}{r} \right| = \frac{m v_{escape}^2}{2}$$

$$v_{escape}^2 = \frac{4K}{m} \frac{q_1 \cdot q_2}{r}.$$

$$v_{escape} = \sqrt{\frac{4 \times 9 \times 10^9 \ (1.6 \times 10^{-19})(2 \times 10^{-9})}{1.67 \times 10^{-27} \quad (5 \times 10^{-3})}} =$$

$$= 1.17 \times 10^6 \ \frac{m}{s}$$

Problem 4

The electric potential is given $V = 3000 - 5x^3 + 10x^2 - 2y^2$ [Volts]

where x and y are measured from the origin of the coordinate system.

A + charge of +1nC with a mass of 1g is located at (x = 2m, y = 2m).

What is its acceleration?

$$\begin{cases} E_x = -\dfrac{\partial V}{\partial x} \\ E_y = -\dfrac{\partial V}{\partial y} \\ E_z = -\dfrac{\partial V}{\partial z} \end{cases}$$

← Think of this partial derivative as if the "function" (V) is dependent on only one variable x

$$\boxed{V = 3000 - 5x^3 + 10x^2 - 2y^2}$$

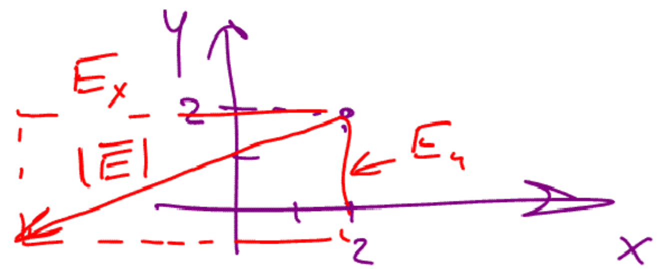
$E_x = -5 \times 3 \times x^2 + 10 \times 2 \times x$

$E_y = -2 \times 2 \times y$ $\qquad \begin{cases} x = 2m \\ y = 2m \end{cases}$

$$\begin{cases} E_x = -5 \times 3 \times 4 + 10 \times 2 \times 2 = -60 + 40 = \\ \qquad = -20 \left[\dfrac{V}{m}\right] \\ E_y = -8 \left[\dfrac{V}{m}\right]. \\ E_z = 0 \end{cases}$$

$\bar{E} = E_x \cdot \hat{i} + E_y \cdot \hat{j} =$

$= -20 \cdot \hat{i} - 8 \cdot \hat{j} \left[\dfrac{V}{m}\right].$



$|\bar{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{20^2 + 8^2} =$

$21.5 \left[\dfrac{V}{m}\right]$

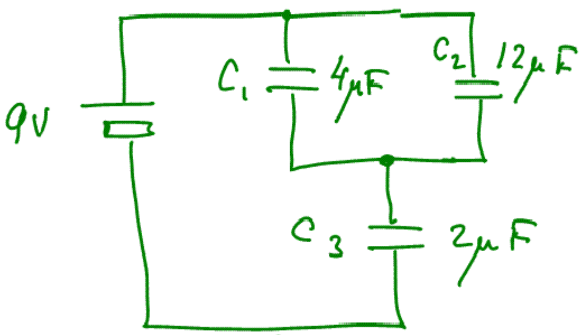
$|a| = \dfrac{F}{m} = \dfrac{q \cdot |\bar{E}|}{m} = \dfrac{21.5 \times 10^{-9}}{10^{-3}} =$

