Relation Algebraic Models for Lambek Calculus with Modalities

Lachlan McPheat (*University College London*)
Mehrnoosh Sadrzadeh (*University College London*)
Jas Semrl (*University College London*)

Lambek Calculus of syntactic types was designed to model the grammatical structure of English. As a result, it does not allow for the structural rules of commutativity, associativity, or contraction. These operations become handy, however, when modelling other languages, and in English for sentential and discourse dependencies. Structural rules are thus added to the base logic in a controlled manner, using modalities.

The first example of modal Lambek calculi had adjoint modalities in the style of tense logic. These were introduced by Moortgat, Morrill, and Jaeger in the 1990s. They have proven to be complete with respect to frame semantics by Valentin and Kurtonina. Recently, in 2017-2022, Kanovitch et. al. introduced a new class of modalities in the style of exponentials of Linear Logic. Finding complete frame or relational semantics for them is an open problem.

The relation-algebraic models come in the form of relational ordered residuated semigroups with modalities. We present some preliminary results as well as develop some tools to reason about these structures. Additionally, we raise a number of open problems.

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