UNC CHARLOTTE ECONOMICS WORKING PAPER SERIES

THE CONVERSION OF MONEY LINES INTO WIN PROBABILITIES: RECONCILIATIONS AND SIMPLIFICATIONS

Jason P. Berkowitz
Craig A. Depken, II
John M. Gandar

Working Paper No. 2016-006

THE UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
BELK COLLEGE OF BUSINESS
DEPARTMENT OF ECONOMICS
9201 University City Blvd
Charlotte, NC 28223-0001
August 2016

ABSTRACT

We contribute to the literature on money line betting markets by investigating the relationships between the various methods used to derive subjective win probabilities from money lines. We show that, although the seven methods described appear to be unique, they actually share many common assumptions and that, surprisingly, they reduce to three distinct estimates of bookmaker commission and subjective win probabilities. We also show that among the three distinct estimates, one is biased when money lines suggest a very heavy favorite in a particular sporting event. Thus, it is important to consider the assumptions for each method when deciding which to use in a particular context. Two empirical examples demonstrate how a market inefficiency, such as a long-shot favorite bias, should influence the choice of methodology.

The Conversion of Money Lines into Win Probabilities: Reconciliations and Simplifications

Jason P. Berkowitz
Department of Economics and Finance
Tobin College of Business
St. John's University
Queens, NY
berkowij@stjohns.edu

Craig A. Depken, II
Department of Economics
Belk College of Business
University of North Carolina Charlotte
Charlotte, NC
cdepken@uncc.edu

John M. Gandar
Department of Economics and Department of Finance
Belk College of Business
University of North Carolina Charlotte
Charlotte, NC
jmgandar@uncc.edu

The Conversion of Money Lines into Win Probabilities: Reconciliations and Simplifications

Abstract

We contribute to the literature on money line betting markets by investigating the relationships between the various methods used to derive subjective win probabilities from money lines. We show that, although the seven methods described appear to be unique, they actually share many common assumptions and that, surprisingly, they reduce to three distinct estimates of bookmaker commission and subjective win probabilities. We also show that among the three distinct estimates, one is biased when money lines suggest a very heavy favorite in a particular sporting event. Thus, it is important to consider the assumptions for each method when deciding which to use in a particular context. Two empirical examples demonstrate how a market inefficiency, such as a long-shot favorite bias, should influence the choice of methodology.

JEL Classifications: Z23, L83

Keywords: betting markets, subjective probabilities, market efficiency

1. Introduction

Money lines are an increasingly popular means by which individuals gamble on the outcomes of sporting events. Rather than betting whether the favorite (underdog) wins (loses) the game by more (less) than a particular point differential given by a spread or sides line, a money line entails a binary choice of which team will win the game regardless of the final point differential. Money lines are generally set by books to reflect the odds that one team or the other will win (with the potential for a draw when possible). One interesting aspect of money lines is that they can be converted to predicted win probabilities for the potential outcomes of the sporting event: favorite wins, underdog wins, or a draw (when possible). The various methods by which money lines are converted to win probabilities is the focus of this paper.

Two distinct areas of academic inquiry are interested in the conversion of money lines into win probabilities. The first sees sports betting markets as convenient laboratories to test the incorporation of information into prices or, more generally, to test the efficient markets hypothesis (EMH). While, the bulk of the literature testing EMH in the context of American sports betting markets focuses on point-spread betting markets, a smaller number of studies examine EMH in money-line betting markets. In order to conduct EMH tests in money-line betting markets, these money lines need to be converted into win probabilities that can be compared to observed win frequencies. However, there appear to be at least three methods of converting money lines into win probabilities in this literature: the method outlined in Woodland and Woodland (1994 and 2001); the modification of the Woodland and Woodland method by Gandar *et al.* (2002 and 2004); and the method used by Sauer (2005). The question of which method is appropriate has yet to be addressed.

The second area of academic inquiry converts money lines into win probabilities to test the short-term uncertainty of outcome hypothesis (UOH) as a determinant of the demand for sporting events. While early studies on this topic use variables such as team win percentages or league/division standings going into the game to capture potential game outcome uncertainty, most recent studies utilize expected team win probabilities derived from betting odds established prior to the start of the game as a measure of

outcome uncertainty. The justification for the use of win probabilities derived from money lines is that these betting market prices serve as efficient and unbiased aggregators of relevant information on game outcomes. Once again, however, there are several, ostensibly different, conversion methods in this UOH literature: Knowles *et al.* (1992) use one conversion method; Rascher (1999) a second method; Lemke *et al.* (2010) and Beckman *et al.* (2012) appear to use a third method; and lastly, Coates and Humphreys (2012) and Coates *et al.* (2014) use the same method employed by Sauer (2005).

The variety of methods used to convert money lines into win probabilities stands in stark contrast to the standardized conversion of betting odds into probabilities used in odds betting markets around the world (in studies of EMH in racetrack betting, association football (soccer), and other odds betting markets and of UOH in studies of game attendance across a variety of sports including English and European football, New Zealand rugby, and Australian rules football.)[1] The contrast becomes even starker when one considers the multiple outcomes common in odds betting relative to the simplicity of the two possible outcomes in money line betting. All of these studies utilizing odds markets routinely use the standard normalization of first converting betting odds into win-odds probabilities and then converting these win-odds probabilities into win probabilities, the same approach used by Sauer (2005). However, because these odds markets frequently exhibit the favorite-longshot bias (hereafter, FLB) whereby the normalized probabilities derived from betting odds on favorites (longshots) tend to understate (overstate) actual winning chances, there is growing recognition that so-called Shin (1992, 1993) probabilities may be preferable.

The present paper explores the interrelationships between the various methods used to derive win probabilities from money lines. Our aims are threefold. First, we explore the relationships between these conversion methods to reconcile and reduce their number as much as possible. Second, we present arguments for the adoption of money line conversion methods compatible with the standard normalization and Shin probabilities used in the literature on odds betting markets. Third, we explore the differences

[1] References to these and other UOH studies are provided in Coates *et al*. (2014). References for recent studies in odds-market betting are included in Vaughan Williams (1999, 2005) and Snowberg and Wolfers (2010).

between the standard normalization and Shin probability estimates in two numerical examples in order to demonstrate their possible uses when a researcher suspects the presence of biases such as the FLB in a particular money line betting market.

The paper finds that the various methods used to convert money lines into win probabilities reduce to just three separate methods. First, we show that the conversion methods of Knowles *et al*. (1992), Woodland and Woodland (1994 and 2001), Lemke *et al*. (2010), and Beckman *et al*. (2012) are identical. Second, we demonstrate that the modified Woodland and Woodland conversion method found in Gandar *et al*. (2002 and 2004) and the standard normalization method used by Sauer (2005), Coates and Humphreys (2012) and Coates *et al*. (2014) are identical. Lastly, we show that the Rascher (1999) conversion method produces probabilities identical to Shin probabilities in the two-outcome case.

We argue that parsimony and a need for compatibility of conversion methods with the racetrack and odds betting markets calls for the use of the standard normalization in preference to the modified Woodland and Woodland conversion method; that the Rascher/Shin conversion method be considered in situations where the researcher has reason to believe that there is a FLB; and that the similarity of the win probabilities produced by the Rascher/Shin method with those derived from the Knowles/Woodland and Woodland conversion method calls for the latter method to be discontinued. Finally, our numerical comparisons provide useful insight into situations where Rascher/Shin probabilities may or may not be preferred to the standard normalized probabilities.

The rest of the paper is organized as follows. Section 2 provides a chronological review of the various conversions of money lines into win probabilities. Section 3 reconciles and reduces the number of conversion methods to three. Section 4 discusses the merits of these methods for researchers seeking an 'off-the-shelf' method to convert money lines into win probabilities. Section 5 provides two empirical examples to aid in this selection process. Conclusions are presented in Section 6.

2. A Review of the Methods used to Convert Money Lines into Win Probabilities

Money Line Betting and Notation

Before proceeding to review the methods of converting money lines into win probabilities used by both the EMH and UOH literature, a brief description of money lines and an introduction to the notation that will be used throughout most of the paper is warranted. Money-line betting entails betting on the game winner; unlike the more familiar point spread (or sides line) betting market, the margin of victory does not matter. Onshore (Las Vegas) and offshore books post money lines on a number of American sports (in addition to the better-known money-line betting markets on Major League Baseball (MLB) and National Hockey League (NHL) games, money lines are also offered on National Football League (NFL), National Basketball Association (NBA), and college football and basketball games).

Money lines are normally quoted as negative numbers for favorites and as positive numbers for underdogs: we label such lines as standard money lines. For example, at a standard money line of (-160, +145), a $160 bet on the favored team wins $100 if the favorite wins the game, while a $100 bet on the underdog wins $145 if the underdog wins the game. Writing these money lines using the notation first used by Woodland and Woodland (1994) and followed by several subsequent studies, the favorite and underdog odds prices at the above money line are respectively, $\beta_1 = 1.60$ and $\beta_2 = 1.45$, and a winning unit bet on the favorite nets $1/\beta_1 = \$0.625$ while a winning unit bet on the underdog nets $\beta_2 = \$1.45$. Occasionally, both money lines are listed as negative numbers. For example, when one team is a very slight favorite over the other, the money line might be (-111, -101) or even (-107, -105) and when both teams are considered equally likely to win (in betting parlance the game is a 'pick-em'), the money line

might be (-110, -110) or possibly (-105, -105). In such instances of double negative lines, winning unit bets net either $(1/\beta_1)$ or $(1/\beta_2)$.[2]

Conversion Method 1: The Knowles et al. (1992) Win Probabilities

We proceed chronologically through our review of the methods used to convert money lines into win probabilities starting with Knowles *et al.* (1992). While the review of the UOH literature provided in Coates *et al*. (2014) identifies Peel and Thomas (1988) as the first study to use betting odds to measure game uncertainty (for English football), the same study identifies Knowles *et al.* as the first study to use win probabilities derived from money lines to measure game uncertainty in their study of baseball attendance (for the 1988 MLB season). Knowles *et al.* start by defining bets and returns for favorites and underdogs as respectively, a bet of β_1 on the favorite team to win \$1, and a bet of \$1 on the underdog to win β_2. Using these bet definitions,[3] expected returns for favorite and underdog bets are

$$E(R_F) = \rho_F(1) + (1-\rho_F)(-\beta_1)$$

and

$$E(R_U) = (1-\rho_F)(\beta_2) + (\rho_F)(-1),$$

where ρ_F is the probability of a favorite win.

