BLAST 2023 Wednesday Schedule

8:45 - 9:00 – coffee

9:00 - 9:50 – Marco Abbadini (plenary talk)
Positive MV-algebras

9:50 - 10:20 – coffee break

10:20 - 10:50 – Richard Ball
Pointless parts of completely regular locales

10:55 - 11:25 – Sebastian Melzer
Algebraic frames in Priestley duality

11:30 - 12:00 – Ranjitha Raviprakash
McKinsey-Tarski algebras: an alternative approach to pointfree topology

12:00 - 1:00 – lunch (provided)

1:00 - 1:50 – Andre Kornell (plenary talk)
On the category of sets and relations

2:00 - 2:30 – Connor Meredith
Nilpotence, localization, and dualizability

10:55 - 11:25 – Xiao Zhuang
Unilinear residuated lattices

3:05 - 3:35 – coffee break

3:35 - 4:05 – Patrick Wynne
Clonoids between abelian groups

4:10 - 5:00 – Keith Kearnes (second tutorial)
Higher commutator theory

Plenary Talk: Positive MV-Algebras

Marco Abbadini (*University of Salerno, Italy*)
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MV-algebras extend the theory of Boolean algebras by replacing the two-element set of truth values $\{0, 1\}$ with the unit interval $[0, 1]$. They provide the algebraic semantics of Lukasiewicz many-valued logic. Inspired by the extensive study of bounded distributive lattices, which are the negation-free subreducts of Boolean algebras, we aim at developing the theory of the negation-free subreducts of MV-algebras, called positive MV-algebras. These algebras can be thought of as the many-valued version of bounded distributive lattices. We axiomatize positive MV-algebras via finitely many quasi-equations. Moreover, generalizing Mundici's celebrated equivalence for MV-algebras [4], we obtain a categorical equivalence between positive MV-algebras and certain lattice-ordered monoids with units.

This talk is based on [2], [3, Ch. 4], and a joint work with P. Jipsen, T. Kroupa and S. Vannucci [1].

References

[1] M. Abbadini. "A finite axiomatization of positive MV-algebras". In: *Algebra Universalis* 83:28.10 (2022), pp. 891–921. DOI: https://doi.org/10.1007/s00012-022-00776-3.

[2] M. Abbadini. "Equivalence á la Mundici for commutative lattice-ordered monoids". In: *Algebra Universalis* 82:45 (2021). DOI: https://doi.org//10/1007/s00012-021-00736-3.

[3] M. Abbadini. "On the axiomatisability of the dual of compact ordered spaces. PhD thesis". In: *University of Milan* (2021). DOI: https://air.unimi.it/retrieve/handle/2434/812809/1698986/phd_unimi_R11882.pdf..

[4] D. Mundici. "Interpretation of AF C^*-algebras in Łukasiewicz sentential calculus. *J. Funct. Anal.*" In: 65(1):15–63 (1986). DOI: https://doi.org/10.1016/0022-1236(86)90015-7.

Pointless Parts of Completely Regular Locales

Richard Ball (*University of Denver*)

(Completely regular) frames generalize (Tychonoff) spaces; indeed, the passage from a frame to its spatial part is a well understood epireflection. But a frame also possesses an equally important pointless part, and with morphisms suitably restricted, the passage of a frame to its pointless part is also an epireflection. Our main theorem is that every frame can be uniquely represented as a subdirect product of its pointless and spatial parts, again with suitably restricted projections. We then exploit this representation by showing that any frame is determined by (what may be described as) the placement of its points in its pointless part.

Algebraic Frames in Priestley Duality

Guram Bezhanishvili (*New Mexico State University*)
Sebastian Melzer (*New Mexico State University*)

By Priestley duality, the category of bounded distributive lattices is dually equivalent to the category of Priestley spaces. Specializing Priestley duality to frames provides a useful tool to study pointfree topology in the language of Priestley spaces. In this talk, we aim to continue this line of research by characterizing Priestley spaces of algebraic, arithmetic, coherent, and Stone frames. Using this machinery, we obtain new proofs of some classic duality results in pointfree topology. This includes the duality between the categories of coherent frames and spectral spaces, as well as the duality between the categories of algebraic frames and compactly based sober spaces.

McKinsey-Tarski Algebras: An Alternative Approach to Pointfree Topology

Guram Bezhanishvili (*New Mexico State University*)
Ranjitha Raviprakash (*New Mexico State University*).

An *interior algebra* is a pair $(B, \Box)$ where B is a boolean algebra and $\Box$ is a unary function on B satisfying the well-known Kuratowski axioms: $\Box a \leq a$, $\Box\Box a \leq \Box a$, $\Box(a \wedge b) = \Box a \wedge \Box b$, and $\Box 1 = 1$. Interior algebras were introduced by McKinsey and Tarski in 1944 and have since been studied extensively by numerous authors. We call a complete interior algebra a *McKinsey-Tarski algebra* or *MT-algebra*, and propose the category MT of MT-algebras as an alternative, more expressive, language to study point-free topology. We show that taking the open elements of an MT-algebra yields an essentially surjective functor from MT to the category Frm of frames. We also show that the well-known dual adjunction between Frm and the category Top of topological spaces extends to a dual adjunction between MT and Top, which restricts to a dual equivalence between Top and the category SMT of spatial MT-algebras. This extends the well-known dual equivalence between the categories of spatial frames and sober spaces. We also present the study of separation axioms in the language of MT-algebras, which is more expressive than the corresponding language of frames. In addition, we develop the Hofmann-Mislove theorem for MT-algebras, which allows us to obtain dual adjunctions and dual equivalences for the categories of locally compact spaces and compact Hausdorff spaces, and their corresponding categories of MT-algebras. This yields an alternative proof of Hofmann-Lawson and Isbell dualities in frame theory. We show that unlike the situation in frames, in MT-algebras spatiality is not a consequence of local compactness. In the talk we explain the reason for this discrepancy and show that it disappears once we add the T_D separation axiom, which is easily expressible in the language of MT-algebras.

