BLAST 2023 Thursday Schedule

8:45 - 9:00 – coffee

9:00 - 9:50 – Dana Bartošová (plenary talk)
Dynamics, Ramsey theory, and model theory

9:50 - 10:20 – coffee break

10:20 - 10:50 – Rodrigo Hernández-Gutiérrez
When is the complement of the diagonal of a LOTS functionally countable?

10:55 - 11:25 – Hector Barriga-Acosta
An old problem in topology coming back to life

11:30 - 12:00 – Alan Dow
Countable Fréchet-Urysohn Spaces

12:00 - 1:00 – lunch (provided)

1:00 - 1:50 – Daniel Herden (plenary talk)
At the boundary of algebra and logic: a history of $\aleph_1$*-free groups*

2:00 - 2:30 – Trevor Jack
Semigroup conjugacy and partition coverings

2:35 - 3:05 – Fred Dashiell
Strongly σ*-complete Boolean algebras: a unicorn?*

3:05 - 3:35 – coffee break

3:35 - 4:05 – Will Brian
Partitioning the real line into Borel sets

4:10 - 5:00 – William Chan (plenary talk)
The cardinality of infinite sets under determinacy

6:00 - ?:?? – Dinner and drinks

Plenary Talk: DYNAMICS, RAMSEY THEORY, AND MODEL THEORY

Dana Bartošová (*University of Florida*)

The 2005 breakthrough paper by Kechris, Pestov, and Todorčević established a striking connection between Ramsey properties of classes of finitely generated first-order structures and dynamical behaviour of groups of automorphisms of countable first-order structures. I will explain this correspondence via Stone duality in the langage of Boolean algebras and notions standard in topological dynamics. I will then discuss recent work with Lynn Scow on a model-theoretic transfer of the Ramsey property from the class of finite substructures of one structure to another and ask the many questions that arise from it, most notably how to understand this transfer by dynamical and group topological means.

WHEN IS THE COMPLEMENT OF THE DIAGONAL OF A LOTS FUNCTIONALLY COUNTABLE?

Rodrigo Hernandez-Gutierrez (*Universidad Autónoma Metropolitana*)
Luis Enrique Gutiérrez-Dominguez (*Universidad Autónoma Metropolitana*)

A space X is functionally countable if every continuous function from X to the reals has its image countable. Recently, Tkachuk asked whether there exist uncountable linearly ordered spaces X such that $X^2 \setminus \{\langle x, x\rangle : x \in X\}$ is functionally countable. In this talk we show that such a space, if it exists, must be a Souslin line. We also show that functional countability of a Souslin line is not sufficient to provide the example required by Tkachuk's question.

AN OLD PROBLEM IN TOPOLOGY COMING BACK TO LIFE

Hector Barriga (*University of North Carolina at Charlotte*)

The box products problem, whether the countable box product of the real line is a normal space, is one of the few problems, if not the oldest, in general topology left unsolved. The question has consistent affirmative answers and it is unknown whether there is a model of ZFC where such space fails to be normal.

Judy Roitman made the last progress in this direction and collected literature in a survey and added two combinatorial statements, MH and Delta, that imply the normality of such space. Some recent findings of these combinatorial statements give this old problem another shot to keep it alive among topologists and set theorists.

Countable Fréchet-Urysohn Spaces

Alan Dow (*University of North Carolina at Charlotte*)

A space is Fréchet-Urysohn if usual converging sequences determine the limit point "operation". It is well-known what countable spaces are. The Fréchet-Urysohn property is hereditary but not even finitely productive. The rich history of their study includes Arhangelskii's highly influential introduction of the $\alpha_0 - \alpha_4$ properties. The notorious Fréchet fan, collapse the x-axis in the subspace $\{\frac{1}{n} : n = 1, \ldots \infty\} \times (\{0\} \cup \{\frac{1}{n} : n = 1, \ldots \infty\})$, is an important test space.

Inspired by the techniques introduced by Hrušak and Ramos-García in their solution of Malykhin's problem about Fréchet-Urysohn topological groups, we pursue the study of the possible values of the π-weight for countable Fréchet spaces. The π-weight of a space is the minimum cardinality of a family of non-empty open sets such that every non-empty open set contains one.

Plenary Talk: At the Boundary of Algebra and Logic: A History of $\aleph_1$-free Groups

Daniel Herden (*Baylor University*)

$\aleph_1$-free groups, abelian groups whose countable subgroups are free, play a central role in Shelah's celebrated proof that the Whitehead problem is undecidable (1974). Since then, set-theoretic constructions of $\aleph_1$-free groups with additional properties have become a staple, providing examples and counterexamples to a number of important questions. In this overview talk, we will trace the history of $\aleph_1$-free groups, discuss some of the set-theoretic techniques used, and tackle the central conceptual question: What makes $\aleph_1$-free groups that particularly susceptible to set theory in the first place?

Semigroup Conjugacy and Partition Coverings

Trevor Jack (*Illinois Wesleyan University*)

We present a solution to Open Problem 6.10 from "Four Notions of Conjugacy for Abstract Semigroups" by João Araújo, et al. The problem asks whether several notions of semigroup conjugacy are partition-covering. The solution in fact answers a generalized version of the problem, which we then use to devise a new definition of semigroup conjugacy that is not partition-covering.

Strongly σ-Complete Boolean Algebras: A Unicorn?

Fred Dashiell (*CECAT: Center for Excellence in Computation, Algebra, and Topology, Chapman University, Orange CA*)

The collection clop(X) of all closed-open sets of any topological space X is a Boolean algebra under finite union and intersection, and any Boolean algebra B is clop(X) in several zero-dimensional topological spaces X, where "zero-dimensional" (ZD) means that X is Hausdorff and clop(X) is a base for the topology. When $B = \text{clop}(X)$ is a complete Boolean algebra, the ZD space X has the property that each open set is a dense subset of some element of B, and conversely. These spaces are called "extremally disconnected". But what if $B = \text{clop}(X)$ is only σ-complete (closed for countable meets and joins)? We know that for some classes of ZD X, every open set of the form $\text{Coz}(f) = x : |f(x)| > 0$ for continuous $f : X \to R$ is a dense subset of some element of B. Spaces with this property are called "basically disconnected". This motivates the following.

Definition: A Boolean algebra B is "strongly σ-complete" if every zero-dimensional space X with clop(X) isomorphic to B is basically disconnected. We easily show that for a Boolean algebra B, complete $\implies$ strongly σ-complete $\implies$ σ-complete.

A strongly σ-complete Boolean algebra is a unicorn, in the sense that every σ-complete B might be strongly σ-complete, in which case the definition would be vacuous. And we have not found one which is not complete. And what could possibly be an algebraic characterization of the strongly σ-complete Boolean algebras?

This talk describes some aspects of these questions, but does not find a unicorn.

Partitioning the Real Line into Borel Sets

Will Brian (*University of North Carolina at Charlotte*)

The Borel partition spectrum is the set of all uncountable cardinals κ such that there is a partition of the real line into precisely κ Borel sets. In this talk I will survey some old and some new results concerning the Borel partition spectrum. I will also outline some of the most important questions surrounding it that still remain unsolved.

Plenary Talk: The Cardinality of Infinite Sets under Determinacy

William Chan (*University of North Texas*)

We will discuss the size of familiar infinite sets under the axiom of determinacy. The talk will especially focus on the known structure of the cardinalities under the injection relation below the power set of the first uncountable cardinal under various determinacy assumptions. We will also formulate a notion of regularity and cofinality for cardinalities in this choiceless setting and survey the known results. This talk includes joint work with Stephen Jackson and Nam Trang.