Setting these expected returns equal to zero and solving for ρ_F produces two estimates for ρ_F, $\beta_1/(1+\beta_1)$ and $\beta_2/(1+\beta_2)$. While Knowles *et al.* acknowledge that averaging the two estimates

[2] For space reasons, the remainder of this section confines the conversion of money lines into win probabilities for standard money lines only. Appendix A provides a derivation of win probabilities for double negative money lines.

[3] Knowles *et al.* (1992) actually use a so-called 'Eastern' line set up for \$5 bets on favorites (or \$5 returns on underdogs). We convert their bet sizes or winnings from \$5 to \$1 and their notation to the (β_1, β_2) notation for standard money lines while leaving their definitions of unit bets and returns unchanged.

would be an "obvious choice" (see Knowles *et al.*, 1992, p. 78), they choose a curious alternative combination of these two probability estimates[4]

$$\rho_F^K = ((\beta_1 + \beta_2)/2)/(((\beta_1 + \beta_2)/2)+1).$$

Following the same procedure, Knowles *et al.* derive the underdog win probability as

$$\rho_U^K = 1/(((\beta_1 + \beta_2)/2)+1).$$

To give the reader an idea of the probabilities resulting from this conversion, Panel A of Table 1 provides numerical estimates of (ρ_F^K, ρ_U^K) for five standard money lines chosen so as to provide favorite probability estimates across roughly equal increments between 0.5 and 0.9.[5]

Conversion Method 2: The Woodland and Woodland (1994, 2001) Win Probabilities

Woodland and Woodland (1994) derive underdog win probabilities from money lines to compare against observed underdog win frequencies in the first tests of EMH in the money line betting market on MLB games. Later, Woodland and Woodland (2001) use the same conversion of money lines into probabilities for testing the efficiency of the money line betting market on NHL games. In both studies, Woodland and Woodland (hereafter WW) find that predicted underdog win probabilities tend to be lower than observed underdog win frequencies, leading them to claim the existence of a reverse favorite-longshot bias in these betting markets. In brief, the WW conversion method is to define unit bets on favorites and underdogs, derive a balanced book estimate of the bookmaker's commission, derive expected returns for favorites and underdogs, set these expected returns equal to the negative of the book's commission, and, finally, derive the underdog (or favorite) win probability.

[4] We use 'curious' in the sense that, despite listing both straightforward averaging and their selected combination method as possible alternatives (see the representations (A-1) and (A-2) in Appendix 1 of their paper) and recognizing that the two methods produce numerically different win probabilities, Knowles *et al*. provide no explanation for their selection of the latter over the former method. We speculate that the explanation may result from a wish to write both ρ_F^K and ρ_U^K in terms of the mid-point of the money lines, $(\beta_1 + \beta_2)/2.$

[5] These money lines are drawn from a sample of nearly 28,000 games for the 2006-07 through 2012-13 college basketball seasons collected from Sports Insight (www.sportsinsight.com). Data from this source are used by a number of other studies including Coates *et al*. (2014). The same data are used in Table 3 below.

Unit bets are defined by WW in the same fashion as Knowles *et al.* (1992): that is, a bet of β_1 on the favorite team to win \$1, and a bet of \$1 on the underdog to win β_2. WW next define *X* and *Y* as the number of unit bets on the favorite and underdog and define net receipts for the bookmaker as $(Y - X)$ if the favorite wins and $(\beta_1 X - \beta_2 Y)$ if the underdog wins. At this balanced-net-receipts ratio of bets on the favorite and underdog, WW derive the book's commission, c^{WW}, as

$$c^{WW} = (Y - X)/(Y + X) = (\beta_1 - \beta_2)/(\beta_1 + \beta_2 + 2).$$

WW then impose the efficient markets requirement that expected returns on favorites and underdogs should both equal the negative of the book's commission. WW define ρ_U as the probability of an underdog win, expected returns on the favorite as $E(R_F) = (1 - \rho_U)(1) + \rho_U(-\beta_1)$, and expected returns on the underdog as $E(R_U) = \rho_U(\beta_2) + (1 - \rho_U)(-1)$. Setting the expected return on the underdog equal to $-c^{WW}$, yields their underdog win probability as:

$$\rho_U^{WW} = 2/(\beta_1 + \beta_2 + 2).$$

Following the equivalent procedure, their favorite win probability is

$$\rho_F^{WW} = (\beta_1 + \beta_2)/(\beta_1 + \beta_2 + 2).$$

Despite the different approach taken by WW, their favorite and underdog win probabilities are identical to the Knowles *et al*. (1992) probabilities discussed above (the reader can easily confirm this by dividing either of the above expressions by two). Nevertheless, we show these win probability estimates for $(\rho_F^{WW}, \rho_U^{WW})$ along with the WW commission, c^{WW}, in Panel B of Table 1

Conversion Method 3: The Rascher (1999) Win Probabilities

Like Knowles *et al*. (1992), Rascher (1999) uses home team win probabilities derived from money lines to measure game uncertainty in a test of UOH for game attendance in baseball (for the 1996 MLB season). He follows both Knowles *et al*. and Woodland and Woodland by defining bets and returns for favorites and underdogs as, respectively, a bet of β_1 on the favorite team to win \$1, and a bet of \$1 on the underdog to win β_2. In the same fashion as Knowles *et al*., Rascher then sets expected returns for favorite and underdog bets equal to zero and derives 'fair bet' favorite win probabilities as $\beta_1/(1+\beta_1)$ and $\beta_2/(1+\beta_2)$. However, unlike Knowles *et al*., Rascher simply averages these fair bet probabilities to estimate his favorite win probability as

$$\rho_F^R = [(\beta_1/(1+\beta_1)) + (\beta_2/(1+\beta_2))]/2.$$

Likewise, Rascher's underdog win probability is estimated as

$$\rho_U^R = [1/(1+\beta_1)) + (1/(1+\beta_2))]/2.$$

Panel C of Table 1 provides numerical estimates of (ρ_F^R, ρ_U^R) for the five selected money lines. While these estimates of (ρ_F^R, ρ_U^R) are very close to (ρ_F^K, ρ_U^K) at each of these money lines, they are not identical. Instead, $\rho_F^K > \rho_F^R$ and $\rho_U^K < \rho_U^R$ across these money lines (a result that holds for all money lines in our sample – see Section 3 below).

Conversion Method 4: The Modified WW Win Probabilities of Gandar et al. (2002, 2004)

The next conversion method we examine is that of Gandar *et al*. (2002 and 2004). Gandar *et al*. (2002) point out that, since β_1 is strictly greater than one, the problem with the WW conversion method is that a unit bet on the favorite is always something more than this. Consequently, they claim that the WW balanced book commission is too large and the underdog win probability is too small. As a result, Gandar *et al*. question whether a reverse favorite-longshot bias exists in the WW sample of MLB games and money lines.

With the exception of a different definition of a unit bet on the favorite, the Gandar *et al*. conversion method follows the same steps taken by Woodland and Woodland. Gandar *et al.* define a unit bet on the favorite as \$1 to win $(1/\beta_1)$, and define X and Y as the number of unit bets on the favorite and underdog, respectively, so that the total dollars bet (or 'handle') is $(X+Y)$. The book's net revenue (or 'hold') is $H_F = (Y-(1/\beta_1)X)$ if the favorite wins or $H_U = (X-\beta_2 Y)$ if the underdog wins. If the bookmaker attracts bets so that net revenue is balanced irrespective of who wins the game, the 'ideal' balanced net revenue X/Y ratio is $(1+\beta_2)/(1+(1/\beta_1))$. At this balanced book X/Y ratio,[6] the book's commission, defined as hold over handle, is

$$c^{MW} = \frac{Y-(1/\beta_1)X}{X+Y} = \frac{X-\beta_2 Y}{X+Y}$$

which reduces to

$$c^{MW} = \frac{\beta_1-\beta_2}{2\beta_1+\beta_1\beta_2+1}.$$

Subjective win probabilities are again calculated under the assumption of market efficiency by setting expected returns from a unit bet on either team (that is, either $E(R_F)$ or $E(R_U)$) equal to the negative of the book's commission at the balanced book X/Y ratio. For example,

$$E(R_F) = \rho_F^{MW}(1/\beta_1) + (1-\rho_F^{MW})(-1),$$

where ρ_F^{MW} is the modified WW favorite subjective win probability. Setting $E(R_F)$ equal to $-c^{MW}$ and solving for ρ_F^{MW} gives

