

# BLAST 2023 Book of Abstracts

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# Positive MV-Algebras

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MV-algebras extend the theory of Boolean algebras by replacing the two-element set of truth values  $\{0, 1\}$  with the unit interval  $[0, 1]$ . They provide the algebraic semantics of Łukasiewicz many-valued logic. Inspired by the extensive study of bounded distributive lattices, which are the negation-free subreducts of Boolean algebras, we aim at developing the theory of the negation-free subreducts of MV-algebras, called positive MV-algebras. These algebras can be thought of as the many-valued version of bounded distributive lattices. We axiomatize positive MV-algebras via finitely many quasi-equations. Moreover, generalizing Mundici's celebrated equivalence for MV-algebras [4], we obtain a categorical equivalence between positive MV-algebras and certain lattice-ordered monoids with units.

This talk is based on [2], [3, Ch. 4], and a joint work with P. Jipsen, T. Kroupa and S. Vannucci [1].

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# Arrow Relations in Integer Partition Lattices

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Edith Vargas-García (*Department of Mathematics, ITAM*)

Andreas Wachtel (*Department of Mathematics, ITAM*)

Integer partitions started to gain interest in 1674 when Leibniz investigated the number of ways one can write (partition) a positive integer  $n$  as a sum of positive integers. Brylawski showed that the set of all partitions of a positive integer  $n$  endowed with the dominance ordering is a complete lattice. These lattices are called lattices of integer partitions, or just partition lattices, for short. The lattices of integer partitions can be viewed as concept lattices; in this contribution we continue the investigations of Behrisch and et al. and Ganter of those lattices, and concentrate on the problem of determining their factor lattices as suggested by Ganter. This question is equivalent to characterising the compatible subcontexts, which in turn have a description in terms of so-called arrow relations. Moreover, we focus on results on these arrow relations and thus shed some light on the structure of lattices of integer partitions.

Keywords: Lattice, Integer partition, Dominance order, Arrow relation

# Pointless Parts of Completely Regular Locales

Richard Ball (*University of Denver*)

(Completely regular) frames generalize (Tychonoff) spaces; indeed, the passage from a frame to its spatial part is a well understood epireflection. But a frame also possesses an equally important pointless part, and with morphisms suitably restricted, the passage of a frame to its pointless part is also an epireflection. Our main theorem is that every frame can be uniquely represented as a subdirect product of its pointless and spatial parts, again with suitably restricted projections. We then exploit this representation by showing that any frame is determined by (what may be described as) the placement of its points in its pointless part.

Keywords: completely regular local, subdirect product, spatial frame, compact frame

# An Old Problem in Topology Coming Back to Life

Hector Barriga (*University of North Carolina at Charlotte*)

The box products problem, whether the countable box product of the real line is a normal space, is one of the few problems, if not the oldest, in general topology left unsolved. The question has consistent affirmative answers and it is unknown whether there is a model of ZFC where such space fails to be normal.

Judy Roitman made the last progress in this direction and collected literature in a survey and added two combinatorial statements, MH and Delta, that imply the normality of such space. Some recent findings of these combinatorial statements give this old problem another shot to keep it alive among topologists and set theorists.

Keywords: MH, Delta, box product problem, box product, nabla product

# Dynamics, Ramsey Theory, and Model Theory

Dana Bartošová (*University of Florida*)

The 2005 breakthrough paper by Kechris, Pestov, and Todorčević established a striking connection between Ramsey properties of classes of finitely generated first-order structures and dynamical behaviour of groups of automorphisms of countable first-order structures. I will explain this correspondence via Stone duality in the language of Boolean algebras and notions standard in topological dynamics. I will then discuss recent work with Lynn Scow on a model-theoretic transfer of the Ramsey property from the class of finite substructures of one structure to another and ask the many questions that arise from it, most notably how to understand this transfer by dynamical and group topological means.

# Algebraic Frames in Priestley Duality

Guram Bezhanishvili (*New Mexico State University*)

Sebastian Melzer (*New Mexico State University*)

By Priestley duality, the category of bounded distributive lattices is dually equivalent to the category of Priestley spaces. Specializing Priestley duality to frames provides a useful tool to study pointfree topology in the language of Priestley spaces. In this talk, we aim to continue this line of research by characterizing Priestley spaces of algebraic, arithmetic, coherent, and Stone frames. Using this machinery, we obtain new proofs of some classic duality results in pointfree topology. This includes the duality between the categories of coherent frames and spectral spaces, as well as the duality between the categories of algebraic frames and compactly based sober spaces.