$= 21.5 \times 10^{-6} \left[\dfrac{m}{s^2}\right]$

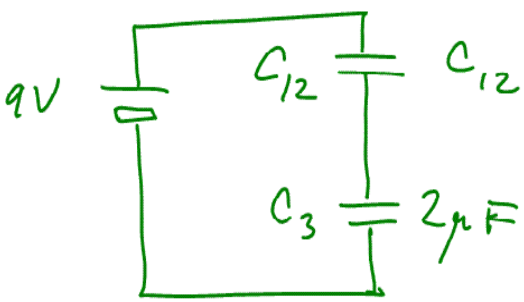
Problem 5

Capacitors in series

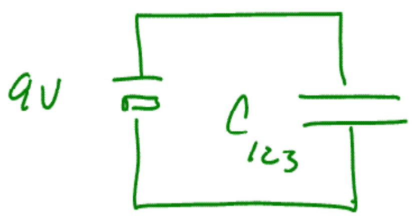
30-64



What are the charge on and the potential difference across each capacitor.



$C_{12} = C_1 + C_2 = 4\mu F + 12\mu F = 16\mu F$



$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{16} + \frac{1}{2} = \frac{1+8}{16} = \frac{9}{16}$$

$\boxed{C = \frac{Q}{\Delta V}} \Rightarrow \boxed{C_{123} = \frac{16}{9}\mu F}$

$Q = C\cdot\Delta V \Rightarrow Q_{123} = C_{123}\cdot(9V) = \left(\frac{16}{9}\mu F\right) \times (9V) = \underline{\underline{16\mu C}}$

$$\boxed{Q_{123} = Q_3 = Q_{12} = 16\mu C}$$

This is a result of "chain reaction" of charging the capacitors C_3 and C_{12} connected in series.

$V_3 = \frac{Q_3}{C_3} = \frac{16\mu C}{2\mu F} = 8V$

$V_{12} = 9 - 8 = 1V$

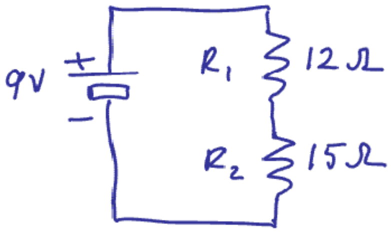
$Q_1 = V_1 \cdot C_1 = 1V \times 4\mu F = 4\mu C$

$Q_2 = V_2 \cdot C_2 = 1V \times 12\mu F = 12\mu C$

C	V [V]	Q [μC]
C_1	1	4
C_2	1	12
C_3	8	16

Problem 6

31–14



How much power is dissipated by each resistor in Figure?

Loop Rule:

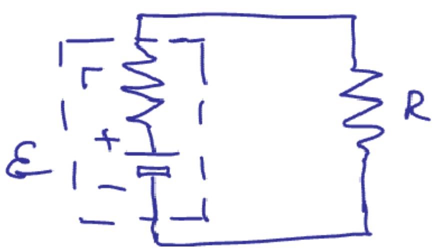
$9 - 12I - 15I = 0$

$I = \frac{9}{12+15} = \frac{9}{27} = \frac{1}{3}\,(A)$, $\quad P = I^2 \cdot R$

$P_{12} = \left(\frac{1}{3}\right)^2 \cdot 12 = 1.33\,W$, $\quad P_{15} = \left(\frac{1}{3}\right)^2 \cdot 15 = 1.67\,W$

31–56a

Load resistor R is attached to the battery of emf $\mathcal{E}$ and internal resistance r. For what value of the resistance R, in terms of $\mathcal{E}$ and r, will the power dissipated by the load resistor be a maximum?



$I = \frac{\mathcal{E}}{R+r}$

$P_R = I^2 \cdot R = \frac{\mathcal{E}^2 \cdot R}{(R+r)^2}$

Max of P_R is characterized by $\frac{dP_R}{dR} = 0$

$\frac{dP_R}{dR} = -2\frac{\mathcal{E}^2 \cdot R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} =$