[6] We note that the 'balanced book' requirement for money lines is not an equal amount of money bet on the two outcomes. While the book's net revenue is balanced at $X/Y = (1+\beta_2)/(1+(1/\beta_1))$, the ratio of monies bet is not. Since $1 \le \beta_2 < \beta_1$ for all but 'pick-em' games (where $\beta_1 = \beta_2$), then this 'balanced' X/Y ratio always lies in the interval $1 \le \beta_2 < X/Y < \beta_1$.

$$\rho_F^{MW} = \frac{(\beta_1 + \beta_1\beta_2)}{(2\beta_1 + \beta_1\beta_2 + 1)}.$$

Likewise, setting $E(R_U)$ equal to $-c^{MW}$ and solving for ρ_U^{MW} gives

$$\rho_U^{MW} = \frac{(1 + \beta_1)}{(2\beta_1 + \beta_1\beta_2 + 1)}.$$

Panel D of Table 1 provides numerical estimates of $(\rho_F^{MW}, \rho_U^{MW})$ along with the commission, c^{MW}, for the five selected money lines. We note that $\rho_F^{MW} < \rho_F^R < \rho_F^K = \rho_F^{MW}$ (or $\rho_U^{MW} > \rho_U^R > \rho_U^K = \rho_U^{WW}$) for all of these money lines. Furthermore, we note that $c^{MW} < c^{WW}$ across all of these money lines, especially so at higher money lines (or higher favorite win probabilities) where the WW commission becomes unrealistically large.[7]

C*onversion Method 5: The Standard Normalized Win Probabilities of Sauer (2005)*

The next method for converting money lines into win probabilities, the standard or basic normalization, has appeared many times in the racetrack and other odds betting markets literature but, as far as we know, was first used by Sauer (2005) for estimating probabilities in a money line betting market. In addition, some recent studies in the UOH literature examining MLB and NHL game attendance (for example, Coates and Humphreys (2012) and Coates *et al.* (2014)) also use this method.[8] In summary, the standard normalization method first converts money lines into so-called decimal win

[7] Consider the money line of (-1150, + 890) in Table 1. At this money line the WW commission is 11.61 percent while the modified WW commission is 2.06 percent. Starting off at a 'dime line' of say (-110, + 100) or $(\beta_1 - \beta_2)$ = 0.10 where c^{MW} = 0.0233, the dime line commission gradually shrinks as one moves to successively higher values of β_1. To re-establish a profit margin for these 'higher' favorite money lines, the book shifts from a 'dime line' to a '15-cent' line, then to a '20-cent' line and so on. As a result, once β_1 gets to values at or above 10 (a favorite money line of -1000), the absolute difference between the favorite and underdog money line has to be large in order to even get a commission close to 2 percent. So WW's 11.61% commission appears to be wildly inflated.

[8] Interestingly, many bookmaker sites describe this method. For example, see the notes explaining 'American' odds and bookmaker margins at the Pinnacle Sportsbook (http://www.pinnaclesports.com).

odds, rearranges these odds into 'less-than fair' win probabilities, and then converts these win probabilities into 'fair' win probabilities.

Using the odds price notation introduced above, the net payoffs to unit bets on the favorite and underdog are respectively, $1/\beta_1$ and β_2, while the gross payoffs or decimal win odds, (o_F, o_U), are respectively, $(1+(1/\beta_1))$ and $(1+\beta_2)$. Inverting these decimal win odds yields the win odds probabilities, $\pi_F = 1/o_F$ and $\pi_U = 1/o_F$. Here the booksum is $(\pi_F + \pi_U)$ and the book 'over-round' or margin is $\pi_F + \pi_U - 1$. Since $\pi_F + \pi_U > 1$, the win odds probabilities are often labeled as the 'less-than-fair' odds probabilities (in the sense that, at most, the teams' chances of winning are π_F and π_U). Dividing both π_F and π_U by $(\pi_F + \pi_U)$ results in the standard normalizations of

$$\rho_F^{SN} = \frac{\pi_F}{(\pi_F + \pi_U)} = \frac{[1/(1+(1/\beta_1))]}{[(1/(1+(1/\beta_1))) + (1/(1+\beta_2))]},$$

and

$$\rho_U^{SN} = \frac{\pi_U}{(\pi_F + \pi_U)} = \frac{[1/(1+\beta_2)]}{[(1/(1+(1/\beta_1))) + (1/(1+\beta_2))]}.$$

Panel E of Table 1 provides numerical estimates of $(\rho_F^{SN}, \rho_U^{SN})$ along with the over-round or margin for the five selected money lines. The reader will note that $(\rho_F^{SN}, \rho_U^{SN}) = (\rho_F^{MW}, \rho_U^{MW})$ for all money lines.

Conversion *Method 6: The Lemke et al. (2010) and Beckman et al. (2012) Win Probabilities*

The final method used to convert money lines into win probabilities appears in both the Lemke *et al.* (2010) and the Beckman *et al.* (2012) studies of MLB game attendance. These studies use the same definition of bets on the favorite and underdog as found in Knowles *et al.* (1992), Woodland and Woodland (1994 and 2001), and Rascher (1999): a bet of β_1 on the favorite team to win \$1, and a bet of

\$1 on the underdog to win β_2. Lemke *et al.* (2010) and Beckman *et al.* (2012) then define a 'fair bet' from the bookmaker's perspective as $R = (\beta_1 + \beta_2)/2,$ and derive the implied favorite and underdog win probabilities as

$$\rho_F^L = R/(R+1) = ((\beta_1 + \beta_2)/2)/[((\beta_1 + \beta_2)/2)+1]$$

and

$$\rho_U^L = 1/(R+1) = 1/[((\beta_1 + \beta_2)/2)+1].$$

In short, while derived somewhat differently, these probability estimates are identical to those of Knowles *et al*. (1992) and Woodland and Woodland (1994 and 2001).

Conversion Method 7: Shin Probabilities

Before attempting to reconcile the above conversion methods it is useful to briefly outline the calculation of Shin probabilities. These probabilities have been used by a number of studies of both racetrack and other odds betting markets (examples include Cain *et al*. (2002, 2003), Smith *et al*. (2009), Strumbelj (2014, 2016) and references therein). However, to our knowledge, Shin probabilities have not been used to derive win probabilities in any money-line betting-market study to date.

The FLB is one of the most longstanding anomalies in the sports betting literature. This bias is the tendency for high odds/low probability betting propositions (i.e., longshots) to have subjective win probabilities above observed win proportions and low odds/high probability propositions (i.e., favorites) to have subjective win probabilities below observed win proportions. As a result, actual returns to favorites tend to exceed actual returns to underdogs. The bias was first identified in horse race betting markets more than sixty years ago.[9] Early explanations of the FLB stressed demand-side explanations such as risk-loving behavior by bettors. More recently, a number of supply-side and market-structure

[9] Griffith (1949) observed a FLB in American pari-mutuel betting markets and Dowie (1976) observed the same bias in the British bookmaking market. See Sauer (1998) for other references to the early literature on this topic.

based explanations have been developed. One of the most interesting of these latter explanations is that of Shin (1992 and 1993).

Shin (1992) develops a theoretical model to explain the FLB. Instead of viewing the FLB as stemming from the inability of books to evaluate the true win probabilities, Shin sees the FLB as a response to an asymmetric and adverse selection problem stemming from the presence of insiders who possess superior information than the book on some horses in the race. As a result of the presence of insiders, books engineer a FLB in order to pass the costs of losses arising from insider betting on to less-informed bettors.

Estimating Shin Probabilities

Shin (1993) develops a method to estimate the proportion of revenue attributable to insider trading (what he denotes as z) which is a measure of any bias present in the betting odds. Jullien and Salanié (1994) show that the Shin procedure can be reversed to extract the bookmaker's underlying probabilistic beliefs by using the betting odds, π_i, for the $i = 1,..,n$ horses in a race with $n \geq 2$, along with the estimate of z for the race. Using the specification provided in Cain, Law, and Peel (2002), these bookmaker probabilistic beliefs, ρ_i^{SH}, are estimated as

$$\rho_i^{SH} = \left[\frac{z^2}{4(1-z)^2} + \frac{\pi_i^2}{\sum_{i=1}^{n} \pi_i (1-z)} \right]^{1/2} - \frac{z}{2(1-z)}.$$

Using the condition $\sum_{i=1}^{n} \rho_i^{SH} = 1$, for $n > 2$, z is computed as

$$z = \frac{\left\{ \sum_{i=1}^{n} \left[z^2 + 4(1-z)(\pi_i^2 / \sum_{n}^{i-1} \pi_i) \right]^{1/2} - 2 \right\}}{n-2}.$$

Solving z for $n > 2$ requires a fixed-point iteration starting at $z_0 = 0$ (for examples, see Cain, et al. (2003), Smith, et al. (2009), and Strumbelj (2014)). Fortunately, as shown by Strumbelj (2014), in the special case with two possible outcomes, z has a much easier solution. Switching to the favorite (F) and underdog (U) notation, Shin's z can be estimated as

$$z = \frac{\left[((\pi_F + \pi_U) - 1)((\pi_F - \pi_U)^2 - (\pi_F + \pi_U))\right]}{\left[(\pi_F + \pi_U)((\pi_F - \pi_U)^2 - 1)\right]}.$$

This estimate of z is then inserted into the equation estimating Shin probabilities.

Panel F of Table 1 provides numerical estimates of $(\rho_F^{SH}, \rho_U^{SH})$ for the five selected money lines along with the estimate of Shin's z. The reader will note that $(\rho_F^{SH}, \rho_U^{SH}) = (\rho_F^R, \rho_U^R)$ for all money lines.

