


**PROBLEMS**

1. Show that when the index dispersion of the semiconducting material is taken into account, the spectral spacing of the longitudinal modes of an injection laser is given by (4.7).

2. The spectral characteristic of a certain injection laser has peaks at 0.67970 μm, 0.67985 μm, 0.68000 μm, 0.68015 μm, and 0.68030 μm. These peaks correspond to the wavelengths of various longitudinal modes. Given that the index is 3.7 at the center wavelength of 0.68000 μm and that the laser cavity is 300 μm long, estimate the index dispersion, \( dn/dλ \), of the semiconductor.

3. The output of a 1-mW source is

\[
J(\theta, \phi) = \begin{cases} 
P_i \cos^2 \theta & 0^\circ \leq \theta \leq 90^\circ, \\
0 & 90^\circ \leq \theta \leq 180^\circ, \\
0 & 0^\circ \leq \phi \leq 360^\circ 
\end{cases}
\]

where \( i = 0, 1, 2, \) and 3.

a. Derive a general expression for \( P_i \) as a function of \( i \).

b. Derive a general expression for the full width between half-intensity points (FWHI). Then calculate the numerical value for the FWHI for the cases of \( i = 0, 1, 2, \) and 3.

c. The source is used to excite a multimode fiber with a numerical aperture of \( NA \). Derive a general expression for the power \( P_{fb} \) coupled into the fiber. Then, plot \( P_{fb} \) vs. \( NA \) for \( 0 < NA < 0.5 \) and \( i = 0, 1, 2, \) and 3.

4. The radiant intensity distribution of an LED is

\[
J(\theta, \phi) = \begin{cases} 
P' \cos^3 \theta & 0^\circ \leq \theta \leq 45^\circ, \\
0 & 45^\circ < \theta \leq 180^\circ, \\
0 & 0^\circ \leq \phi \leq 360^\circ 
\end{cases}
\]

where \( P' \) is a constant. A power of 0.2 mW is coupled into a multimode fiber with an \( NA \) of 0.15. When the fiber is replaced by another fiber with an unknown \( NA \), a power of 0.4 mW is coupled to the unknown fiber. Calculate the \( NA \) of the unknown fiber.
5. The radiant intensity distribution of a source is of the form

\[ J(\theta, \phi) = \begin{cases} P' \cos^2 \theta & 0^\circ \leq \theta \leq 60^\circ \\ 0 & 60^\circ < \theta \leq 180^\circ \end{cases} \quad 0^\circ \leq \phi \leq 360^\circ \]

a. What is the beam width (full angle between half-intensity points) of this source?

b. The source is used to excite a multimode fiber with an NA of 0.15, and a total power of 0.05 mW is coupled into the fiber. What is the total power radiated by the source?

6. Figure 4.30b depicts an LED with a hemispherical dome, which has a radius \( R_{dn} \) and refractive index \( n_{sc} \). Radiation, with radiant intensity \( J_o \), is emitted isotropically in all directions from all points on an active junction area of radius \( R_j \). Show that if \( R_{dn} \geq R_j n_{sc} \), then all radiation from the active junction area will emerge from the dome and none will be internally reflected.

7. A flat, undomed LED is immersed in a medium with an index of \( n \) and \( n < n_{sc} \). Suppose \( n_{sc} = 3.6 \) and the radiant intensity distribution in the semiconductor is

\[ J(\theta_{sc}, \phi_{sc}) = \begin{cases} J_o & 0^\circ \leq \theta_{sc} \leq 90^\circ, \\ 0 & 90^\circ < \theta_{sc} \leq 180^\circ, \end{cases} \quad 0^\circ \leq \phi_{sc} \leq 360^\circ \]

The attenuation in the semiconductor region is negligible. Estimate the minimum value of \( n \) if 50 percent or more of the total power is transmitted through the flat interface.

