Countable Fréchet-Urysohn Spaces

Alan Dow (University of North Carolina at Charlotte)

A space is Fréchet-Urysohn if usual converging sequences determine the limit point "operation". It is well-known what countable spaces are. The Fréchet-Urysohn property is hereditary but not even finitely productive. The rich history of their study includes Arhangelskii's highly influential introduction of the $\alpha_0 - \alpha_4$ properties. The notorious Fréchet fan, collapse the x-axis in the subspace $\{\frac{1}{n} : n = 1, \dots, \infty\} \times (\{0\} \cup \{\frac{1}{n} : n = 1, \dots, \infty\})$, is an important test space.

Inspired by the techniques introduced by Hrušak and Ramos-García in their solution of Malykhin's problem about Fréchet-Urysohn topological groups, we pursue the study of the possible values of the π -weight for countable Fréchet spaces. The π -weight of a space is the minimum cardinality of a family of non-empty open sets such that every non-empty open set contains one.