3. Reconciliations and Simplifications

The numerical estimates of the various favorite win probabilities for five standard money lines shown in Table 1 reveal that the various conversion methods reduce to only three separate numerical estimates of win probabilities for any specific money line. We have already observed that the Knowles probabilities (ρ_F^K, ρ_U^K), the original WW probabilities $(\rho_F^{WW}, \rho_U^{WW})$, and the Lemke/Beckman probabilities, (ρ_F^L, ρ_U^L) are identical. The table also reveals that: the modified WW probabilities $(\rho_F^{MW}, \rho_U^{MW})$ and the standard normalized probabilities $(\rho_F^{SN}, \rho_U^{SN})$ are identical; and the Rascher probabilities (ρ_F^R, ρ_U^R) and the Shin probabilities $(\rho_F^{SH}, \rho_U^{SH})$ are identical. This section explores the connections between the conversion methods to explain these two simplifications.

Reconciling the Modified WW and the Standard Normalization Conversion Methods

It is straightforward, if somewhat tedious, to show that the modified WW probabilities $(\rho_F^{MW}, \rho_U^{MW})$ and the standard normalized probabilities $(\rho_F^{SN}, \rho_U^{SN})$ are identical. We illustrate this using the underdog probability, ρ_U^{SN}. The denominator of ρ_U^{SN} is the book's over-round, $(\pi_F + \pi_U)$. Noting that $1/(1+(1/\beta_1))$ can be rewritten as $\beta_1/(1+\beta_1)$, we can rearrange the denominator of ρ_U^{SN} to read as

$$(1/(1+(1/\beta_1)) + (1/(1+\beta_2)) = (2\beta_1 + \beta_1\beta_2 + 1)/((1+\beta_1)(1+\beta_2)).$$

This rearrangement allows us to simplify ρ_U^{SN} to

$$\rho_U^{SN} = \frac{1/(1+\beta_2)}{(2\beta_1 + \beta_1\beta_2 + 1)/((1+\beta_2)(1+\beta_1))} = \frac{(1+\beta_1)}{(2\beta_1 + \beta_1\beta_2 + 1)} = \rho_U^{MW}.$$

Likewise, it is straightforward to show that

$$\rho_F^{SN} = \frac{1/(1+(1/\beta_1)}{(2\beta_1 + \beta_1\beta_2 + 1)/((1+\beta_2)(1+\beta_1))} = \frac{(\beta_1 + \beta_1\beta_2)}{(2\beta_1 + \beta_1\beta_2 + 1)} = \rho_F^{MW}.$$

That is, by proportionately distributing the bookmaker's over-round, $((\pi_F + \pi_U) - 1)$, between π_F and π_U, the resulting normalized probabilities, $(\rho_F^{SN}, \rho_U^{SN})$, are identical to $(\rho_F^{MW}, \rho_U^{MW})$.

It is useful here to briefly explore the difference between the book's margin and the book's commission, c^{MW}. At the money line of (-160, +145) shown in Table 1, the book's commission, c^{MW} is 0.0230 while the book's margin, m, estimated directly from the win odds probabilities as $[(\pi_F + \pi_U) - 1]$, is 0.0235. While the differences are small, the two estimates are not identical. The discrepancy between the two methods of estimating the book's commission is easily resolved via the same manipulation used above. Writing the booksum as

$$(\pi_F + \pi_U) = (1/(1+(1/\beta_1)) + (1/(1+\beta_2) = (2\beta_1 + \beta_1\beta_2 + 1)/((1+\beta_1)(1+\beta_2))$$

allows the book's margin to be written as

$$[(\pi_F + \pi_U) - 1] = \left[(2\beta_1 + \beta_1\beta_2 + 1)/((1+\beta_1)(1+\beta_2)) - 1\right] = (\beta_1 - \beta_2)/((1+\beta_1)(1+\beta_2)).$$

Dividing the margin by the over-round results in

$$\frac{[(\pi_F + \pi_U) - 1]}{(\pi_F + \pi_U)} = \frac{(\beta_1 - \beta_2)}{(1+\beta_1)(1+\beta_2)} \cdot \frac{(1+\beta_1)(1+\beta_2)}{(2\beta_1 + \beta_1\beta_2 + 1)} = \frac{(\beta_1 - \beta_2)}{(2\beta_1 + \beta_1\beta_2 + 1)} = c^{MW}.$$

That is, the percentage margin found by dividing the book's margin by the book's over-round is identical to the balanced book commission.

Reconciling the Rascher and Shin Conversion Methods

The second reconciliation, that between the Rascher probabilities (ρ_F^R, ρ_U^R) and the Shin probabilities $(\rho_F^{SH}, \rho_U^{SH})$, is much less straightforward. We use the Simplify function of the Maple software package to establish that the Shin probabilities $(\rho_F^{SH}, \rho_U^{SH})$ in the two outcome case of

$$\rho_F^{SH} = \left[\frac{z^2}{4(1-z)^2} + \frac{\pi_F^2}{(\pi_F + \pi_U)} \cdot \frac{1}{(1-z)}\right]^{1/2} - \frac{z}{2(1-z)},$$

and

$$\rho_U^{SH} = \left[\frac{z^2}{4(1-z)^2} + \frac{\pi_U^2}{(\pi_F + \pi_U)} \cdot \frac{1}{(1-z)}\right]^{1/2} - \frac{z}{2(1-z)},$$

reduce to the Rascher probabilities (ρ_F^R, ρ_U^R). Details of this simplification for both standard and double negative money lines are provided in Appendix B.

An Additional Simplification

We have already noted that the standard normalization method proportionally reduces the decimal win odds (π_F, π_U) to the win probabilities $(\rho_F^{SN}, \rho_U^{SN})$. In a similar fashion, it is straightforward to show that the Rascher/Shin conversion method simply deducts an equal amount from the decimal win odds, π_F and π_U, to arrive at the win probabilities (ρ_F^R, ρ_U^R) or $(\rho_F^{SH}, \rho_U^{SH})$ in the two outcome case. We illustrate this point by rewriting Rascher's favorite win probability, ρ_F^R, in terms of the favorite's decimal win odds and over-round. This probability is defined as

$$\rho_F^R = \frac{1}{2}\left[(\beta_1/(1+\beta_1)) + (\beta_2/((1+\beta_2))\right] = \frac{1}{2}\left[\frac{\beta_1+\beta_2+2\beta_1\beta_2}{(1+\beta_1)(1+\beta_2)}\right],$$

which can be rewritten as

$$\rho_F^R = \frac{\beta_1}{(1+\beta_1)} - \frac{1}{2}\left[\frac{(\beta_1-\beta_2)}{(1+\beta_1)(1+\beta_2)}\right] = \frac{1}{(1+(1/\beta_1))} - \frac{1}{2}\left[\frac{(\beta_1-\beta_2)}{(1+\beta_1)(1+\beta_2)}\right].$$

Since $\pi_F = 1/(1+(1/\beta_1))$ and $[(\pi_F+\pi_U)-1] = (\beta_1-\beta_2)/((1+\beta_1)(1+\beta_2))$,

$$\rho_F^R = \pi_F - \frac{1}{2}[(\pi_F+\pi_U)-1].$$

Likewise

$$\rho_U^R = \pi_U - \frac{1}{2}[(\pi_F+\pi_U)-1].$$

That is, in the two-outcome case, the Rascher (and Shin) favorite and underdog win probabilities are simply derived by deducting one-half of the book's over-round from the favorite and underdog decimal win odds.

4. Which Conversion Method to Use?

How should a researcher, seeking an 'off-the-shelf' method to convert money lines into win probabilities, pick between the three conversion methods? Over all money lines, the SN/modified WW favorite win probabilities provide the lowest probability estimates and the Knowles/original WW probabilities the highest with the Rascher/Shin probabilities falling just below the latter. Moreover, while the differences between these probabilities are small over most of the probability range, these probability estimates noticeably diverge toward the extremes of the probability range. Hence, we do not believe that the answer to the question of which conversion method should be used is "it does not matter." Rather, we believe that simplicity of assumptions and compatibility of methods with those used in other odds betting markets calls for the adoption of, at most, two of the above conversion methods.

We begin by suggesting that one of the three methods, specifically the Knowles/WW method, be dropped from consideration. As we have noted earlier, there is no obvious rationale for the Knowles' choice of combining the two probability estimates emerging from setting their version of expected returns to favorites and underdogs equal to zero. The WW version of the method, in addition to a misspecification of unit bets on the favorite (and hence the book's commission – see Gandar, *et al*., 2002), requires both the assumption of a balanced book and an efficient market such that expected returns to either the favorite or the underdog equal the negative of the book's commission. Both of these assumptions are controversial. The idea that bookmakers can balance the book on any individual game has been shown to be highly unlikely.[10] Likewise, it is not clear that in all money line betting markets actual returns to bets on favorites and underdogs are either equal to each other or equal to the negative of the book's commission. Finally, the Knowles/WW method produces win probabilities that are very close to Rascher/Shin probabilities (usually the same to the fourth significant digit). For these reasons, we advise against using the Knowles/WW method.

[10] Indications that bookmakers run unbalanced books come from studies examining the numbers of bets on both sides of the proposition in point spread betting markets: these studies clearly show that these bet numbers are almost always unequal. However, because information on actual monies bet is unavailable, in order for unequal numbers of bets to translate into unequal monies bet, these studies must assume that average bet sizes on both propositions are equal. As far as we are aware, this assumption remains untested. As well, we are unaware of any similar studies of the number of bets in money line betting markets.