Keywords: Pointfree topology, Duality theory, Algebraic frames

# McKinsey-Tarski Algebras: An Alternative Approach to Pointfree Topology

Guram Bezhanishvili (*New Mexico State University*)

Ranjitha Raviprakash (*New Mexico State University*).

An *interior algebra* is a pair  $(B, \Box)$  where  $B$  is a boolean algebra and  $\Box$  is a unary function on  $B$  satisfying the well-known Kuratowski axioms:  $\Box a \leq a$ ,  $\Box \Box a \leq \Box a$ ,  $\Box(a \wedge b) = \Box a \wedge \Box b$ , and  $\Box 1 = 1$ . Interior algebras were introduced by McKinsey and Tarski in 1944 and have since been studied extensively by numerous authors. We call a complete interior algebra a *McKinsey-Tarski algebra* or *MT-algebra*, and propose the category **MT** of MT-algebras as an alternative, more expressive, language to study point-free topology. We show that taking the open elements of an MT-algebra yields an essentially surjective functor from **MT** to the category **Frm** of frames. We also show that the well-known dual adjunction between **Frm** and the category **Top** of topological spaces extends to a dual adjunction between **MT** and **Top**, which restricts to a dual equivalence between **Top** and the category **SMT** of spatial MT-algebras. This extends the well-known dual equivalence between the categories of spatial frames and sober spaces. We also present the study of separation axioms in the language of MT-algebras, which is more expressive than the corresponding language of frames. In addition, we develop the Hofmann-Mislove theorem for MT-algebras, which allows us to obtain dual adjunctions and dual equivalences for the categories of locally compact spaces and compact Hausdorff spaces, and their corresponding categories of MT-algebras. This yields an alternative proof of Hofmann-Lawson and Isbell dualities in frame theory. We show that unlike the situation in frames, in MT-algebras spatiality is not a consequence of local compactness. In the talk we explain the reason for this discrepancy and show that it disappears once we add the  $T_D$  separation axiom, which is easily expressible in the language of MT-algebras.

Keywords: Pointfree topology, Duality theory, Interior algebras

# Partitioning the Real Line into Borel Sets

Will Brian (*University of North Carolina at Charlotte*)

The Borel partition spectrum is the set of all uncountable cardinals  $\kappa$  such that there is a partition of the real line into precisely  $\kappa$  Borel sets. In this talk I will survey some old and some new results concerning the Borel partition spectrum. I will also outline some of the most important questions surrounding it that still remain unsolved.

# That Damned Problem of Eilenberg and Schützenberger

Emily Carlson

Mehul Gupta (*University of Toronto*)

George McNulty (*University of South Carolina*)

Ross Willard (*University of Waterloo*)

In a 1976 paper, Samuel Eilenberg and M.P. Schützenberger posed a provocative problem about finite semigroups, which Mark Sapir solved in 1988: if the pseudovariety generated by a finite semigroup  $S$  is finitely axiomatizable (relative to the class of all finite semigroups), is this so because the variety generated by  $S$  is finitely axiomatizable? (Sapir’s answer: yes). The same question can be posed for any finite algebra in a finite signature, and this expanded question is what universal algebraists call the Eilenberg-Schützenberger problem.

In this talk I will state the problem more carefully and explain just how far we are from a solution. I will describe an “obvious reason” for a positive answer which, so far, works in all special cases for which the problem has been answered. Time permitting, I will describe our recent work on this problem in the context of finite-dimensional nonassociative algebras over finite fields.

Keywords: universal algebra, variety, pseudovariety, finitely axiomatizable

# The Cardinality of Infinite Sets under Determinacy

William Chan (*University of North Texas*)

We will discuss the size of familiar infinite sets under the axiom of determinacy. The talk will especially focus on the known structure of the cardinalities under the injection relation below the power set of the first uncountable cardinal under various determinacy assumptions. We will also formulate a notion of regularity and cofinality for cardinalities in this choiceless setting and survey the known results. This talk includes joint work with Stephen Jackson and Nam Trang.

