Infinite-dimensional Ramsey Theory for Binary Free-amalgamation Classes

Andy Zucker (University of Waterloo) Natasha Dobrinen (University of Notre Dame)

When one attempts to generalize Ramsey's theorem to color the infinite subsets of the natural numbers, one must place definability constraints on the colorings considered. Ellentuck introduces a finer topology on the infinite subsets of naturals and shows that for any finite coloring of this space which has the Property of Baire with respect to this topology, one can find an infinite subset of naturals all of whose further infinite subsets receive the same color. Upon attempting to generalize this result to countably infinite structures, then in addition to these definability constraints, the theory of big Ramsey degrees places a fundamental limitation on how large of a subspace of structures one can expect such a generalization to hold. In recent joint work with Natasha Dobrinen, we state and prove such a generalization for Fraïssé limits of finitely constrained binary free amalgamation classes and show that this result is in many ways best possible.

Keywords: Structural Ramsey theory, Ramsey spaces. binary structures