What can we say about the choice between the modified WW and standard normalization (SN) methods? While the two methods produce identical estimates of win probabilities and the book's commission, the assumptions employed by the two methods are very different. The SN method converts bookmaker odds into win probabilities utilizing only the assumption that the book proportionately allocates its margin over the two outcomes. In contrast, the modified WW method, like the original WW method, requires both the assumption of a balanced book and an efficient market. Parsimony (or Occam's razor) argues that a derivation using fewer assumptions is preferable to one involving more (and more restrictive) assumptions. The controversial nature of these additional assumptions leads us to strongly prefer the standard normalization approach for the derivation of win probabilities from money lines over the modified WW approach. Using the SN method also has the benefit of congruence with the extensive literature converting odds into win probabilities.

That the complex Shin conversion method reduces to the simplicity of the Rascher method in the two-outcome case is a complete, but nonetheless welcome, surprise to us and, we suspect, to the rest of the field. We speculate that the explanation may lie in the Shin (1993) model's untested assumption that competition between bookmakers drives expected profits (the book's commission) to zero. As a result, expected returns to a bet on either the favorite or underdog, the mirror image of book expected profits, should both equal zero. This, of course, is the assumption employed by Rascher (1999) in his calculation of win probabilities. Irrespective of its cause, the result enables us to suggest that, unless the researcher is also interested in deriving Shin's z, the simplicity of the Rascher method is appealing.

We next turn to the choice between the SN and Rascher/Shin probabilities. As we have demonstrated, both methods make assumptions about the allocation of the book's over-round: the SN method assumes that the book's over-round is proportionally allocated across the possible outcomes while the Rascher/Shin method assumes that the over-round is equally distributed across the two outcomes. We are not aware of any evidence that books actually allocate the over-round in either manner. As a result, the choice between the SN and Rascher/Shin methods is likely an empirical matter: it may reduce to a

simple comparison of the forecast accuracy of the resulting probabilities. We note here that several recent studies of odds betting markets (Cain *et al*. 2002 and 2003, Smith *et al*. 2009, and Strumbelj, 2014) have established that Shin probabilities are more accurate forecasts than are the SN probabilities for a large number of odds betting markets. We suspect that the same may be true in at least some money line betting markets.

Moreover, the question of which method to use might actually have qualitative results. For instance, Strumbelj (2014) shows that using Shin probabilities leads to a change in the implications for the UOH in the case of European football attendance when compared to the results found when using the SN probabilities. While this alone does not indicate which method is appropriate, combined with other evidence concerning short-run UOH in a particular sport might provide guidance as to which procedure is most appropriate for a given context. It would appear that the most appropriate outside information would be some indication of the pre-game odds of home team winning by attendees (although this data requirement might prove too expensive to be practical).

Both for simplicity and congruence with the existing literature on other odds betting markets, this paper advocates that studies wishing to convert money lines into win probabilities use the standard normalization and/or the Rascher/Shin methods. There is an additional benefit to this recommendation: future studies can cease using the rather awkward WW (β_1, β_2) notation and standardize the notation to that used in studies of other odds betting markets.[11]

5. Two Empirical Examples Comparing SN and Rascher/Shin Probabilities

This section provides two empirical examples comparing SN and Rascher/Shin probabilities in money-line betting markets. The first example uses the sample of closing money lines for Major League Baseball (MLB) games used by Woodland and Woodland (1994) and Gandar *et al*. (2002). We have already noted there is a close fit between SN win probabilities and actual

[11] We acknowledge this suggestion by an anonymous referee.

win proportions in this sample. As a result, using Rascher/Shin probabilities may be a way of confirming the interpretation of market efficiency in this case. The second example examines closing money lines in a sample of recent college basketball games. The comparison of SN probabilities with observed win frequencies reveals a noticeable FLB in these money lines. Estimating Shin probabilities reduces but does not eliminate this bias. In addition, the comparison of SN and Rascher/Shin probabilities may reveal additional insight into the question of why this bias occurs. However, we emphasize that both of these examples are illustrative rather than exhaustive examinations of the presence and possible causes of biases in these money-line betting markets.

Comparing Probability Estimates in the MLB Money Line Betting Market

Both the original analysis of individual MLB money lines carried out by Woodland and Woodland (1994) and the subsequent correction of methodology and analysis of the same MLB lines by Gandar *et al*. (2002), note the close correspondence between probabilities derived from closing money lines and actual win proportions at the great majority of individual money lines. Additionally, Gandar *et al*. (2002) show that actual returns on all underdog bets are not significantly different from expected returns (the negative of their revised commission), indicating that there is little evidence of a bias (either a reverse FLB as claimed by Woodland and Woodland or a FLB), a result confirmed by a subsequent study by Cain, Law, and Peel (2003). Does this impression of the absence of bias remain when Rascher/Shin probabilities are used in place of SN probabilities?

Table 2 shows SN, Rascher/Shin, and WW favorite win probabilities for the 26 individual money lines of the original WW sample of money lines (for the 1978-1989 MLB seasons).[12] While the absolute differences between these win probability estimates are small, it is always the case that $\rho_F^{WW} > \rho_F^{SH} > \rho_F^{SN}$. As a result, the comparison of Z-test results for the null hypothesis of no difference between the three win probability estimates and the observed win frequencies is interesting. Using the 10 percent level of significance, the null hypothesis of no difference between the SN win probabilities and observed win frequencies is rejected for only two of the 26 individual lines – slightly less than the frequency one might expect by chance. Using the same significance level, the same null hypothesis is rejected at one additional line for both the Rascher/Shin and WW win probabilities, a rejection frequency slightly greater than one might expect by chance. We do not wish to put too much emphasis on this slight change in frequency: when there is little or no bias, as appears to be the case here, the Rascher/Shin probabilities serve mainly to confirm results obtained for SN probabilities.[13]

Lastly, we stress that this result holds for the Woodland and Woodland MLB data sample but may not hold for a more recent sample of MLB games. One of the issues with the WW sample is the limited range of observed win probability values (from 0.27 to 0.73). In the seasons since 1989 the MLB money line betting market has changed considerably. Now, both onshore and offshore books post a much greater variety of opening money lines and change these lines frequently over the trading period. One of the consequences of this activity is that closing lines reflect a much wider probability range. And, as shown previously, differences between the

[12] While we have chosen to use favorite win probabilities and observed win frequencies in this table rather than the equivalent underdog probabilities and frequencies used in the Woodland & Woodland (1994, 2001) and Gandar et al. (2002, 2004) studies, none of the Z-tests results are changed by this choice.

[13] Nevertheless, the slightly different results for the Rascher/Shin probabilities as compared to the SN probabilities or the original WW probabilities raise an interesting question about the original Woodland and Woodland (1994) results. By correcting for a FLB in their sample of games when this bias probably does not exist, did the WW probabilities inadvertently produce the impression of a reverse FLB?

SN and Rascher/Shin probabilities are most evident toward the extremes of the win probability range. A reexamination of money lines in this and other money-line betting markets is overdue.

Comparing Probability Estimates for Closing College Basketball Money Lines

This section provides an empirical illustration of the possible differences between the SN and Rascher/Shin probabilities and how such differences might guide the researcher in the appropriate choice of conversion method. We do this using a sample of games drawn from the money line betting market on American college basketball games. This market has at least two advantages relative to the money-line betting markets on professional sports such as baseball. First, there are a very large number of games played per season (roughly 4,000 games compared to MLB's approximately 2,000 games). Second, the range of money lines and hence, of win probability estimates, is much greater in our sample of college basketball games than the very limited range in the Woodland and Woodland sample of MLB games. If the restricted probability range of that sample is the cause of the failure to find a FLB in those data (as claimed by Cain, Law, and Peel, 2003), then the much wider range of probabilities apparent in college basketball money lines may provide evidence of a bias toward the ends of the probability spectrum. However, a possible drawback to using the college basketball money lines is that, for at least some games, this betting market may be thin (in terms of limited handle) relative to the equivalent markets on professional sports.

Our sample is for seven recent college basketball seasons (2006-07 through 2012-13) and contains 27,925 games.[14] After eliminating all games with missing money lines (6,048 games)

[14] These data are obtained from SportsInsight.com and represent money lines from four large offshore bookmakers. Data from this source are used by a number of other studies: recent examples include Humphreys (2010), Paul and Weinbach (2012), and Paul (2014). Unfortunately, our sample is missing some early season games for the 2007-08 season (all November and December games plus some January games). To check that these missing

and all 'pick-em' games, i.e., those without a favorite (85 games), our final sample contains 21,792 games with a clear closing money line favorite.

Table 3 shows observed favorite win proportions and SN and Rascher/Shin win probabilities for closing money lines.[15] This table uses probability intervals instead of computing probabilities at individual money lines because of the proliferation of closing money lines in our sample of games. In the past, books tended to post a limited number of opening money lines, usually lines ending in either '0' or '5,' and when they changed money lines, books tended to move to other lines ending with the same two digits. Today, books, especially offshore books, post highly individualized opening money lines: in our sample there are 1,069 different opening money lines. Furthermore, books change these money lines frequently; in our sample, money lines change from open to close in 90.58 percent of the 21,792 games. As a result, there are 1,506 individual closing money lines ranging from (-107,-105) at the low end of the favorite range to (-50,000, +25,000) at the top end (in SN win probability terms this is a favorite range from 0.5023 to 0.9960). Consequently, rather than use individual closing money lines, Table 3 shows 25 equally sized favorite probability intervals.[16]

Comparing SN favorite win probabilities against observed favorite win frequencies across these probability intervals reveals a FLB, especially toward the upper end of the probability spectrum. Favorite win frequencies exceed the SN win probabilities for 18 of the 25 intervals, significantly so for 8 of these intervals (at ρ-values less than 0.10). This bias is also apparent in a

games do not have any discernible impact, we dropped all games for the 2007-08 season and found no appreciable differences from the results reported here for the full sample of games.