## Strongly $\sigma$ -Complete Boolean Algebras: A Unicorn?

Fred Dashiell (*CECAT: Center for Excellence in Computation, Algebra, and Topology, Chapman University, Orange CA*)

The collection  $\text{clop}(X)$  of all closed-open sets of any topological space  $X$  is a Boolean algebra under finite union and intersection, and any Boolean algebra  $B$  is  $\text{clop}(X)$  in several zero-dimensional topological spaces  $X$ , where “zero-dimensional” (ZD) means that  $X$  is Hausdorff and  $\text{clop}(X)$  is a base for the topology. When  $B = \text{clop}(X)$  is a complete Boolean algebra, the ZD space  $X$  has the property that each open set is a dense subset of some element of  $B$ , and conversely. These spaces are called “extremally disconnected”. But what if  $B = \text{clop}(X)$  is only  $\sigma$ -complete (closed for countable meets and joins)? We know that for some classes of ZD  $X$ , every open set of the form  $\text{Coz}(f) = \{x : |f(x)| > 0\}$  for continuous  $f : X \rightarrow \mathbb{R}$  is a dense subset of some element of  $B$ . Spaces with this property are called “basically disconnected”. This motivates the following.

Definition: A Boolean algebra  $B$  is “strongly  $\sigma$ -complete” if every zero-dimensional space  $X$  with  $\text{clop}(X)$  isomorphic to  $B$  is basically disconnected. We easily show that for a Boolean algebra  $B$ , complete  $\implies$  strongly  $\sigma$ -complete  $\implies$   $\sigma$ -complete.

A strongly  $\sigma$ -complete Boolean algebra is a unicorn, in the sense that every  $\sigma$ -complete  $B$  might be strongly  $\sigma$ -complete, in which case the definition would be vacuous. And we have not found one which is not complete. And what could possibly be an algebraic characterization of the strongly  $\sigma$ -complete Boolean algebras?

This talk describes some aspects of these questions, but does not find a unicorn.

Keywords: Boolean algebra, zero-dimensional, strongly zero-dimensional, extremally disconnected, basically disconnected, cozero set, complete Boolean algebra, sigma-complete Boolean algebra

# Countable Fréchet-Urysohn Spaces

Alan Dow (*University of North Carolina at Charlotte*)

A space is Fréchet-Urysohn if usual converging sequences determine the limit point “operation”. It is well-known what countable spaces are. The Fréchet-Urysohn property is hereditary but not even finitely productive. The rich history of their study includes Arhangel'skii's highly influential introduction of the  $\alpha_0 - \alpha_4$  properties. The notorious Fréchet fan, collapse the  $x$ -axis in the subspace  $\{\frac{1}{n} : n = 1, \dots, \infty\} \times (\{0\} \cup \{\frac{1}{n} : n = 1, \dots, \infty\})$ , is an important test space.

Inspired by the techniques introduced by Hrušak and Ramos-García in their solution of Malykhin's problem about Fréchet-Urysohn topological groups, we pursue the study of the possible values of the  $\pi$ -weight for countable Fréchet spaces. The  $\pi$ -weight of a space is the minimum cardinality of a family of non-empty open sets such that every non-empty open set contains one.

# Decidability of Distributive $\ell$ -pregroups

Nick Galatos (*University of Denver*)

Isis Gallardo (*University of Denver*)

We show that every distributive lattice-ordered pregroup can be embedded into a functional algebra over an integral chain, thus improving the existing Cayley/Holland- style embedding theorem. We use this to show that the variety of all distributive lattice-ordered pregroups is generated by the single functional algebra on the integers. Finally, we show that the equational theory of the variety is decidable.

Keywords: Lattice-ordered pregroups, Decidability, Equational theory, Variety generation, Residuated lattices, Lattice-ordered groups, Diagrams

# Unilinear Residuated Lattices

Nikolaos Galatos (*University of Denver*)  
Xiao Zhuang (*University of Denver*).

We characterize all residuated lattices that have height equal to 3 and show that the variety they generate has continuum-many subvarieties. More generally, we study unilinear residuated lattices: their lattice is a union of disjoint incomparable chains, with bounds added. We give two general constructions of unilinear residuated lattices, provide an axiomatization and a proof-theoretic calculus for the variety they generate, and prove the finite model property for various subvarieties.