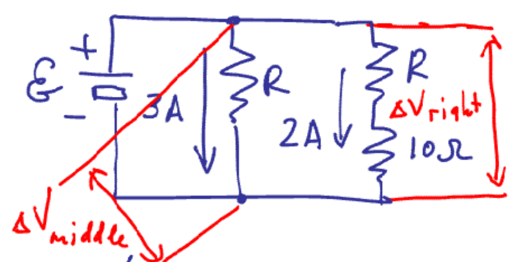
$$\boxed{\frac{d(u \cdot v)}{dx} = \frac{du}{dx} \cdot v + \frac{dv}{dx} \cdot u \qquad u = \frac{\mathcal{E}^2}{(R+r)^2}, \quad v = R}$$

$= \frac{-2R\mathcal{E}^2 + \mathcal{E}^2 \cdot R + \mathcal{E}^2 \cdot r}{(R+r)^3} = \frac{\mathcal{E}^2(r-R)}{(R+r)^3} = 0$

The power is maximized if $r = R$

In addition study series and parallel resistors.

31–54



What are the resistance R and emf of the battery?

$\Delta V_{midde} = 3 \cdot R$

$\Delta V_{right} = 2 \cdot (R+10)$

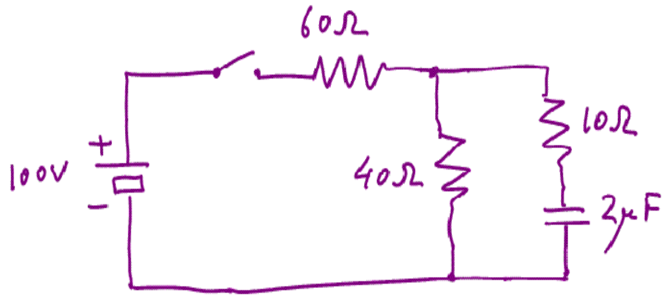
$\Delta V_{middle} = \Delta V_{right}$

$3R = 2(R+10) \Rightarrow 3R = 2R + 20 \Rightarrow R = 20\Omega$

$\mathcal{E} = \Delta V_{middle} = (3A) \times (20\Omega) = 60V$

Problem 7

31-77



The switch has been closed for a long time

a) what is Q_C?

b) S-is open at $t=0$. At what time Q_C decreased to 10% of its initial value?

a) In steady state ($t \to \infty$) the capacitor behaves as a break $\Rightarrow$

$$I = \frac{100V}{60\Omega + 40\Omega} = 1A$$

$$V_{40} = I \cdot R_{40} = (1A)\cdot(40\Omega) = 40V$$

This voltage across 40Ω resistor is applied to the capacitor: $V_{40} = V_C$

$$Q_C = V_C \cdot C = (40V)\cdot(2\times10^{-6}F) = 80\mu C$$

b) After opening the switch:

$\Longleftrightarrow$

The discharge of capacitor in RC-circuit is described by:

$$Q_C(t) = Q_C(0)\cdot e^{-\frac{t}{\tau}} \quad (1), \quad \text{where } \tau = RC$$

$Q_C(0) = 80\mu C$

$\tau = RC = (50\Omega)\cdot(2\times10^{-6}) = 10^{-4}s = 0.1ms$

We are asked to find t_1 when

$Q_C(t_1) = 0.1 \cdot Q_C(0)$ - plug into Eq. (1)

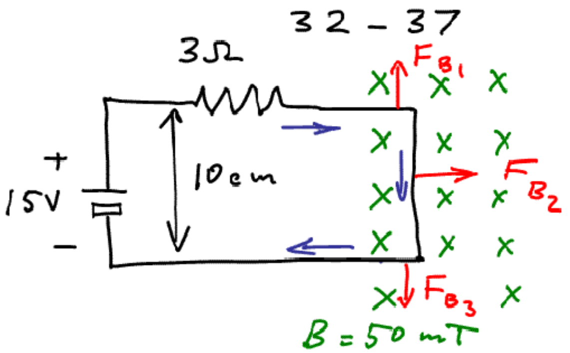
$$0.1\cdot \cancel{Q_C(0)} = \cancel{Q_C(0)}\cdot e^{-\frac{t_1}{\tau}}$$

$$\ln(0.1) = -\frac{t_1}{\tau} \Rightarrow t_1 = -\tau\cdot\ln(0.1) = \underline{0.23ms}$$

In addition study series and parallel capacitors.