[15] Future research might fully develop the differences between standard normalization and Shin probabilities associated with opening and closing lines. A replication of Table 3 using opening lines for the same sample is available from the authors upon request.

[16] We tried a larger number of smaller intervals (such as 50 equal sized favorite intervals) but game numbers in the intervals at the upper end of the probability spectrum are insufficient to meet the standard cutoffs for the use of the normal approximation to the binomial distribution.

comparison of actual returns to unit bets on favorites against expected returns (measured as the negative of the estimated commission or $-c^{MW}$. Favorite actual returns exceed expected returns for the same 18 probability intervals (with 8 of these being significant at ρ-values less than 0.10), and favorite actual returns are positive for 10 of these intervals. Over all games, favorite actual returns are -0.0097, significantly higher than both the average expected return of -0.0245 and the overall underdog actual return of -0.1069.

Computing Rascher/Shin probabilities modifies this conclusion. While actual favorite win proportions still exceed these win probabilities for the same 18 probability intervals, these differences are much smaller and are significant (at ρ-values of 0.10 or lower) for only 4 of the 25 intervals. The Rascher/Shin probabilities reduce, but do not entirely eliminate, the evidence of a FLB bias in this sample.

To firmly establish that a FLB (among other possible biases) exists in this betting market would require much more evidence than provided in this simple illustration. A researcher investigating this topic should probably also examine opening lines to see if differences between SN and Rascher/Shin probabilities indicate that (a) a FLB is deliberately built into money lines by bookmakers attempting to protect their profits in the presence of better informed traders; (b) is the result of inadvertent errors on the part of the bookmaker in collecting and processing information; or (c) stems from the shading of lines to profit from known bettor biases. Additionally, such an examination should also focus on line changes and betting patterns over the trading period.

What this empirical illustration shows is the need for caution in converting money lines into win probabilities. We would argue that any researcher carrying out this conversion should derive both standard normalized and Rascher/Shin probabilities from money lines. If there is

little difference between these probability estimates (as is the case in our first example), any bias is likely small and unimportant and the choice of conversion method likely does not matter. However, if differences exist between these probabilities (as is the case in our second sample), then the researcher likely needs to further explore the causes of the differences in order to justify the choice of conversion method uses for subsequent analysis.

6. Conclusions

This paper has three principal purposes. First, we explore the relationships between the multiple methods of converting money lines into win probabilities that appear in both the literature examining EMH in money line markets and in the literature testing the UOH in game attendance. Our aim is to reconcile and reduce the number of conversion methods as much as possible. While the various methods appear to be unique, we find that they actually reduce to three: the Knowles/Woodland and Woodland method; the standard normalized/ modified WW method; and the Rascher/Shin method.[17] While this result is somewhat surprising, we show that these various methods often share common assumptions, even if these commonalities are not obvious.

Secondly, we argue that researchers looking for an 'off-the shelf' method of obtaining win probabilities from money lines should: (i) pass over the Knowles/WW method because it has no obvious rationale (Knowles, et al.) and requires restrictive and quite possibly unjustified assumptions about the efficiency of these markets (Woodland and Woodland); (ii) adopt the standard normalized method in preference to the modified WW method, both for reasons of compatibility with methods used in odds-betting markets and because the latter requires the same restrictive and unjustified assumptions as the original WW approach; and (iii), use the Rascher/Shin method when the researcher has reason to believe that market inefficiencies, such as a FLB, may be present.

[17] The results remind us of the contribution by Siegfried (1970).

Lastly, we provide two empirical examples to demonstrate how the standard normalization and the Shin probabilities perform in a market without a suspected favorite longshot bias and in a market in which such a bias is suspected. In our first empirical example, the money line market for Major League Baseball utilized by Woodward and Woodward (1994), the standard normalization performs as well as the Shin probabilities with a significant reduction in computational costs; we submit this is because this particular betting market does not suffer from economically meaningful inefficiency. However, in our second empirical example, the money line market for NCAA Men's Basketball games, the standard normalization performs considerably worse than the Shin probabilities; we submit this is because this particular betting market suffers from economically meaningful inefficiency such as a long-shot favorite bias.[18]

When money lines often reflect heavy favorites, as is the case with college basketball games but not the case with Major League Baseball games, the Woodward and Woodward probability estimates are dramatically different and overstated (for the favorite) compared to those obtained using the standard normalization or the Shin approach. This suggests that when investigating markets with heavy favorites (including college basketball, college football, and perhaps tennis) researchers should be more careful about which methods to use and which to avoid.

[18] Berkowitz, Depken, and Gandar (2016) document a FLB in NCAA men's basketball games and football games.

References

Beckman E. M., W. Cai, R. M. Esrock, and R, J, Lemke (2012). "Explaining game-to-game ticket sales for Major League Baseball over time." *Journal of Sports Economics*, 13(5), 536-553.

Berkowitz, J. P., C. A. Depken, and J. M. Gandar (2016). "A favorite-longshot bias in fixed-odds betting markets: Evidence from college basketball and college football," *The Quarterly Review of Economics and Finance,* in press, http://dx.doi.org/10.1016/j.qref.2015.11.011.

Cain, M., D. Law, and D. Peel (2002). "Is one price enough to value a state-contingent asset correctly? Evidence from a gambling market." *Applied Financial Economics*, 12(1), 33-38.

Cain, M., D. Law, and D. Peel (2003). "The favourite-longshot bias, bookmaker margins and insider trading in a variety of betting markets." *Bulletin of Economic Research*, 55(3), 263-273.

Coates, D., and B. R. Humphreys (2012). "Game attendance and outcome uncertainty in the National Hockey League." *Journal of Sports Economics*, 13(4), 364-377.

Coates, D., B. R. Humphreys, and L. Zhou (2014). "Reference dependent preferences, loss aversion, and live game attendance." *Economic Inquiry*, 52(2), 959-973.

Dowie, J. (1976). "On the efficiency and equity of betting markets." *Economica*, 43(170), 139-150.

Gandar, J. M., R. A. Zuber, and R. S. Johnson (2002). "Reexamining the betting market of Major League Baseball: Is there a favorite-longshot bias?" *Applied Economics*, 34(10), 1309-1317.

Gandar, J. M., R. A. Zuber, R. S. Johnson, and W. Dare (2004). "A reexamination of the efficiency of the betting market on National Hockey League games." *Journal of Sports Economics*, 5(2), 152-168.

Griffith, R. M. (1949). "Odds adjustment by American horse race bettors." *American Journal of Psychology*, 62(2), 13-26.

Humphreys, B. R. (2010). 'Point spread shading and behavioral biases in NBA betting markets.' *Revista di Dirotto ed Ecoconmia dello Sport*, 6(1), 13-26.

Jullien, B. and B. Salanie (1994). "Measuring the incidence of insider trading: a comment on Shin." *Economic Journal*, 104(427), 1418-1419.

Knowles, G., K. Sherony, and M. Haupert (1992). "The demand for Major League Baseball: a test of the uncertainty of outcome hypothesis." *The American Economist*, 35(2), 72-80.

Lemke, R. J., M. Leonard, and K. Tlhokwane (2010). "Estimating attendance at Major League Baseball games for the 2007 season." *Journal of Sports Economics*, 11(3), 316-346.

Paul, R. J. (2014). 'The Relationship between Sportsbook Volume, Forecast Accuracy, and Market Efficiency: the NFL and NCAA.' *Journal of Prediction Markets*, 8(2), 30-43.

Paul, R. J. and A. P. Weinbach (2012). "Behavioral biases and sportsbook pricing in Major League Baseball." *Journal of Economics and Finance*, 36(1), 128-135.

Peel, D. and D. Thomas (1988). "Outcome Uncertainty and the Demand for Football: An Analysis of Match Attendances in the English Football League," *Scottish Journal of Political Economy,* 35(3), 242-249.

Rascher, D. A. (1999). "A test of the optimal positive production network externality in Major League Baseball." In *Sports Economics: Current Research*, edited by J. Fizel, E. Gustafson, and L. Hadley. Westport, CT: Praeger, 27-45.

Sauer, R. D. (1998). "The economics of wagering markets." *Journal of Economic Literature*, 36(4), 2121–2064.

Sauer, R. D. (2005). "The state of research on markets for sports betting and suggested future directions." *Journal of Economics and Finance*, 29(3), 416-426.

Shin, H. S. (1992). "Prices of state contingent claims with insider trading and the favourite-longshot bias." *Economic Journal*, 102(411), 426-435.

Shin, H. S. (1993). "Measuring the incidence of insider trading in a market for state contingent claims." *Economic Journal*, 104(420), 1141-1153.

Siegfried, J. (1970). "A First Lesson in Econometrics." *The Journal of Political Economy,* 78(6): 1378-1379.

Smith, M. A., D. Paton, and L. Vaughan Williams (2009). "Do bookmakers possess superior skills to bettors in predicting outcomes?" *Journal of Economic Behavior and Organization*, 71(2), 539-549.

