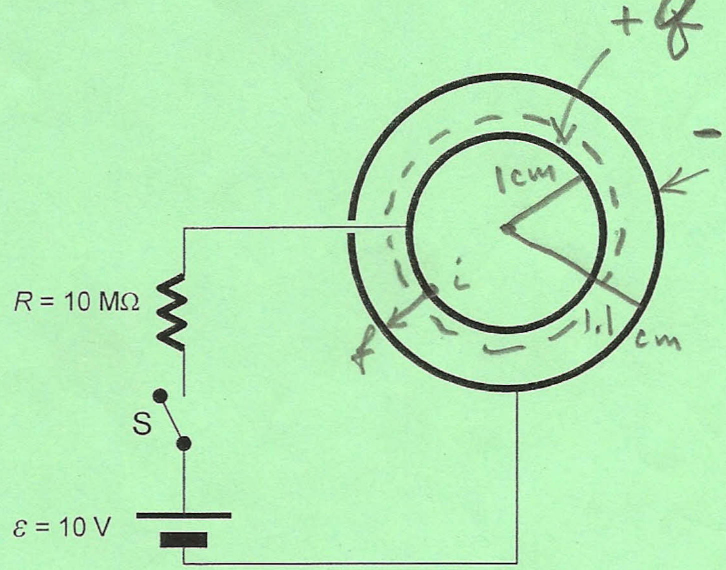
1. Figure shows a spherical capacitor. The inner sphere has a radius $a = 1$ cm and the outer sphere has a radius $b = 1.1$ cm. Assume that a maximum charge of q Coulombs can accumulate on the plates of capacitor when switch S has been closed for a long time, determine.

a) Using Gauss's law, derive an expression for the electric field inside the capacitor.
b) Determine an expression for the potential difference across the capacitor in terms of a, b, q and $K=1/(4\pi\varepsilon_0)$.
c) Derive an expression for the capacitance of the spherical capacitor in terms of a, b, q and $K=1/(4\pi\varepsilon_0)$.
d) Determine the time constant of the RC circuit.



a) $\oint_G \bar{E}\cdot d\bar{A} = \frac{q_{enc}}{\varepsilon_0}$ 4

$\boxed{E \parallel d\bar{A} \Rightarrow \bar{E}\cdot d\bar{A} = |\bar{E}|\cdot|d\bar{A}|\cdot \cos 0^\circ = E\cdot dA}$

$\oint_G \bar{E}\cdot d\bar{A} = \int_G E\cdot dA = E(r)\cdot \int_G dA = E(r)\cdot 4\pi r^2,$ $\quad q_{enc} = +q.$

$E(r)\cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E(r) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

b) $V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{s}.$ 4

$\Delta V = V_i - V_f = \int_i^f \bar{E}\cdot d\bar{s} = \int_a^b \frac{1}{4\pi\varepsilon_0}\frac{q\,dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} =$

$= \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_a^b = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\,\frac{b-a}{ab}.$

c) $C = \frac{q}{\Delta V} = \frac{\cancel{q} \cdot 4\pi\varepsilon_0 \cdot ab}{\cancel{q}(b-a)} = 4\pi\varepsilon_0 \frac{ab}{b-a}$ 1

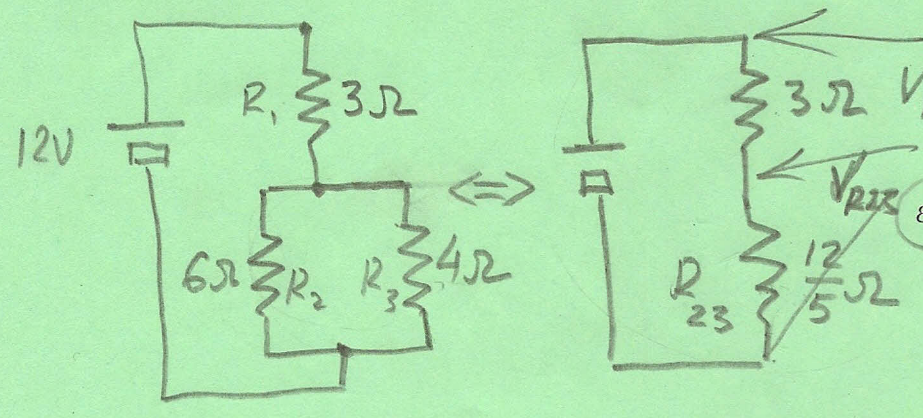
d) $\tau = RC = 4\pi\varepsilon_0 \frac{abR}{b-a} =$ 1