Keywords: Unilinear residuated lattices, Axiomatization, Subvarieties, Finite embeddability property

# Locally Integral Involutive Po-monoids

José Gil-Férez (*Chapman University*)

Peter Jipsen (*Chapman University*)

Siddhartha Lodhia (*Chapman University*)

We introduce and study locally integral involutive partially ordered monoids (locally integral ipo-monoids, for short). Their relevance, among other things, resides in the fact that they constitute semantics for some nonclassical logics. We will demonstrate that every locally integral ipo-monoid  $A$  decomposes in a unique way into a family of integral ones, which we call its integral components. Moreover, we will associate to  $A$  a family of monoid homomorphisms (indexed on the order of the positive cone of  $A$ ) so that the structure of  $A$  can be recovered as a glueing of its integral components along that family. Reciprocally, we will give necessary and sufficient conditions so that the Płonka sum of any family of integral ipo-monoids (indexed on a lower-bounded join-semilattice) along a family of monoid homomorphisms is an ipo-monoid.

# At the Boundary of Algebra and Logic: A History of $\aleph_1$ -free Groups

Daniel Herden (*Baylor University*)

$\aleph_1$ -free groups, abelian groups whose countable subgroups are free, play a central role in Shelah's celebrated proof that the Whitehead problem is undecidable (1974). Since then, set-theoretic constructions of  $\aleph_1$ -free groups with additional properties have become a staple, providing examples and counterexamples to a number of important questions. In this overview talk, we will trace the history of  $\aleph_1$ -free groups, discuss some of the set-theoretic techniques used, and tackle the central conceptual question: What makes  $\aleph_1$ -free groups that particularly susceptible to set theory in the first place?

# When is the complement of the diagonal of a LOTS functionally countable?

Rodrigo Hernandez-Gutierrez (*Universidad Autónoma Metropolitana*)  
Luis Enrique Gutiérrez-Domínguez (*Universidad Autónoma Metropolitana*)

A space  $X$  is functionally countable if every continuous function from  $X$  to the reals has its image countable. Recently, Tkachuk asked whether there exist uncountable linearly ordered spaces  $X$  such that  $X^2 \setminus \{(x, x) : x \in X\}$  is functionally countable. In this talk we show that such a space, if it exists, must be a Souslin line. We also show that functional countability of a Souslin line is not sufficient to provide the example required by Tkachuk's question.

Keywords: functionally countable, linearly ordered space, Aronszajn line, Souslin line

# Semigroup Conjugacy and Partition Coverings

Trevor Jack (*Illinois Wesleyan University*)

We present a solution to Open Problem 6.10 from “Four Notions of Conjugacy for Abstract Semigroups” by João Araújo, et al. The problem asks whether several notions of semigroup conjugacy are partition-covering. The solution in fact answers a generalized version of the problem, which we then use to devise a new definition of semigroup conjugacy that is not partition-covering.

Keywords: Semigroups, Conjugacy, Partitions

# On Representations of Distributive Involutive Residuated Lattices by Binary Relations

Peter Jipsen (*Chapman University*)  
Jaš Šemrl (*University College London*)

The collection of all binary relations on a set is a representable relation algebra, and if we replace the complementation and the converse operation with the operation of complement-converse then we obtain an interesting class of subreducts, known as representable weakening relation algebras, that are distributive involutive residuated lattices.

A binary relation defined on a poset is a weakening relation if the partial order acts as a both-sided compositional identity. We present a two-player game for the class of representable weakening relation algebras akin to that for the class of representable relation algebras. This enables us to define classes of abstract weakening relation algebras that approximate the quasivariety of representable weakening relation algebras and produce explicit finite axiomatizations for some of these classes. We define the class of diagonally representable weakening relation algebras and prove that it is a discriminator variety. We also provide representations for several small weakening relation algebras.

Keywords: distributive residuated lattices, weakening relation algebras, binary relations, two-player representation game

# Higher Commutator Theory

Keith Kearnes (*University of Colorado*)

The commutator operation of group theory has been generalized in many different ways. These generalizations permit one to import ideas from group theory into other areas of mathematics, but the subject has developed beyond that to include original concepts. I will start by discussing the intuition behind, and the scope of, the theory of the commutator of two or more variables and end with applications and problems.