Snowberg, E., and J. Wolfers (2010). "Explaining the favorite-longshot bias: is it risk-love or misperception?" *Journal of Political Economy*, 118(4), 723-746.

Strumbelj, E. (2014). "On determining probability forecasts from betting odds." *International Journal of Forecasting*, 30(4), 934-943.

Strumbelj, E. (2016). "A Comment on the Bias of Probabilities Derived from Betting Odds and Their Use in Measuring Outcome Uncertainty," *Journal of Sports Economics,* in press, DOI: 10.1177/1527002513519329.

Vaughan Williams, L. (1999). "Information efficiency in betting markets." *Bulletin of Economic Research*, 51(1), 1 -30.

Vaughan Williams L. (Editor) (2005). *Information efficiency in financial and betting markets*. Cambridge: Cambridge University Press.

Woodland L. M., and W. M. Woodland (1994). "Market efficiency and the favorite-longshot bias: the baseball betting market." *Journal of Finance*, 49(1), 269-280.

Woodland L. M., and W. M. Woodland (2001). "Market efficiency and profitable wagering in the National Hockey League: Can bettors score on longshots?" *Southern Economic Journal*, 67(4), 983-995.

Table 1
Alternative Estimates of Favorite Win Probabilities at Selected Money Lines

Money Lines	(-110, +100)	(-160, +145)	(-265, +230)	(-450, +390)	(-1150, +890)
Panel A: Knowles *et al*. (1992) Conversion Method					
ρ_F^K	0.5122	0.6040	0.7122	0.8077	0.9107
ρ_U^K	0.4878	0.3960	0.2878	0.1923	0.0893
Panel B: Woodland and Woodland (1994, 2001) Conversion Method					
ρ_F^{WW}	0.5122	0.6040	0.7122	0.8077	0.9107
ρ_U^{WW}	0.4878	0.3960	0.2878	0.1923	0.0893
c^{WW}	0.0244	0.0297	0.0504	0.0577	0.1161
Panel C: Rascher (1999) Conversion Method					
ρ_F^R	0.5119	0.6036	0.7115	0.8071	0.9095
ρ_U^R	0.4881	0.3964	0.2885	0.1929	0.0905
Panel D: Modified Woodland & Woodland Conversion Method of Gandar *et al*. (2002, 2004)					
ρ_F^{MW}	0.5116	0.6012	0.7055	0.8004	0.9011
ρ_U^{MW}	0.4884	0.3988	0.2945	0.1996	0.0989
c^{MW}	0.0233	0.0230	0.0282	0.0218	0.0206
Panel E: Standard Normalized Conversion Method used by Sauer (2005)					
ρ_F^{SN}	0.5116	0.6012	0.7055	0.8004	0.9011
ρ_U^{SN}	0.4884	0.3988	0.2945	0.1996	0.0989
Over-round or margin	0.0238	0.0235	0.0291	0.0223	0.0210
Panel F: Shin Conversion Method					
ρ_F^{SH}	0.5119	0.6036	0.7115	0.8071	0.9095
ρ_U^{SH}	0.4881	0.3964	0.2885	0.1929	0.0905
Shin's 'z'	0.0238	0.0236	0.0292	0.0226	0.0219

Table 2
Market Efficiency Results for Three Favorite Win Probability Estimates
Closing Money Lines for the 1978-1989 MLB Seasons

For each of the 26 individual money lines in the original Woodland and Woodland sample of MLB games over the 1978-1989 seasons, the table shows: the number of games (*N*); observed favorite wins (*FW*); observed underdog wins (*UW*); the observed favorite win proportion (*FWP*); and the favorite win probability and associated *Z*-statistic testing the null hypothesis of equal proportions for the standard normalized, Rascher/Shin, and original Woodland & Woodland methods of estimating win probabilities from money lines. Bold font indicates statistical significance at the 10% level.

Money Lines	Game Numbers & Favorite Win Proportions				Standard Normalized Win Probabilities		Rascher/Shin Win Probabilities		Original Woodland & Woodland Win Probabilities	
(β_1, β_2)	*N*	*FW*	*UW*	*FWP*	ρ_F^{SN}	Z-stat	ρ_F^{SH}	Z-stat	ρ_F^{WW}	Z-stat
(1.10, 1.00)	2,626	1,276	1,350	0.5141	0.5116	0.2524	0.5119	0.2240	0.5122	0.1942
(1.15, 1.05)	1,906	944	962	0.5047	0.5230	-1.5991	0.5235	**-1.6449**	0.5238	**-1.6685**
(1.20, 1.10)	2,151	1,005	1,146	0.5328	0.5339	-0.0144	0.5346	-0.1726	0.5349	-0.1960
(1.25, 1.15)	1,969	928	1,041	0.5287	0.5443	-1.3907	0.5452	-1.4726	0.5455	-1.4936
(1.30, 1.20)	2,053	932	1,121	0.5460	0.5543	-0.7506	0.5553	-0.8485	0.5556	-0.8686
(1.35, 1.25)	1,686	757	929	0.5510	0.5638	-1.0596	0.5650	-1.1598	0.5652	-1.1769
(1.40, 1.30)	1,820	737	1,083	0.5951	0.5730	**1.9061**	0.5743	**1.7929**	0.5745	**1.7763**
(1.45, 1.35)	1,432	644	788	0.5503	0.5817	**-2.4130**	0.5832	**-2.5231**	0.5833	**-2.5371**
(1.50, 1.40)	1,467	621	846	0.5767	0.5902	-1.0496	0.5917	-1.1673	0.5918	-1.1806
(1.55, 1.45)	1,164	485	679	0.5833	0.5983	-1.0393	0.5998	-1.1495	0.6000	-1.1607
(1.60, 1.50)	1,086	430	656	0.6041	0.6061	-0.1355	0.6077	-0.2457	0.6078	-0.2559
(1.65, 1.55)	790	319	471	0.5962	0.6136	-1.0020	0.6152	-1.0999	0.6154	-1.1082
(1.70, 1.60)	766	287	479	0.6253	0.6208	0.2589	0.6225	0.1610	0.6226	0.1533
(1.75, 1.65)	511	176	335	0.6556	0.6278	1.3013	0.6295	1.2205	0.6296	1.2146
(1.80, 1.70)	471	177	294	0.6242	0.6345	-0.4624	0.6362	-0.5431	0.6364	-0.5486
(1.85, 1.75)	220	70	150	0.6818	0.6409	1.2637	0.6427	1.2095	0.6429	1.2060
(1.90, 1.80)	234	82	152	0.6496	0.6472	0.0759	0.6490	0.0179	0.6491	0.0144
(2.00, 1.85)	516	170	346	0.6705	0.6552	0.7346	0.6579	0.6056	0.6581	0.5949
(2.10, 1.90)	132	43	89	0.6742	0.6627	0.2810	0.6663	0.1936	0.6667	0.1846
(2.20, 2.00)	331	105	226	0.6828	0.6735	0.3612	0.6771	0.2216	0.6774	0.2086
(2.30, 2.10)	62	14	48	0.7742	0.6836	1.5337	0.6872	1.4775	0.6875	1.4727
(2.40, 2.20)	189	64	125	0.6614	0.6931	-0.9469	0.6967	-1.0562	0.6970	-1.0648
(2.50, 2.30)	21	6	15	0.7143	0.7021	0.1218	0.7056	0.0871	0.7059	0.0845
(2.60, 2.40)	127	38	89	0.7008	0.7106	-0.2441	0.7141	-0.3308	0.7143	-0.3367
(2.80, 2.60)	59	16	43	0.7288	0.7262	0.0446	0.7295	-0.0124	0.7297	-0.0158
(3.00, 2.60)	35	10	25	0.7143	0.7297	-0.2057	0.7361	-0.2930	0.7368	-0.3030
	23,824	10,336	13,488	0.5662						

Table 3
Standard Normalized and Rascher/Shin Favorite Win Probabilities at Closing Money Lines for the 2006-07 through 2012-13 College Basketball Seasons

This table shows 25 equally sized favorite probability intervals for both the standard normalized favorite win probabilities, ρ^{n}_{F}, and the Shin favorite win probabilities, ρ^{SH}_{F}. For each probability interval:,*N* is the number of games; *FW* is the observed number of favorite wins; *FWP* is the observed favorite win proportion; and the Z-statistics test the null hypothesis that there is no difference between the observed *FWP* and the subjective favorite win probability (either ρ^{NS}_{F} or ρ^{SH}_{F}). Z-statistics in bold indicate significant at the 10% significance level or lower.