8. Two injection laser diodes (diodes A and B) are made of the same materials and have similar configurations. For both diodes, mirror 1 is perfectly reflecting and mirror 2 has a reflectivity of 0.3. The attenuation constant of the semiconductor is about 15 cm\(^{-1}\). Diode A has an optical cavity volume of \( 500 \times 30 \times 0.2 \) \( \mu \)m\(^3\), while the dimensions of diode B are \( 400 \times 30 \times 0.2 \) \( \mu \)m\(^3\). The PI characteristic of diode A is shown in the figure. Estimate the threshold current (in mA) of diode B.

9. The PI characteristic of an injection laser is shown in the figure on the next page. The laser has an optical cavity volume of \( 500 \times 30 \times 0.2 \) \( \mu \)m\(^3\) and emits at a wavelength of 0.9 \( \mu \)m. Other parameters are: attenuation constant \( \alpha = 15 \) cm\(^{-1}\), mirror reflectivity \( \Gamma_1 = \Gamma_2 = 0.35 \), and internal resistance 2 \( \Omega \). Calculate:
a. The threshold current.
b. The external quantum efficiency.
c. The internal quantum efficiency.
d. The power efficiency at $I = 30$ mA.

The PI characteristic of a fictitious injection laser is shown in the figure. The laser emits at a wavelength of 1.3 $\mu$m, and the optical cavity is 400 $\mu$m long. The semiconductor has an attenuation constant $\alpha = 12$ cm$^{-1}$. The internal resistance of the diode is 2.5 $\Omega$. Mirror 1 is perfectly reflecting, that is, $\Gamma_1 = 1.0$, but the reflectivity of mirror 2 is not known. However, it is known that the internal quantum efficiency is 95 percent. Calculate:
a. The threshold current.
b. The external quantum efficiency.
c. The reflectivity of mirror 2.

The PI characteristic of a fictitious injection laser is shown in the figure for problem 10. Other parameters known include: $d = 400$ $\mu$m, $\lambda = 1.2$ $\mu$m, $\Gamma_1 = 0.3$, $\Gamma_2 = 1.0$, and $\eta_{int} = 0.5$. Calculate the attenuation constant $\alpha$ of the semiconductor.

An injection laser diode has an active junction region of $300 \times 5 \times 0.2$ $\mu$m$^3$, and $\tau_e = 3$ ns, $\tau_{ph} = 2$ ps, $g_1 \approx 5 \times 10^{-7}$ cm$^3$ s$^{-1}$, $\gamma = 0$, and $N_{min} = 5 \times 10^{17}$ cm$^{-3}$.
a. Calculate the threshold current (in mA) of the diode.
b. The diode is driven with a dc current of 50 mA. Calculate the steady-state electron density and photon density in the pn junction.
c. An RF current of frequency \( f_m \) is superimposed onto the dc current of 30 mA. Sketch the response curve, that is, \(| e \Phi_1 \delta f / J_1 |\) vs. \( f_m \).
d. The response curve has a peak at frequency \( f_{mo} \). What is \( f_{mo} \) in GHz?
e. Would \( f_{mo} \) be different if \( \gamma \approx 10^{-3} \)? Why? How about \( 10^{-2} \)? Why?

13. The PI and the spectral characteristics of a visible injection laser diode are shown in the figure. The laser diode has a stripe geometry with a length \( d \), width \( s \), and thickness \( w \). The index and the dispersion of the medium is such that \( n - \lambda \frac{dn}{d\lambda} \approx 4.48 \) at the center wavelength of 683.5 nm. As shown in the figure, in the wavelength range between 681 nm and 686 nm there are 25 longitudinal modes. The threshold current of 48 mA, as read from the PI characteristic, corresponds to a threshold current density of 3.2 kA/cm².

![Graph](image)

a. Calculate the length \( d \) (in \( \mu \text{m} \)) of the diode.
b. Calculate the width \( s \) (in \( \mu \text{m} \)) of the stripe.
c. Calculate the external quantum efficiency.

Clearly, the strengths of the longitudinal modes are different. To simplify the problem in parts d and e, assume that the six longitudinal modes near the center wavelength are of equal strength and each has an output of 0.2 mW.
d. Suppose that six longitudinal modes near the center wavelength are excited and mode locked and all other modes are suppressed. Calculate the peak power of the mode locked pulses.
e. Calculate the full width between half-power points of the pulses.