BLAST 2023 Friday Schedule

8:45 - 9:00 – coffee

9:00 - 9:50 – David Stanovský (plenary talk)
Commutator theory for specific classes of algebras: a case study

9:50 - 10:20 – coffee break

10:20 - 10:50 – Peter Mayr
Systems of term equations over finite algebras

2:35 - 3:05 – Edith Vargas-García
Arrow relations in integer partition lattices

11:30 - 12:00 – Peter Jipsen
On representation of distributive involutive risiduated lattices by binary relations

12:00 - 1:00 – lunch (provided)

1:00 - 1:30 – Amanda Tran
Structural analysis on the projective space of $\mathbb{Z}_4^2$

1:35 - 2:05 – M. Hossein Shahzamanian
On pseudovarieties satisfying $\mathsf{V} \circledast \mathsf{G} = \mathsf{EV}$

2:10 - 2:40 – Joseph McDonald
Direct products of bounded fuzzy lattices and ordinal products of linear fuzzy posets

2:40 - 3:25 – coffee break

3:25 - 3:55 – Albert Madinya
Topologizing the space of minimal primes of an M*-frame*

4:00 - 4:30 – José Gil-Férez
Locally integral involutive Po-monoids

4:30 - 5:00 – Jaš Šemrl
Relation algebraic models for Lambek calculus with modalities

Plenary Talk: COMMUTATOR THEORY FOR SPECIFIC CLASSES OF ALGEBRAS: A CASE STUDY

David Stanovský (*Charles University, Prague*)

What is the center of an inverse semigroup? Are Moufang loops of odd order solvable? Why there are no left self-distributive quasigroups of order $4k+2$? The abstract commutator theory has been developed for congruence modular varieties, and successfully applied to many problems of general nature, such as the finite basis problem. However, applying the general theory to a specific class of algebras is not always straightforward. I will summarize our attempts to adapt the commutator theory to loops, inverse semigroups and quandles, answering the aforementioned questions.

SYSTEMS OF TERM EQUATIONS OVER FINITE ALGEBRAS

Peter Mayr (*University of Colorado Boulder*)

For a fixed finite algebra A, we consider the decision problem $\mathrm{SysTerm}(A)$: does a given system of term equations have a solution in A? This can be formulated as a constraint satisfaction problem (CSP) with relations the graphs of the basic operations of A. From the complexity dichotomy for CSP due to Bulatov and Zhuk, it follows that $\mathrm{SysTerm}(A)$ for a finite algebra A is in P if A has a not necessarily idempotent Taylor polymorphism and is NP-complete otherwise. We show more explicitly that for a finite algebra A in a congruence modular variety, $\mathrm{SysTerm}(A)$ is in P if the core of A is abelian and is NP-complete otherwise. Given A by the graphs of its basic operations, this condition can be decided in quasi-polynomial time.

ARROW RELATIONS IN INTEGER PARTITION LATTICES

Asma Almazaydeh (*Department of Mathematics, Tafila Technical University*)
Mike Behrisch (*Institute of Discrete Mathematics and Geometry, Technische Universität Wien*)
Edith Vargas-García (*Department of Mathematics, ITAM*)
Andreas Wachtel (*Department of Mathematics, ITAM*)

Integer partitions started to gain interest in 1674 when Leibniz investigated the number of ways one can write (partition) a positive integer n as a sum of positive integers. Brylawski showed that the set of all partitions of a positive integer n endowed with the dominance ordering is a complete lattice. These lattices are called lattices of integer partitions, or just partition lattices, for short. The lattices of integer partitions can be viewed as concept lattices; in this contribution we continue the investigations of Behrisch and et al. and Ganter of those lattices, and concentrate on the problem of determining their factor lattices as suggested by Ganter. This question is equivalent to characterising the compatible subcontexts, which in turn have a description in terms of so-called arrow relations. Moreover, we focus on results on these arrow relations and thus shed some light on the structure of lattices of integer partitions.

On Representations of Distributive Involutive Residuated Lattices by Binary Relations

Peter Jipsen (*Chapman University*)
Jaš Šemrl (*University College London*)

The collection of all binary relations on a set is a representable relation algebra, and if we replace the complementation and the converse operation with the operation of complement-converse then we obtain an interesting class of subreducts, known as representable weakening relation algebras, that are distributive involutive residuated lattices.

A binary relation defined on a poset is a weakening relation if the partial order acts as a both-sided compositional identity. We present a two-player game for the class of representable weakening relation algebras akin to that for the class of representable relation algebras. This enables us to define classes of abstract weakening relation algebras that approximate the quasivariety of representable weakening relation algebras and produce explicit finite axiomatizations for some of these classes. We define the class of diagonally representable weakening relation algebras and prove that it is a discriminator variety. We also provide representations for several small weakening relation algebras.

