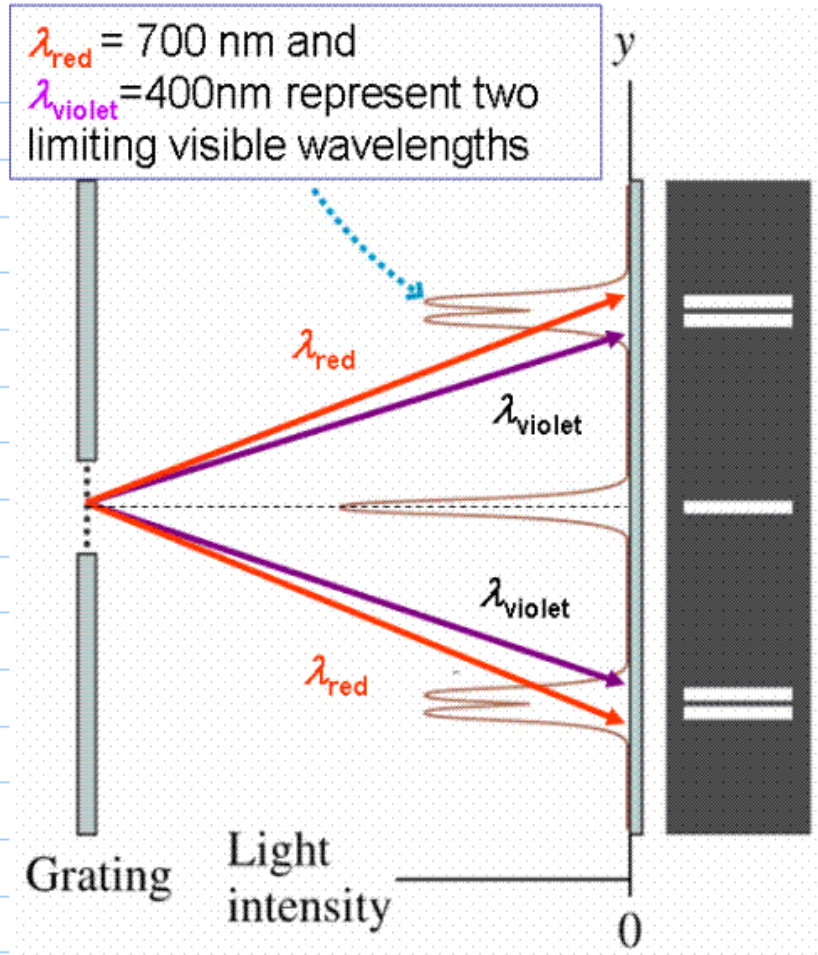
Review for Quiz 10

Problem 5: Chapter 22, problem 42. Additional: problem 30, same chapter



White light (400-700 nm) is incident on a 600 line/mm diffraction grating. What is the width of the first-order rainbow on a screen 2.0 m behind the grating?

a) Show the width of the first-order rainbow in Figure.

b) Where would you expect to see the second-order rainbow?

Main equations for grating:

$$\begin{cases} y_m(\lambda) = L \cdot \tan\theta_m(\lambda) \\ d \cdot \sin\theta_m(\lambda) = m\lambda, \quad m = 0, 1, 2, 3, \ldots \end{cases}$$

The slit spacing:

$$d = \frac{1}{600 \frac{\text{line}}{\text{mm}}} = \frac{1\,\text{mm}}{600} = \frac{10^{-3}\,\text{m}}{600} = 1.667 \cdot 10^{-6}\,\text{m}$$

For the red light ($\lambda = 700\,\text{nm}$) and $m = 1$:

$$d \cdot \sin\theta_{red} = \lambda_{red} \Rightarrow$$

$$\theta_{red} = \arcsin\frac{\lambda_{red}}{d} = \arcsin\frac{700 \cdot 10^{-9}\,\text{m}}{1.667 \cdot 10^{-6}\,\text{m}} = 24.83^\circ$$

$$y_{red} = L \cdot \tan\theta_{red} = (2.0\,\text{m}) \cdot \tan(24.83^\circ) = 92.56\,\text{cm}$$

For the violet wavelength ($\lambda = 400\,\text{nm}$) and $m = 1$:

$$\theta_{violet} = \arcsin\frac{400 \cdot 10^{-9}\,\text{m}}{1.667 \cdot 10^{-6}\,\text{m}} = 13.88^\circ \Rightarrow$$

$$y_{violet} = (2.0\,\text{m}) \cdot \tan(13.88^\circ) = 49.42\,\text{cm}$$

The width of the rainbow:

$$w = y_{red} - y_{violet} = 92.56\,\text{cm} - 49.42\,\text{cm} = 43.1\,\text{cm}$$

As an additional exercise find θ_{red} and θ_{violet} for a second order rainbow ($m = 2$).