Problem 8

32 – 37



The right edge of the circuit in Figure extends into 50mT uniform $\overline{B}$. What is the magnitude and direction of the net force on the circuit?

$$\overline{F}_B = I \cdot \underset{\underset{I\ \text{comp}}{\uparrow}}{\overline{L}} \times \overline{B} \quad (\nwarrow \text{ II comp})$$

, vector $\overline{L}$ has the same direction as current in a wire

Study right-hand rule

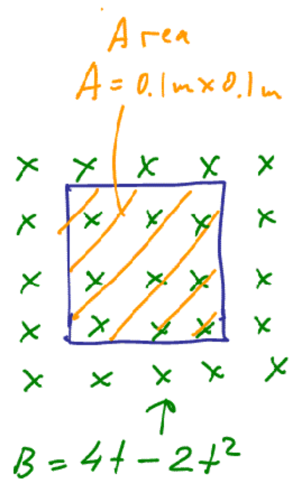
1. Align fingers with the I component
2. Find orientation of the palm which allows swipping fingers toward the II component
3. The direction of thumb represents the direction of magnetic force $\overline{F_B}$

$|\overline{F}_{B_1}| = |\overline{F}_{B_3}|$, but they have opposite direction, so they are cancelling each other.

$$F_{B_2} = I \cdot |\overline{L}| \cdot |\overline{B}| \cdot \underset{\varphi = 90^\circ}{\sin\varphi} = \frac{\mathcal{E}}{R} L \cdot B = \frac{15V}{3\Omega}(0.1m)\cdot(50mT) = \underline{0.025\,N}$$

Problem 9

33 – 25

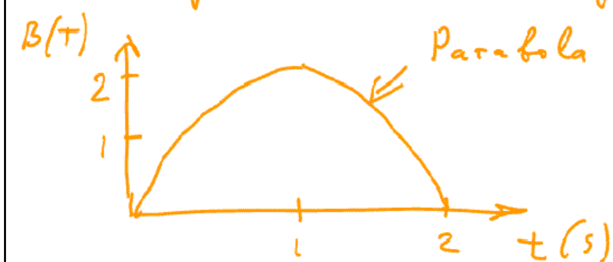


A 20cm × 20cm square loop has a resistance of 0.1Ω. A magnetic field ⊥ to the loop is $B = 4t - 2t^2$, where B is in tesla and t is in seconds

a) Determine B, $\mathcal{E}$, and I at half-second intervals from 0s to 2s.

b) Use your results to draw graphs of B and I versus time.

Let us first sketch B(t) given in the problem



Faraday's Law gives the magnitude of the $\mathcal{E}_{ind}$:

$$\mathcal{E}_{ind} = \left|\frac{d\Phi}{dt}\right| = A \cdot \left|\frac{dB}{dt}\right| = (0.2\,m^2)\cdot\left|(4 - 4t)\frac{T}{S}\right| =$$

$$\boxed{\begin{array}{l}\Phi = \overline{B}\cdot\overline{A} = |\overline{B}|\cdot|\overline{A}|\cdot\cos\varphi = B\cdot A \\ \varphi = 0^\circ \text{ since } \overline{B} \parallel \overline{A}\end{array}}$$

$= 0.16 \cdot |(1-t)| [V]$ (1)

$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R} |1.6(1-t)| [A]$ (2)

The Eq (1) and (2) give only the magnitude, but not the direction of $\mathcal{E}_{ind}$ and I.

In order to find the direction we should use Lenz's Law.