Keywords: abelian, algebra, commutator, congruence, supernilpotence

# On the Category of Sets and Relations

Andre Kornell (*Dalhousie University*)

Sets and relations form the dagger symmetric monoidal category **Rel**. I will present a category-theoretic axiomatization of **Rel**. Then, I will argue that **Rel** can be regarded as an algebraization of classical predicate logic. Finally, I will draw an analogy between **Rel** and **Hilb**, the dagger symmetric monoidal category of Hilbert spaces and bounded operators.

# Topologizing the Space of Minimal Primes of an M-Frame

Albert Madinya (*Florida Atlantic University*)  
Papiya Bhattacharjee (*Florida Atlantic University*)

An M-frame is an algebraic frame possessing a unit and satisfying the Finite Intersection Property. Given an M-frame, call it  $L$ , we can topologize the set of minimal prime elements of  $L$ , which we will denote by  $\text{Min}(L)$ . One such way we could topologize  $\text{Min}(L)$  is with the Zariski topology as is done with the prime ideals of a commutative ring. The other is the inverse topology which has a similar construction to that of the Zariski topology. Our aim in this talk is to study these topological spaces and the interplay that exists between the topological properties of  $\text{Min}(L)$  and the frame-theoretic properties of  $L$ .

Keywords: Prime elements, compact elements, algebraic frame, hull-kernel topology, inverse topology

# Systems of Term Equations Over Finite Algebras

Peter Mayr (*University of Colorado Boulder*)

For a fixed finite algebra  $A$ , we consider the decision problem  $\text{SysTerm}(A)$ : does a given system of term equations have a solution in  $A$ ? This can be formulated as a constraint satisfaction problem (CSP) with relations the graphs of the basic operations of  $A$ . From the complexity dichotomy for CSP due to Bulatov and Zhuk, it follows that  $\text{SysTerm}(A)$  for a finite algebra  $A$  is in P if  $A$  has a not necessarily idempotent Taylor polymorphism and is NP-complete otherwise. We show more explicitly that for a finite algebra  $A$  in a congruence modular variety,  $\text{SysTerm}(A)$  is in P if the core of  $A$  is abelian and is NP-complete otherwise. Given  $A$  by the graphs of its basic operations, this condition can be decided in quasi-polynomial time.

Keywords: satisfiability, solvability of equations, complexity

# Direct Products of Bounded Fuzzy Lattices and Ordinal Products of Linear Fuzzy Posets

Joseph McDonald (*University of Alberta*)

We continue the work of Chon, as well as Mezzomo, Bedregal, and Santiago, by studying algebraic operations on bounded fuzzy lattices arising from fuzzy partially ordered sets. Chon in [1] proved that fuzzy lattices are closed under taking direct products defined using the minimum triangular norm operator. Mezzomo, Bedregal, and Santiago in [3] introduced further operations on fuzzy lattices and extended Chon's result to the case of bounded fuzzy lattices under the same minimum triangular norm direct product construction. The first contribution of this study is to strengthen their result by showing that bounded fuzzy lattices are closed under a much more general construction of direct products; namely direct products whose underlying triangular norm operators have no zero divisors. We then introduce the concept of triangular norm based ordinal products of fuzzy posets, which is then investigated within the setting of linear (totally ordered) fuzzy posets. Time permitting, various applications of these results will be discussed.

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Keywords: Fuzzy relations, Fuzzy partially ordered set, Bounded fuzzy lattice, Direct product, Triangular norm, Zero divisor, Fuzzy linear poset, Ordinal product

# Relation Algebraic Models for Lambek Calculus with Modalities

Lachlan McPheat (*University College London*)  
Mehrnoosh Sadrzadeh (*University College London*)  
Jas Semrl (*University College London*)

Lambek Calculus of syntactic types was designed to model the grammatical structure of English. As a result, it does not allow for the structural rules of commutativity, associativity, or contraction. These operations become handy, however, when modelling other languages, and in English for sentential and discourse dependencies. Structural rules are thus added to the base logic in a controlled manner, using modalities.