Structural Analysis on the Projective Space of $\mathbb{Z}_2^4$

Amanda Tran (*University of Massachusetts, Boston*)

Given the projective space of $\mathbb{Z}_2^4$, there are fifteen projective points and thirty-five projective lines. The projective line complex admissibility problem seeks to describe and generalize the underlying structures that separate admissible (linearly independent) versus inadmissible (linearly redundant) complexes. Specifically, the line complex problem is used in the context of Radon Transforms over these projective points. This project addresses necessary conditions that contribute to admissible and inadmissible linear structures using discrete analysis, vector analysis, linear algebra, and discrete geometry. In particular, we are interested in generalized classes of minimally inadmissible collections of lines, their associated geometry, and its dependence on the "Even Incident Condition" (which is proven and explored in this project.)

On Pseudovarieties Satisfying $\mathsf{V} \textcircled{m} \mathsf{G} = \mathsf{EV}$

M. Hossein Shahzamanian (*CMUP, Departamento de Matemática, Faculdade de Ciências, Universidade do Porto*)
Jorge Almeida (*CMUP, Departamento de Matemática, Faculdade de Ciências, Universidade do Porto*)

By a pseudovariety we mean a class of finite semigroups that is closed under homomorphic images, subsemigroups and finite products. A relational morphism between semigroups S and T is a relation $S \to T$ with domain S which, as a subset of $S \times T$, is a subsemigroup. The group kernel $K(S)$ of S is the intersection of the subsemigroups $\tau^{-1}(1)$ over all relational morphisms $\tau\colon S \to G$ into finite groups. For finite semigroups, $K(S)$ is a subsemigroup containing the idempotents that is closed under weak conjugation: if s' is a weak inverse of s, that is, if $s'ss' = s'$, then $sK(S)s' \cup s'K(S)s \subseteq K(S)$. Rhodes' Type II Conjecture states that, in case S is finite, the group kernel $K(S)$ is the smallest subsemigroup of S containing the idempotents and closed under weak conjugation. The Mal'cev product of two pseudovarieties V and W is defined as follows:

$$\mathsf{V} \textcircled{m} \mathsf{W} = \{M \mid \text{there is a relational morphism } \tau\colon M \to N \text{ with } N \in \mathsf{W} \text{ and such that } \tau^{-1}(e) \in \mathsf{V} \text{ for every idempotent } e \in N\}.$$

Let S be a finite semigroup and V be a pseudovariety. We have $S \in \mathsf{V} \textcircled{m} \mathsf{G}$ if and only if $K(S) \in \mathsf{V}$ where G is the pseudovariety of all finite groups. In addition, $S \in \mathsf{EV}$ if and only if the subsemigroup $\langle E(S) \rangle$ generated by the idempotents belongs to V. Hence, the inclusion $\mathsf{V} \textcircled{m} \mathsf{G} \subseteq \mathsf{EV}$ always holds. The purpose of this presentation is to discuss the results of a study that focused on cases when the pseudovariety equality

$$\mathsf{V} \textcircled{m} \mathsf{G} = \mathsf{EV} \tag{1}$$

holds.

A related problem, which has also been investigated is to consider instead the equation

$$\mathsf{V} * \mathsf{G} = \mathsf{EV}, \tag{2}$$

where the star denotes semidirect product of pseudovarieties. There are connections between the two problems since $\mathsf{V} * \mathsf{G}$ is always contained in $\mathsf{V} \textcircled{m} \mathsf{G}$, with equality holding whenever (but not exclusively) V is local. Thus, for a pseudovariety V to be a solution of the equation (1) is a weaker property than to be a solution of (2). For instance, while the fact that the pseudovariety J (of all finite $\mathcal{J}$-trivial semigroups), which is not local, satisfies (1) is not very difficult, it is a much deeper fact that J also satisfies (2). On the other hand, for the pseudovarieties DS and DA, of all finite semigroups in which all regular elements respectively lie in groups or are idempotents, are both local and solutions of both equations. A whole interval of solutions of both equations is $[\mathsf{Sl}, \mathsf{Com}]$ where Sl and Com are the pseudovarieties of all finite, respectively, semilattices and commutative semigroups. Indeed, by a theorem of Ash, Sl is a solution of (2) and, clearly, $\mathsf{ESl} = \mathsf{ECom}$. While the pseudovariety Sl is local, Com is not. During the presentation, we will derive pseudoidentities that can function as either solutions of (1) for all pseudovarieties that adhere to them or as counterexamples. Initially, we were motivated to investigate the pseudovariety equation (1) for a pseudovariety V satisfying $\mathsf{SI} \subseteq \mathsf{V} \subseteq \mathsf{DA}$. However, we later found out that there are subpseudovarieties within J, situated between SI and DA, which do not satisfy the pseudovariety equation (1). We provide a set of semigroups where a pseudovariety includes at least one of the semigroups mentioned in the set if and only if the pseudovariety comprises a subpseudovariety that does not satisfy the pseudovariety equation (1). Moreover, a pseudovariety does not include any semigroups of the set if and only if the pseudovariety is in EB where $\mathsf{B} = [\![x^2 = x]\!]$.