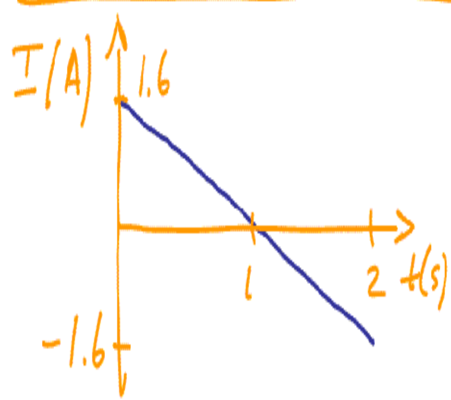
During $0 < t < 1s$ $B\uparrow \Rightarrow \Phi = A \cdot B \uparrow \Rightarrow$ B_{ind} has a direction opposite to B.

On contrary, at $1s < t < 2s$ $B\downarrow \Rightarrow \Phi = AB\downarrow \Rightarrow$ B_{ind} has the same direction as B

Since the direction of B_{ind} is related to the direction of I by the right hand rule we have to conclude that $\mathcal{E}_{ind}$ and I must have opposite signs in the second half of the time interval.

If we assume that $\mathcal{E}_{ind}$ and I were positive at $0s < t < 1s$ than they should become negative at $1s < t < 2s$

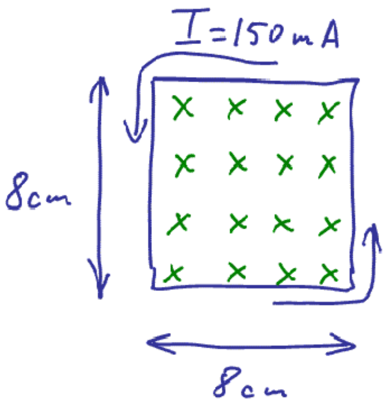
Time (s)	B (T)	$\mathcal{E}$ (V)	I (A)
0	0	0.16	1.6
0.5	1.5	0.08	0.8
1	2	0	0
1.5	1.5	-0.08	-0.8
2	0	-0.16	-1.6



Additionally, you can solve 33-66

Problem 10

33-13



The resistance of the loop is 0.1Ω. Is the magnetic field strength increasing or decreasing? At what rate $\left(\frac{T}{s}\right)$?

Since I (or I_{ind}) is CCW by using right hand rule we can find that B_{ind} is out of page.

Lenz's rule says that the direction of I_{ind} is such that B_{ind} always resists changes of the Φ (flux of the external magnetic field).

From the sketch we know that:

$$\begin{cases} B(\text{external}) \text{ is in page} \\ B_{ind} \text{ is out of page} \end{cases}$$

Using Lenz's rule we can conclude that B(external) is <u>increasing</u>.

Let us use Faraday's Law to calculate the magnitude of the induced $\mathcal{E}$ and I:

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = A\left|\frac{dB}{dt}\right| \Rightarrow \left|\frac{dB}{dt}\right| = \frac{\mathcal{E}}{A}$$

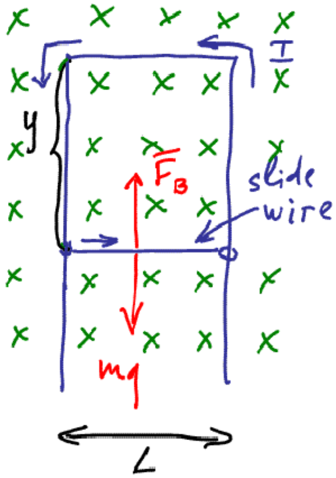
On the other hand $\mathcal{E} = I \cdot R$

$$\left|\frac{dB}{dt}\right| = \frac{I \cdot R}{A} = \frac{(150\times10^{-3}A)\cdot(0.1\Omega)}{(0.08m)^2} =$$

$$= 2.34\,\frac{T}{s}$$

Problem 11

33 - 49



U-shaped conducting rail is oriented vertically in a horizontal $\overline{B}$. The rail has no electric resistance and does not move. A slide wire with mass m and resistance R can slide up and down without friction while maintaining el. contact with the rail.

a) Show that the slide wire reaches a terminal velocity v_{term}, and find an expression for v_{term}

b) Determine the value of v_{term} if $L = 20cm$, $m = 60g$, $R = 0.1\,\Omega$ and $B = 0.5T$

Faraday's Law gives us the magnitudes of induced $\mathcal{E}$ and I

Lenz's Law gives us the direction of induced $\mathcal{E}$ and I

Let us start from the Lenz's Law.