The first example of modal Lambek calculi had adjoint modalities in the style of tense logic. These were introduced by Moortgat, Morrill, and Jaeger in the 1990s. They have proven to be complete with respect to frame semantics by Valentin and Kurtonina. Recently, in 2017-2022, Kanovitch et. al. introduced a new class of modalities in the style of exponentials of Linear Logic. Finding complete frame or relational semantics for them is an open problem.

The relation-algebraic models come in the form of relational ordered residuated semigroups with modalities. We present some preliminary results as well as develop some tools to reason about these structures. Additionally, we raise a number of open problems.

Keywords: algebras of relations, ordered residuated semigroups with modalities, Lambek calculus with modalities

# Nilpotence, Localization, and Dualizability

Connor Meredith (*University of Colorado Boulder*)

It is currently unknown what types of solvable interval may occur in the congruence lattice of a finite, dualizable, algebra in a congruence modular variety. Recently, we have investigated the dualizability of nilpotent nonabelian algebras that belong to congruence modular varieties. A classic result of Quackenbush and Szabó and a more recent result of Nickodemus shows that a finite group is dualizable if and only if it does not have a nonabelian Sylow subgroup. A finite group is nilpotent if and only if it is a direct product of groups of prime power order, but the connection between nilpotence and prime power direct decomposition seen in groups does not extend to the general setting. Instead, there is a second, distinct, form of nilpotence called supernilpotence. Supernilpotence was defined by Aichinger and Mudrinski for algebras in congruence permutable varieties and makes use of the higher commutator defined by Bulatov. There is no general implication between nilpotence and supernilpotence, but in the case of finite algebras, supernilpotence implies nilpotence. It is known that if  $A$  is a finite nilpotent algebra in a congruence modular variety, then the presence of a supernilpotent nonabelian congruence in any algebra in the prevariety generated by  $A$  prevents  $A$  from being dualizable.

In this talk, we will discuss several new results concerning the relationship between nilpotence and dualizability. One such result is:

**Theorem.** *Let  $N > 0$ . There exists a dualizable nilpotent nonabelian algebra of size  $N$  (of finite type in a congruence modular variety) if and only if  $N$  is not a prime power.*

The forward direction of this equivalence is a special case of a Theorem of Bentz and Mayr. We will present algebras that witness the converse. The exact role that supernilpotence plays in preventing the dualizability of finite nilpotent algebras has not been fully determined. Let  $A$  be a finite nilpotent algebra. On one hand, if  $\text{ISP}(A)$  contains an algebra with a nonabelian supernilpotent congruence, then  $A$  is nondualizable. Bentz and Mayr ask if the absence of such an algebra is enough to guarantee the dualizability  $A$ . We provide a negative answer:

**Theorem.** *There exists a finite nilpotent algebra  $A$  such that*

1. *for each  $B$  in  $\text{ISP}(A)$ , every  $k$ -supernilpotent congruence of  $B$  with  $k \geq 2$  is Abelian and*
2.  *$A$  is (inherently) nondualizable.*

Our construction of such an algebra relies on the existence of a supernilpotent nonabelian localization and an appeal to the following theorem.

**Theorem.** *Let  $A$  be a finite algebra and let  $e$  be an idempotent unary term operation of  $A$ . If the localization of  $A$  to the neighborhood  $e(A)$  is nondualizable, then  $A$  is nondualizable.*

We will finish the talk by discussing the limitations of our technique that are brought about by the interaction between localization and the higher commutator.

**Theorem.** *Let  $A$  be a Mal'cev algebra with an idempotent unary term operation  $e$ . Suppose  $e(A)$  is a generating set for  $A$ . For each binary relation  $\theta$  of  $A$ , let  $\theta^*$  denote the congruence of  $A$  generated by  $\theta$  and let  $\theta_*$  denote the restriction of  $\theta$  to  $e(A)$ . Then for any congruences  $\alpha_1, \dots, \alpha_n$  of  $e(A)$ ,*

$$[\alpha_1^*, \dots, \alpha_n^*]_* = [\alpha_1, \dots, \alpha_n].$$

Acknowledgments: This is a joint work with Dr. Keith Kearnes.