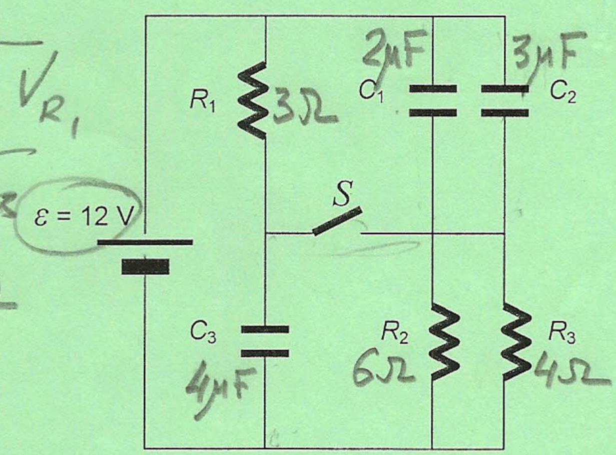
$= \left(\frac{1}{4\pi\varepsilon_0}\right)^{-1} \times \frac{10^{-2} \cdot 1{,}1 \times \cancel{10^{-2}} \cdot 10^{7}}{0.1 \cdot \cancel{10^{-2}}} = \left(9 \times 10^{9}\right)^{-1} \frac{1.1 \cdot 10^{5}}{0.1} =$

$= 0.11 \times 10^{-9} \times 11 \times 10^{5} = 1.2 \times 10^{-4}\ s = \underline{\underline{0.12\ ms}}$

2. In the circuit shown, $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 4\ \Omega$, $C_1 = 2\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 4\ \mu F$ and $\varepsilon = 12$ V. Determine amount of charge accumulated on each capacitor, C_1, C_2 and C_3, after the switch S has been (a) open for a long time, and (b) closed for a long time.

b) Switch closed for a long time





6

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{4} = \frac{2+3}{12} = \frac{5}{12}$$

$$R_{23} = \frac{12}{5}\ \Omega$$

$$V_{R_1} = \mathcal{E}\,\frac{R_1}{R_1 + R_{23}} = \frac{12 \cdot 3}{3 + \frac{12}{5}} = \frac{36}{\frac{15+12}{5}} = \frac{180}{27} = 6.666\ V$$

$$V_{R_{23}} = \mathcal{E}\,\frac{R_{23}}{R_1 + R_{23}} = \frac{12 \cdot \frac{12}{5}}{\frac{27}{5}} = \frac{\cancel{5} \cdot 12 \cdot 12 \cdot \cancel{(1/5)}}{27} = \frac{144}{27} = 5.333\ V$$

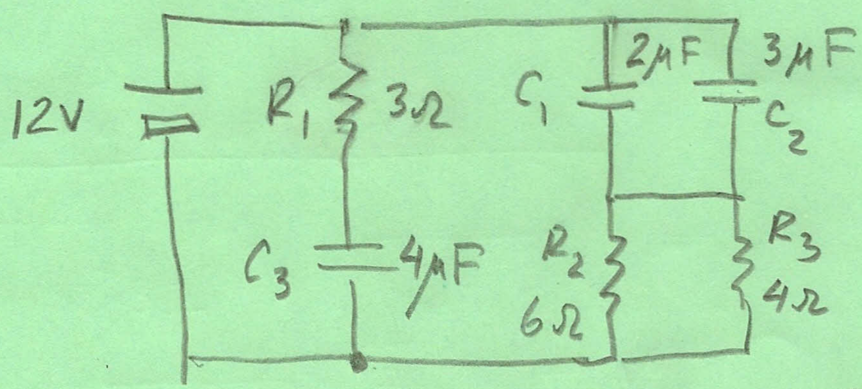
$$C = \frac{Q}{V} \Rightarrow Q = C \cdot V$$

$$Q_{C_1} = C_1 \times V_{R_1} = 2\mu F \cdot 6.666 V = 13.33\ \mu C$$

$$Q_{C_2} = C_2 \times V_{R_1} = 3\mu F \cdot 6.666 V = \mathbf{20.0}\ \mu C$$

$$Q_{C_3} = C_3 \times V_{R_{23}} = 4\mu F \cdot 5.333 V = 21.33\ \mu C$$

a) Switch open for long time. (4)