The slide-wire tends to fall $\Rightarrow$ See the sketch $y\uparrow$

What is happening to Φ?

$\Phi = \bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cdot\cos\varphi = A\cdot B = L\cdot B\cdot y$

$\boxed{\varphi = 0^\circ}$ $\boxed{A = L\cdot y}$

Thus, the flux of the external B is increasing.

Lenz's rule says that the direction of I and B_{ind} are such that they resist the changes of the external flux.

Φ(external) – in page and increasing

Due to Lenz's Law this means that

B_{ind} – should be out of page

Using right hand rule one can find that I (induced) is counter clock wise (CCW)

There will be a magnetic force $\overline{F}_B$ acting on a slide-wire due to this current: $\overline{F}_B = I\,\overline{L}\times\overline{B}$

Using right hand rule one can see that the direction of this force is upwards.

As we will see from calculation

$F_B \sim v$, where v – velocity of wire

Initially, the wire falls with the acceleration determined by the gravity (g). However due to increase of v and, consequently, F_B eventually the condition $|\overline{F}_B| = mg$ is met.

This means that the total force becomes to be zero. The velocity of falling reaches a constant value, called a terminal velocity v_{term}.

Now, let use use Faraday's Law to calculate the following:

$$\mathcal{E} \rightarrow I \rightarrow F_B \rightarrow v_{term}$$

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d(B \cdot L \cdot y)}{dt}\right| = B \cdot L \cdot \frac{dy}{dt} = B \cdot L \cdot v$$

$$I = \frac{\mathcal{E}}{R} = \frac{B \cdot L \cdot v}{R}$$

$$\overline{F}_B = I \cdot \overline{L} \times \overline{B} = I \cdot L \cdot B = \frac{B \cdot L \cdot v}{R} \cdot L \cdot B = \frac{B^2 \cdot L^2 \cdot v}{R},$$

$|F_B| = mg$ (condition for v_{term})

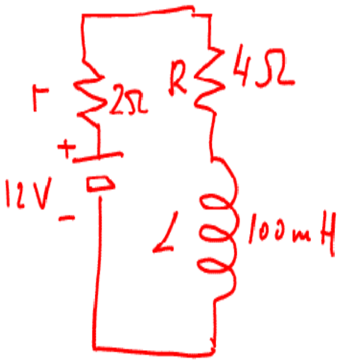
$$\frac{B^2 \cdot L^2 \cdot v_{term}}{R} = mg$$

$$v_{term} = \frac{mg\,R}{B^2 \cdot L^2}$$

$$v_{term} = \frac{(0.01) \cdot (9.8) \cdot (0.1)}{(0.2)^2 \cdot (0.5)^2} = 0.98\ \frac{m}{s}$$

Problem 12

33-52



A 100mH inductor whose windings have a resistance of 4 Ω is connected across a 12V battery having an internal resistance of 2Ω. How much energy is stored in the inductor?

$$I = \frac{\mathcal{E}}{R+r} = \frac{12V}{4\Omega + 2\Omega} = 2A$$

$$U_L = \frac{1}{2} L I^2 = \frac{1}{2}(100 \times 10^{-3}) \cdot (2)^2 = \underline{\underline{0.2\,J}}$$

Additionally study the behavior of an inductor L in circuits in two cases:

a) In a steady state

b) Immediately after the switch is closed or open

Problems 33-73, 33-74 and 33-75