Keywords: Supernilpotence, Higher Commutator, Dualizability, Localization

# Algebras from Finite Group Actions and a Question of Eilenberg and Schützenberger

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In 1976, S. Eilenberg and M.P. Schützenberger posed the following Diabolical question: if  $A$  is a finite algebraic structure,  $\Sigma$  is the set of all identities true in  $A$ , and there exists a finite subset  $F$  of  $\Sigma$  such that  $F$  and  $\Sigma$  have exactly the same finite models, must there also exist a finite subset  $F'$  of  $\Sigma$  such that  $F'$  and  $\Sigma$  have exactly the same finite and infinite models? (That is, must the identities of  $A$  be “finitely based”?). It is known that any counter example to their question must be inherently nonfinitely based (INFB) but not inherently nonfinitely based in the finite sense (INFBfin). In this talk, I will show that the algebras constructed by Lawrence and Willard from group action do not provide a counter example to this question. If time permits, I will give the first known examples of inherently nonfinitely based “automatic algebras” constructed from group actions.

Keywords: Finite Algebras, inherently nonfinitely based, Group Action

## On pseudovarieties satisfying $V \circledast G = EV$

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By a pseudovariety we mean a class of finite semigroups that is closed under homomorphic images, subsemigroups and finite products. A relational morphism between semigroups  $S$  and  $T$  is a relation  $S \rightarrow T$  with domain  $S$  which, as a subset of  $S \times T$ , is a subsemigroup. The group kernel  $K(S)$  of  $S$  is the intersection of the subsemigroups  $\tau^{-1}(1)$  over all relational morphisms  $\tau: S \rightarrow G$  into finite groups. For finite semigroups,  $K(S)$  is a subsemigroup containing the idempotents that is closed under weak conjugation: if  $s'$  is a weak inverse of  $s$ , that is, if  $s'ss' = s'$ , then  $sK(S)s' \cup s'K(S)s \subseteq K(S)$ . Rhodes' Type II Conjecture states that, in case  $S$  is finite, the group kernel  $K(S)$  is the smallest subsemigroup of  $S$  containing the idempotents and closed under weak conjugation. The Mal'cev product of two pseudovarieties  $V$  and  $W$  is defined as follows:

$$V \circledast W = \{M \mid \text{there is a relational morphism } \tau: M \rightarrow N \text{ with } N \in W \\ \text{and such that } \tau^{-1}(e) \in V \text{ for every idempotent } e \in N\}.$$

Let  $S$  be a finite semigroup and  $V$  be a pseudovariety. We have  $S \in V \circledast G$  if and only if  $K(S) \in V$  where  $G$  is the pseudovariety of all finite groups. In addition,  $S \in EV$  if and only if the subsemigroup  $\langle E(S) \rangle$  generated by the idempotents belongs to  $V$ . Hence, the inclusion  $V \circledast G \subseteq EV$  always holds. The purpose of this presentation is to discuss the results of a study that focused on cases when the pseudovariety equality

$$V \circledast G = EV \tag{1}$$

holds.

A related problem, which has also been investigated is to consider instead the equation

$$V * G = EV, \tag{2}$$

where the star denotes semidirect product of pseudovarieties. There are connections between the two problems since  $V * G$  is always contained in  $V \circledast G$ , with equality holding whenever (but not exclusively)  $V$  is local. Thus, for a pseudovariety  $V$  to be a solution of the equation (1) is a weaker property than to be a solution of (2). For instance, while the fact that the pseudovariety  $J$  (of all finite  $\mathcal{J}$ -trivial semigroups), which is not local, satisfies (1) is not very difficult, it is a much deeper fact that  $J$  also satisfies (2). On the other hand, for the pseudovarieties  $DS$  and  $DA$ , of all finite semigroups in which all regular elements respectively lie in groups or are idempotents, are both local and solutions of both equations. A whole interval of solutions of both equations is  $[Sl, Com]$  where  $Sl$  and  $Com$  are the pseudovarieties of all finite, respectively, semilattices and commutative semigroups. Indeed, by a theorem of Ash,  $Sl$  is a solution of (2) and, clearly,  $ESl = ECom$ . While the pseudovariety  $Sl$  is local,  $Com$  is not. During the presentation, we will derive pseudoidentities that can function as either solutions of (1) for all pseudovarieties that adhere to them or as counterexamples. Initially, we were motivated to investigate the pseudovariety equation (1) for a pseudovariety