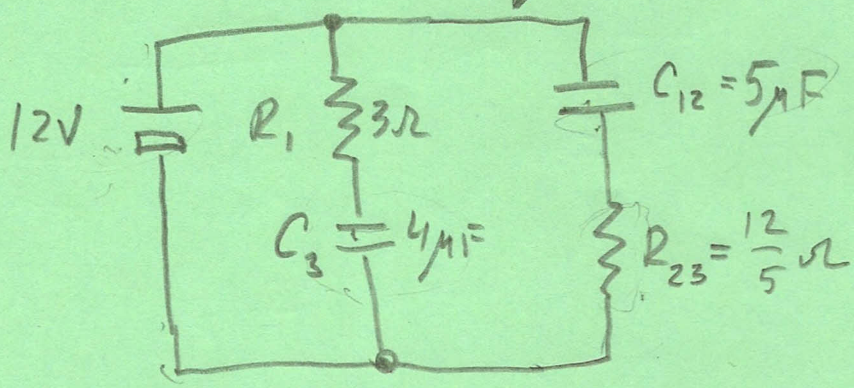


$C_{12} = C_1 + C_2 = 2+3 = 5\mu F$

$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = $

$= \frac{1}{6} + \frac{1}{4} = \frac{2+3}{12} = \frac{5}{12}.$

$R_{23} = \frac{12}{5}\,\Omega$

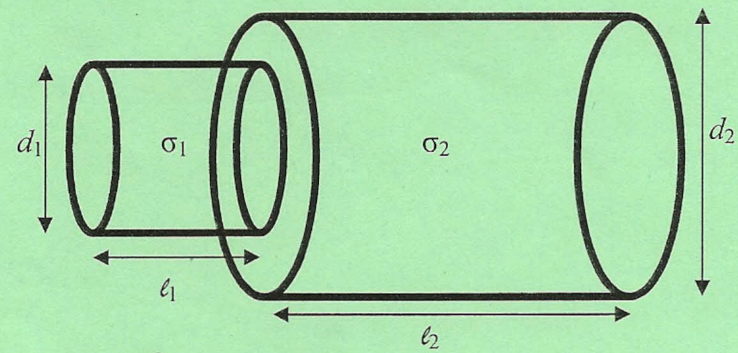


$V_{C_3} = 12V, \quad Q_{C_3} = C_3 \cdot V_{C_3} = 4\mu F \cdot 12V = \underline{\underline{48\mu C}}$

$V_{C_{12}} = 12V, \quad Q_{C_1} = C_1 \cdot V_{C_{12}} = 2\mu F \cdot 12V = \underline{\underline{24\mu C}}$

$Q_{C_2} = C_2 \cdot V_{C_{12}} = 3\mu F \cdot 12V = \underline{\underline{36\mu C}}$

3. A wire is made of two different metal segments with conductivities σ_1 and σ_2 respectively, as shown in Figure below. The two segments have different diameters and different lengths. When the wire is connected to a battery, the potential drop across each segment is the same. What is the ratio of σ_1 / σ_2 expressed in terms of ℓ_1, ℓ_2, d_1, and d_2 where ℓ and d stand for length and diameter of wire segments respectively?