Plenary Talk: On the Category of Sets and Relations

Andre Kornell (*Dalhousie University*)

Sets and relations form the dagger symmetric monoidal category **Rel**. I will present a category-theoretic axiomatization of **Rel**. Then, I will argue that **Rel** can be regarded as an algebraization of classical predicate logic. Finally, I will draw an analogy between **Rel** and **Hilb**, the dagger symmetric monoidal category of Hilbert spaces and bounded operators.

Nilpotence, Localization, and Dualizability

Connor Meredith (*University of Colorado Boulder*)

It is currently unknown what types of solvable interval may occur in the congruence lattice of a finite, dualizable, algebra in a congruence modular variety. Recently, we have investigated the dualizability of nilpotent nonabelian algebras that belong to congruence modular varieties. A classic result of Quackenbush and Szabó and a more recent result of Nickodemus shows that a finite group is dualizable if and only if it does not have a nonabelian Sylow subgroup. A finite group is nilpotent if and only if it is a direct product of groups of prime power order, but the connection between nilpotence and prime power direct decomposition seen in groups does not extend to the general setting. Instead, there is a second, distinct, form of nilpotence called supernilpotence. Supernilpotence was defined by Aichinger and Mudrinski for algebras in congruence permutable varieties and makes use of the higher commutator defined by Bulatov. There is no general implication between nilpotence and supernilpotence, but in the case of finite algebras, supernilpotence implies nilpotence. It is known that if A is a finite nilpotent algebra in a congruence modular variety, then the presence of a supernilpotent nonabelian congruence in any algebra in the prevariety generated by A prevents A from being dualizable.

In this talk, we will discuss several new results concerning the relationship between nilpotence and dualizability. One such result is:

Theorem. *Let $N > 0$. There exists a dualizable nilpotent nonabelian algebra of size N (of finite type in a congruence modular variety) if and only if N is not a prime power.*

The forward direction of this equivalence is a special case of a Theorem of Bentz and Mayr. We will present algebras that witness the converse. The exact role that supernilpotence plays in preventing the dualizability of finite nilpotent algebras has not been fully determined. Let A be a finite nilpotent algebra. On one hand, if $\mathrm{ISP}(A)$ contains an algebra with a nonabelian supernilpotent congruence, then A is nondualizable. Bentz and Mayr ask if the absence of such an algebra is enough to guarantee the dualizability A. We provide a negative answer:

Theorem. *There exists a finite nilpotent algebra A such that*

1. *for each B in* $\mathrm{ISP}(A)$*, every k-supernilpotent congruence of B with $k \geq 2$ is Abelian and*
2. *A is (inherently) nondualizable.*

Our construction of such an algebra relies on the existence of a supernilpotent nonabelian localization and an appeal to the following theorem.

Theorem. *Let A be a finite algebra and let e be an idempotent unary term operation of A. If the localization of A to the neighborhood* $e(A)$ *is nondualizable, then A is nondualizable.*

We will finish the talk by discussing the limitations of our technique that are brought about by the interaction between localization and the higher commutator.

Theorem. *Let A be a Mal'cev algebra with an idempotent unary term operation e. Suppose* $e(A)$ *is a generating set for A. For each binary relation θ of A, let θ^* denote the congruence of A generated by θ and let θ_* denote the restriction of θ to* $e(A)$*. Then for any congruences $\alpha_1, \dots, \alpha_n$ of* $e(A)$*,*

$$[\alpha_1^*, \dots, \alpha_n^*]_* = [\alpha_1, \dots, \alpha_n].$$

Acknowledgments: This is a joint work with Dr. Keith Kearnes.

Unilinear Residuated Lattices

Nikolaos Galatos (*University of Denver*)
Xiao Zhuang (*University of Denver*).

We characterize all residuated lattices that have height equal to 3 and show that the variety they generate has continuum-many subvarieties. More generally, we study unilinear residuated lattices: their lattice is a union of disjoint incomparable chains, with bounds added. We give two general constructions of unilinear residuated lattices, provide an axiomatization and a proof-theoretic calculus for the variety they generate, and prove the finite model property for various subvarieties.

Clonoids Between Abelian Groups

Patrick Wynne (*University of Colorado Boulder*)
Peter Mayr (*University of Colorado Boulder*)

A clonoid from an algebra $\mathbb{A}$ to an algebra $\mathbb{B}$ is a set of functions from finite powers of A into B that is closed first with respect to the operations of $\mathbb{A}$ and next with respect to the operations of $\mathbb{B}$. We investigate clonoids from one finite abelian group to another. These structures arise in the description of nilpotent algebras in congruence modular varieties. If the abelian groups are of non-coprime order then the number of clonoids from $\mathbb{A}$ to $\mathbb{B}$ is countably infinite. For distinct primes p and q we show that every clonoid from $\mathbb{Z}_{p^n}$ to $\mathbb{Z}_q$ is generated by the subset of n-ary functions. Thus there are finitely many such clonoids. This is joint work with Peter Mayr.

Tutorial: Higher Commutator Theory

Keith Kearnes (*University of Colorado*)

The commutator operation of group theory has been generalized in many different ways. These generalizations permit one to import ideas from group theory into other areas of mathematics, but the subject has developed beyond that to include original concepts. I will start by discussing the intuition behind, and the scope of, the theory of the commutator of two or more variables and end with applications and problems.