$\mathcal{V}$  satisfying  $SI \subseteq \mathcal{V} \subseteq DA$ . However, we later found out that there are subpseudovarieties within  $\mathcal{J}$ , situated between  $SI$  and  $DA$ , which do not satisfy the pseudovariety equation (1). We provide a set of semigroups where a pseudovariety includes at least one of the semigroups mentioned in the set if and only if the pseudovariety comprises a subpseudovariety that does not satisfy the pseudovariety equation (1). Moreover, a pseudovariety does not include any semigroups of the set if and only if the pseudovariety is in  $EB$  where  $B = \llbracket x^2 = x \rrbracket$ .

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Keywords: semigroup, pseudovariety, Mal’cev product, relational morphism, idempotent-generated subsemigroup

# Commutator Theory for Specific Classes of Algebras: A Case Study

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What is the center of an inverse semigroup? Are Moufang loops of odd order solvable? Why there are no left self-distributive quasigroups of order  $4k+2$ ? The abstract commutator theory has been developed for congruence modular varieties, and successfully applied to many problems of general nature, such as the finite basis problem. However, applying the general theory to a specific class of algebras is not always straightforward. I will summarize our attempts to adapt the commutator theory to loops, inverse semigroups and quandles, answering the aforementioned questions.

# Structural Analysis on the Projective Space of $\mathbb{Z}_2^4$

Amanda Tran (*University of Massachusetts, Boston*)

Given the projective space of  $\mathbb{Z}_2^4$ , there are fifteen projective points and thirty-five projective lines. The projective line complex admissibility problem seeks to describe and generalize the underlying structures that separate admissible (linearly independent) versus inadmissible (linearly redundant) complexes. Specifically, the line complex problem is used in the context of Radon Transforms over these projective points. This project addresses necessary conditions that contribute to admissible and inadmissible linear structures using discrete analysis, vector analysis, linear algebra, and discrete geometry. In particular, we are interested in generalized classes of minimally inadmissible collections of lines, their associated geometry, and its dependence on the "Even Incident Condition" (which is proven and explored in this project.)

Keywords: projective geometry, finite geometry, discrete geometry, algebraic geometry, incidence geometry

# Clonoids Between Abelian Groups

Patrick Wynne (*University of Colorado Boulder*)

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A clonoid from an algebra  $\mathbb{A}$  to an algebra  $\mathbb{B}$  is a set of functions from finite powers of  $A$  into  $B$  that is closed first with respect to the operations of  $\mathbb{A}$  and next with respect to the operations of  $\mathbb{B}$ . We investigate clonoids from one finite abelian group to another. These structures arise in the description of nilpotent algebras in congruence modular varieties. If the abelian groups are of non-coprime order then the number of clonoids from  $\mathbb{A}$  to  $\mathbb{B}$  is countably infinite. For distinct primes  $p$  and  $q$  we show that every clonoid from  $\mathbb{Z}_{p^n}$  to  $\mathbb{Z}_q$  is generated by the subset of  $n$ -ary functions. Thus there are finitely many such clonoids. This is joint work with Peter Mayr.

Keywords: clonoids, clones, abelian groups, nilpotent algebras

# Infinite-dimensional Ramsey Theory for Binary Free-amalgamation Classes

Andy Zucker (*University of Waterloo*)  
Natasha Dobrinen (*University of Notre Dame*)

When one attempts to generalize Ramsey's theorem to color the infinite subsets of the natural numbers, one must place definability constraints on the colorings considered. Eleuthuck introduces a finer topology on the infinite subsets of naturals and shows that for any finite coloring of this space which has the Property of Baire with respect to this topology, one can find an infinite subset of naturals all of whose further infinite subsets receive the same color. Upon attempting to generalize this result to countably infinite structures, then in addition to these definability constraints, the theory of big Ramsey degrees places a fundamental limitation on how large of a subspace of structures one can expect such a generalization to hold. In recent joint work with Natasha Dobrinen, we state and prove such a generalization for Fraïssé limits of finitely constrained binary free amalgamation classes and show that this result is in many ways best possible.

Keywords: Structural Ramsey theory, Ramsey spaces. binary structures