$$j = \boxed{\frac{I}{A} = \sigma E}, \quad V = E \cdot \ell, \quad E = \frac{I}{A \cdot \sigma}$$

$$I_1 = I_2 \quad 2$$

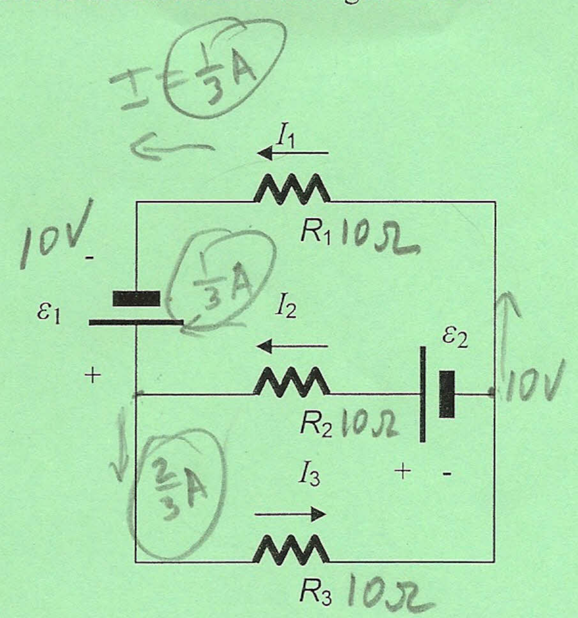
$$V_1 = V_2 \Rightarrow E_1 \cdot \ell_1 = E_2 \cdot \ell_2$$

$$\frac{\cancel{I_1} \cdot \ell_1}{A_1 \cdot \sigma_1} = \frac{\cancel{I_2} \cdot \ell_2}{A_2 \cdot \sigma_2} \quad 5$$

$$\frac{\sigma_1}{\sigma_2} = \frac{\ell_1 \cdot A_2}{A_1 \ \ell_2} = \left(\frac{\ell_1}{\ell_2}\right) \times \left(\frac{A_2}{A_1}\right) = \frac{\ell_1}{\ell_2}\left(\frac{\pi d_2^2}{\pi d_1^2}\right) = \frac{\ell_1}{\ell_2}\left(\frac{d_2}{d_1}\right)^2$$

3

4. In the circuit shown, $R_1 = R_2 = R_3 = 10\ \Omega$, and $\varepsilon_1 = \varepsilon_2 = 10$ V. Using Kirchhoff's loop and junction laws, determine the currents I_1, I_2 and I_3 flowing through R_1, R_2 and R_3 respectively, where the direction of each current is as indicated in the figure below.



Properly written equations give 7

$$\begin{cases} -I_1\cdot 10 + 10 + I_2\cdot 10 - 10 = 0 & (1) \Rightarrow I_1 = I_2 \\ -I_3\cdot 10 + 10 - I_2\cdot 10 = 0 & (2) \Rightarrow 10I_2 = 10 - 10I_3 \\ I_1 + I_2 = I_3 & (3) \end{cases}$$

$I_2 = 1 - I_3$

$\cancel{I_2 = 10I_1 + 10}$

$I_2 + I_2 = I_3, \quad 2I_2 = I_3$

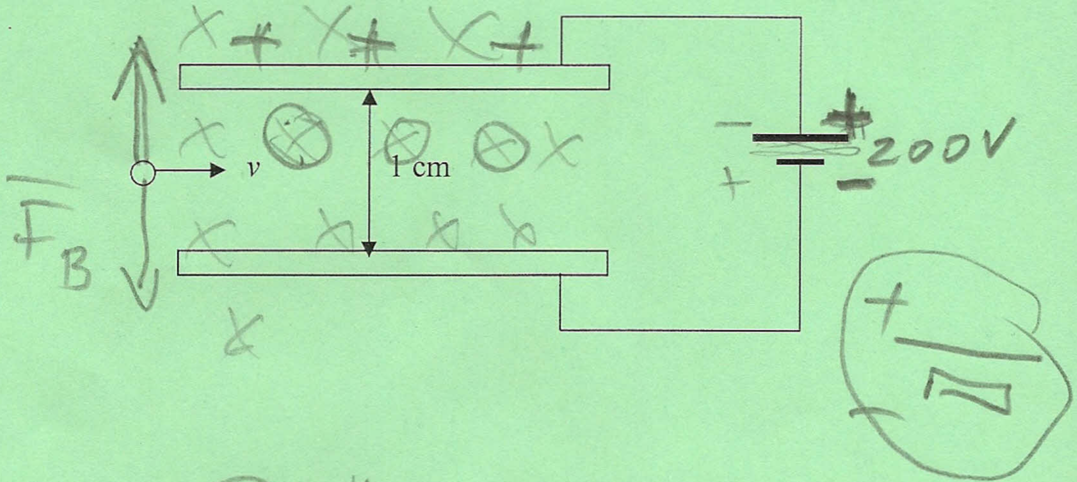
$2(1 - I_3) = I_3$.