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Direct Products of Bounded Fuzzy Lattices and Ordinal Products of Linear Fuzzy Posets

Joseph McDonald (*University of Alberta*)

We continue the work of Chon, as well as Mezzomo, Bedregal, and Santiago, by studying algebraic operations on bounded fuzzy lattices arising from fuzzy partially ordered sets. Chon in [1] proved that fuzzy lattices are closed under taking direct products defined using the minimum triangular norm operator. Mezzomo, Bedregal, and Santiago in [3] introduced further operations on fuzzy lattices and extended Chon's result to the case of bounded fuzzy lattices under the same minimum triangular norm direct product construction. The first contribution of this study is to strengthen their result by showing that bounded fuzzy lattices are closed under a much more general construction of direct products; namely direct products whose underlying triangular norm operators have no zero divisors. We then introduce the concept of triangular norm based ordinal products of fuzzy posets, which is then investigated within the setting of linear (totally ordered) fuzzy posets. Time permitting, various applications of these results will be discussed.

References:
[1] Chon, I.: Fuzzy partial order relations and fuzzy lattices. Korean Journal of Mathematics. 17, 361–374 (2009)

[2] Klement, E., Mesiar, R., Pap, E.: Triangular Norms. Trends in Logic, vol. 8, Springer (2000)

[3] Mezzomo, I., Bedregal, B., and Santiago, R.: On some operations on bounded fuzzy lattices. The Journal of Fuzzy Mathematics. 22, 853–878 (2014)

Topologizing the Space of Minimal Primes of an M-Frame

Albert Madinya (*Florida Atlantic University*)
Papiya Bhattacharjee (*Florida Atlantic University*)

An M-frame is an algebraic frame possessing a unit and satisfying the Finite Intersection Property. Given an M-frame, call it L, we can topologize the set of minimal prime elements of L, which we will denote by $\mathrm{Min}(L)$. One such way we could topologize $\mathrm{Min}(L)$ is with the Zariski topology as is done with the prime ideals of a commutative ring. The other is the inverse topology which has a similar construction to that of the Zariski topology. Our aim in this talk to is to study these topological spaces and the interplay that exists between the topological properties of $\mathrm{Min}(L)$ and the frame-theoretic properties of L.

Locally Integral Involutive Po-monoids

José Gil-Férez (*United States Chapman University*)
Peter Jipsen (*United States Chapman University*)
Siddhartha Lodhia (*United States Chapman University*)

We introduce and study locally integral involutive partially ordered monoids (locally integral ipo-monoids, for short). Their relevance, among other things, resides in the fact that they constitute semantics for some nonclassical logics. We will demonstrate that every locally integral ipo-monoid A decomposes in a unique way into a family of integral ones, which we call its integral components. Moreover, we will associate to A a family of monoid homomorphisms (indexed on the order of the positive cone of A) so that the structure of A can be recovered as a glueing of its integral components along that family. Reciprocally, we will give necessary and sufficient conditions so that the Płonka sum of any family of integral ipo-monoids (indexed on a lower-bounded join-semilattice) along a family of monoid homomorphisms is an ipo-monoid.

Relation Algebraic Models for Lambek Calculus with Modalities

Lachlan McPheat (*University College London*)
Mehrnoosh Sadrzadeh (*University College London*)
Jas Semrl (*University College London*)

Lambek Calculus of syntactic types was designed to model the grammatical structure of English. As a result, it does not allow for the structural rules of commutativity, associativity, or contraction. These operations become handy, however, when modelling other languages, and in English for sentential and discourse dependencies. Structural rules are thus added to the base logic in a controlled manner, using modalities.

The first example of modal Lambek calculi had adjoint modalities in the style of tense logic. These were introduced by Moortgat, Morrill, and Jaeger in the 1990s. They have proven to be complete with respect to frame semantics by Valentin and Kurtonina. Recently, in 2017-2022, Kanovitch et. al. introduced a new class of modalities in the style of exponentials of Linear Logic. Finding complete frame or relational semantics for them is an open problem.

The relation-algebraic models come in the form of relational ordered residuated semigroups with modalities. We present some preliminary results as well as develop some tools to reason about these structures. Additionally, we raise a number of open problems.