Locally Integral Involutive Po-monoids

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We introduce and study locally integral involutive partially ordered monoids (locally integral ipo-monoids, for short). Their relevance, among other things, resides in the fact that they constitute semantics for some nonclassical logics. We will demonstrate that every locally integral ipo-monoid A decomposes in a unique way into a family of integral ones, which we call its integral components. Moreover, we will associate to A a family of monoid homomorphisms (indexed on the order of the positive cone of A) so that the structure of A can be recovered as a glueing of its integral components along that family. Reciprocally, we will give necessary and sufficient conditions so that the Płonka sum of any family of integral ipo-monoids (indexed on a lower-bounded join-semilattice) along a family of monoid homomorphisms is an ipo-monoid.