$2 - 2I_3 = I_3$

$2 = 3I_3 \Rightarrow I_3 = \frac{2}{3}A$

$I_2 = 1 - \frac{2}{3} = \frac{1}{3}A, \quad I_1 = \frac{1}{3}A$

5. An electron travels with speed 1.0×10^7 m/s between the two parallel charged plates shown in Figure below. The plates are separated by 1.0 cm and are charged by a 200 V battery. What magnetic field strength and direction will allow the electron to pass between the plates without being deflected?



$v = 1.0 \times 10^7 \ \frac{m}{s}$

If B is in page $\otimes$ then F_B is down ↓
In this case + of the battery should be connected to the top electrode to compensate: $\bar{F}_E + \bar{F}_B = 0.$ ⟵ 5

$|F_e| = eE = e \cdot \frac{V}{d}$

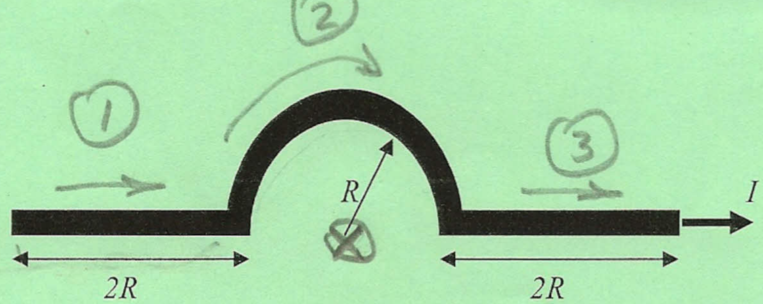
$|F_B| = |e\bar{v} \times \bar{B}| = evB.$ ⟵ 3

$\cancel{e}\frac{V}{d} = \cancel{e}vB \Rightarrow B = \frac{V}{d \cdot v} = \frac{200}{10^{-2} \cdot 10^7} =$

$= \frac{2 \cdot 10^2}{10^{-2} \cdot 10^7} = \frac{2 \cdot 10^4}{10^7} = 2 \cdot 10^{-3} \ (T).$ ⟵ 2

However the second way is also possible if B is Out of page $\odot$. In this case the top electrode is negative.

6. Current I flows in the wire shown in Figure below. Use the Biot-Savart law to find an expression for the magnetic field strength at the center of the semicircle. What is the direction of the magnetic field at this point?



$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{I\,d\bar{s}\times\hat{r}}{r^2}.$$

For regions 1 and 3

$d\bar{s} \parallel d\hat{r}$ or antiparallel. In any case

$d\bar{s}\times\hat{r} = |d\bar{s}|\cdot|\hat{r}|\cdot\sin 0^\circ = 0 \Rightarrow$ ← 2

$dB_1 = dB_3 = 0 \Rightarrow$

Total magnetic field from 1 and 3 is zero.

For region 2:

$d\bar{s} \perp d\hat{r} \Rightarrow d\bar{s}\times\hat{r} = |d\bar{s}|\times\underbrace{|\hat{r}|}_{1}\times\underbrace{\sin 90^\circ}_{1} = ds.$

$dB_2 = \frac{\mu_0}{4\pi}\frac{I\,ds}{r^2} = \frac{\mu_0}{4\pi}\frac{IR}{R^2}\cdot d\theta = \frac{I\mu_0}{4\pi\cdot R}\cdot d\theta$ ← 3

$\boxed{ds = R\cdot d\theta}$ $\quad B_2 = \int dB_2 = \int \frac{\mu_0 I}{4\pi R}\cdot d\theta =$

$= \frac{\mu_0 I}{4\pi R}\int_0^\pi d\theta = \frac{\mu_0 I\cdot\pi}{4\pi R} = \frac{\mu_0 I}{4R}$ ← 5