

MATH RESEARCH AT UNC CHARLOTTE 2023

Project 3: Higher order Turán inequality for restricted partitions

Mentor: Dr. Arindam Roy

Project description

Recently, a study of higher order Turán inequalities for restricted partition functions $p_A(n)$ brought a lot of attention. The restricted partition function $p_A(n)$ counts the total number of partitions whose parts are in the set A . For example, if A denotes the set of perfect square numbers, then $p_A(4) = 2, p_A(5) = 2, p_A(6) = 2$, etc. A theory developed by Griffin, Ono, Rolen, and Zagier can prove the hyperbolicity of Jensen polynomials for $p_A(n)$, that is, the roots of these Jensen polynomials are real, for sufficiently large values of n and a suitable set A . Jensen polynomial of degree d associated to $p_A(n)$ is a polynomial whose coefficients are $\binom{d}{j} p_A(n+j)$ for $0 \leq j \leq d$. In this project we will consider various subsets A of \mathbb{N} and will determine the exact value N_d so that for each $n > N_d$ the Jensen polynomial of degree d associated with the partition function $p_A(n)$ is hyperbolic. Beside obtaining numerical evidence, we will explore the circle method as a tool for this project. A basic computational skill and some background of complex analysis are required for this project.