Probability Range	Standard Normalized Win Probabilities					Rascher/Shin Win Probabilities				
	N	*FW*	*FWP*	ρ_F^{SN}	Z-Stat	*N*	*FW*	*FWP*	ρ_F^{SH}	Z-Stat
$0.50 < \rho \leq 0.52$	844	424	0.5024	0.5117	-0.5405	803	406	0.5056	0.5115	-0.3365
$0.52 < \rho \leq 0.54$	1,149	604	0.5257	0.5305	-0.3271	1,120	582	0.5196	0.5302	-0.7099
$0.54 < \rho \leq 0.56$	1,180	626	0.5305	0.5506	-1.3840	1,152	612	0.5313	0.5504	-1.3059
$0.56 < \rho \leq 0.58$	1,078	607	0.5631	0.5702	-0.4696	1,095	623	0.5689	0.5701	-0.0796
$0.58 < \rho \leq 0.60$	1,075	665	0.6186	0.5900	**1.9095**	1,039	619	0.5958	0.5904	0.3523
$0.60 < \rho \leq 0.62$	1,116	691	0.6192	0.6093	0.6769	1,102	696	0.6316	0.6098	1.4855
$0.62 < \rho \leq 0.64$	1,184	763	0.6444	0.6295	1.0613	1,098	703	0.6403	0.6295	0.7373
$0.64 < \rho \leq 0.66$	919	617	0.6714	0.6502	1.3496	954	632	0.6625	0.6498	0.8219
$0.66 < \rho \leq 0.68$	1,009	671	0.6650	0.6692	-0.2792	987	665	0.6738	0.6696	0.2751
$0.68 < \rho \leq 0.70$	984	704	0.7154	0.6892	**1.7790**	993	692	0.6969	0.6897	0.4903
$0.70 < \rho \leq 0.72$	1,254	901	0.7185	0.7107	0.6115	1,000	714	0.7140	0.7101	0.2746
$0.72 < \rho \leq 0.74$	919	690	0.7508	0.7303	1.4034	1,121	841	0.7502	0.7298	1.5406
$0.74 < \rho \leq 0.76$	1,024	786	0.7676	0.7504	1.2694	883	667	0.7554	0.7501	0.3644
$0.76 < \rho \leq 0.78$	779	611	0.7843	0.7690	1.0164	998	773	0.7745	0.7695	0.3758
$0.78 < \rho \leq 0.80$	1,018	836	0.8212	0.7897	**2.4669**	826	681	0.8245	0.7912	**2.3528**
$0.80 < \rho \leq 0.82$	869	728	0.8377	0.8101	**2.0813**	814	664	0.8157	0.8093	0.4652
$0.82 < \rho \leq 0.84$	780	661	0.8474	0.8303	1.2757	750	639	0.8520	0.8273	**1.7889**
$0.84 < \rho \leq 0.86$	723	607	0.8396	0.8505	-0.8220	774	657	0.8488	0.8479	0.0711
$0.86 < \rho \leq 0.88$	878	771	0.8781	0.8691	0.7901	924	804	0.8701	0.8700	0.0117
$0.88 < \rho \leq 0.90$	926	846	0.9136	0.8895	**2.3443**	857	760	0.8868	0.8909	-0.3827
$0.90 < \rho \leq 0.92$	847	794	0.9374	0.9100	**2.7856**	787	723	0.9187	0.9103	0.8235
$0.92 < \rho \leq 0.94$	693	672	0.9697	0.9286	**4.2031**	852	818	0.9601	0.9292	**3.5194**
$0.94 < \rho \leq 0.96$	362	348	0.9613	0.9495	1.0275	517	500	0.9671	0.9491	**1.8605**
$0.96 < \rho \leq 0.98$	140	139	0.9929	0.9684	**1.6555**	292	288	0.9863	0.9681	**1.7734**
$0.98 < \rho < 1.00$	44	43	0.9773	0.9855	-0.4558	56	55	0.9821	0.9866	-0.2893
	21,794	15,805	0.7252			21,794	15,814	0.7256		

Appendix A: Win Probabilities for Double Negative Money Lines

This appendix derives win probabilities from so-called double negative money lines. For space reasons, we show these derivations only for the standard normalization, modified WW, and Rascher conversion methods.

Standard Normalization Win Probabilities

When books post so-called double negative money lines such as (-110, -101) or (-107, -105), net payoffs to unit bets on the favorite and underdog are respectively, $(1/\beta_1)$ and $(1/\beta_2)$, gross payoffs or decimal win odds, (o_F, o_U) are respectively, $(1+(1/\beta_1))$ and $(1+(1/\beta_2))$. Inverting these decimal win odds results in the win odds probabilities, $\pi_F = 1/o_F$ and $\pi_U = 1/o_F$. As before, since the booksum of $(\pi_F + \pi_U)$ exceeds one, the win probabilities are found by dividing both π_F and π_U by $(\pi_F + \pi_U)$. That is,

$$\rho_F^{SN} = \frac{\pi_F}{(\pi_F + \pi_U)} = \frac{1/(1+(1/\beta_1))}{(1/(1+(1/\beta_1))) + (1/(1+(1/\beta_2)))},$$

and

$$\rho_U^{SN} = \frac{\pi_U}{(\pi_F + \pi_U)} = \frac{1/(1+(1/\beta_2))}{(1/(1+(1/\beta_1))) + (1/(1+(1/\beta_2)))}.$$

Modified Woodland and Woodland Win Probabilities

When the book posts double negative money lines, the book's net revenue is $H_F = (Y - (1/\beta_1)X)$, if the favorite wins, or $H_U = (X - (1/\beta_2)Y)$, if the underdog wins. If the net revenue is balanced regardless of who wins the game, the book's commission is

$$c^{MW} = \frac{Y-(1/\beta_1)X}{X+Y} = \frac{X-(1/\beta_2)Y}{X+Y} = \frac{1-(1/\beta_1)(1/\beta_2)}{(1/\beta_1)+(1/\beta_2)+2}.$$

Win probabilities are again found by setting the expected return from a unit bet on F or U (that is, either $E(R_F)$ or $E(R_U)$) equal to the negative of the book's commission. For example,

$$E(R_U) = \rho_U(1/\beta_2) + (1-\rho_U)(-1) = \rho_U(1+(1/\beta_2)) - 1,$$

where ρ_U is the underdog win probability. Setting $E(R_U)$ equal to $-c^{MW}$ and solving for ρ_U produces the modified WW win probability

$$p_U^{MW} = \frac{(1+(1/\beta_1)}{((1/\beta_1)+(1/\beta_2)+2)}.$$

Likewise, setting $E(R_F)$ equal to $-c^{MW}$ and solving for ρ_F^{MW} gives

$$\rho_F^{MW} = \frac{(1+(1/\beta_2)}{((1/\beta_1)+(1/\beta_2)+2)}.$$

As for standard money lines, straightforward manipulations can be used to show that, in the case of double negative money lines, $(\rho_F^{SN}, \rho_U^{SN})$ are identical to $(\rho_F^{MW}, \rho_U^{MW})$ and that the book's commission equals the margin divided by the booksum.

Rascher Win Probabilities

Rascher win probabilities for double negative money lines are again found by setting expected returns to bets on favorites and underdogs equal to zero and averaging the resulting 'fair bet' probabilities. The resulting favorite and underdog win probabilities are

$$\rho_F^R = [(\beta_1/(1+\beta_1)) + (1/(1+\beta_2))]/2$$

and

$$\rho_U^R = [1/(1+\beta_1)) + (\beta_2/(1+\beta_2))]/2.$$

As shown in Appendix B, for the same double-negative money lines the Rascher win probabilities, (ρ_F^R, ρ_U^R), are identical to the Shin win probabilities, $(\rho_F^{SH}, \rho_U^{SH})$.

Appendix B: Shin Probabilities are Rascher Probabilities

We used the Maple symbolic processor to show that the Shin probability for the favorite winning reduces to the Rascher probability for the favorite winning both when standard money lines (negative for favorite, positive for underdog) and non-standard money lines (negative for both favorite and underdog) are used.

With standard money lines (negative for favorite and positive for underdog), define π_F = b1/(1+b1) and π_U = 1/(1+b2). Define the standard commission as:

$$c = \frac{b1 - b2}{b1\,b2 + 2\,b1 + 1}\ .$$

Substituting π_F, π_U, and c into Shin's z yields:

$$z = \frac{(b1 - b2)\left(\left(\frac{b1}{1 + b1} - \frac{1}{1 + b2}\right)^2 - \frac{b1}{1 + b1} - \frac{1}{1 + b2}\right)}{(b1\,b2 + 2\,b1 + 1)\left(\left(\frac{b1}{1 + b1} - \frac{1}{1 + b2}\right)^2 - 1\right)}\ .$$

We then substitute z, π_F, and π_U into the Shin probability of the favorite winning to obtain:

$$\left(\frac{1}{4}\left((b1-b2)^2\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2\right)\Big/\left((b1\,b2+2\,b\right.\right.$$

$$\left.+1)^2\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-1\right)^2\left(1\right.\right.$$

$$\left.\left.-\frac{(b1-b2)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-\frac{b1}{1+b1}-\frac{1}{1+b2}\right)}{(b1\,b2+2\,b1+1)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-1\right)}\right)^2\right)+b1^2\Big/$$

$$\left((1+b1)^2\left(\frac{b1}{1+b1}+\frac{1}{1+b2}\right)\left(1\right.\right.$$

$$\left.\left.\left.-\frac{(b1-b2)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-\frac{b1}{1+b1}-\frac{1}{1+b2}\right)}{(b1\,b2+2\,b1+1)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-1\right)}\right)\right)\right)^{0.5}-\left((b1\right.$$

$$\left.-b2)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-\frac{b1}{1+b1}-\frac{1}{1+b2}\right)\right)\Big/\left((b1\,b2+2\,b1\right.$$

$$\left.+1)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-1\right)\left(2\right.\right.$$

$$\left.\left.-\frac{2\,(b1-b2)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-\frac{b1}{1+b1}-\frac{1}{1+b2}\right)}{(b1\,b2+2\,b1+1)\left(\left(\frac{b1}{1+b1}-\frac{1}{1+b2}\right)^2-1\right)}\right)\right)$$

Using Maple's simplify command, the Shin probability reduces to:

$$\rho_F^{SH} = \frac{1}{2}\,\frac{2\,b1\,b2+b1+b2}{b1\,b2+b1+b2+1} = \rho_F^R\,.$$

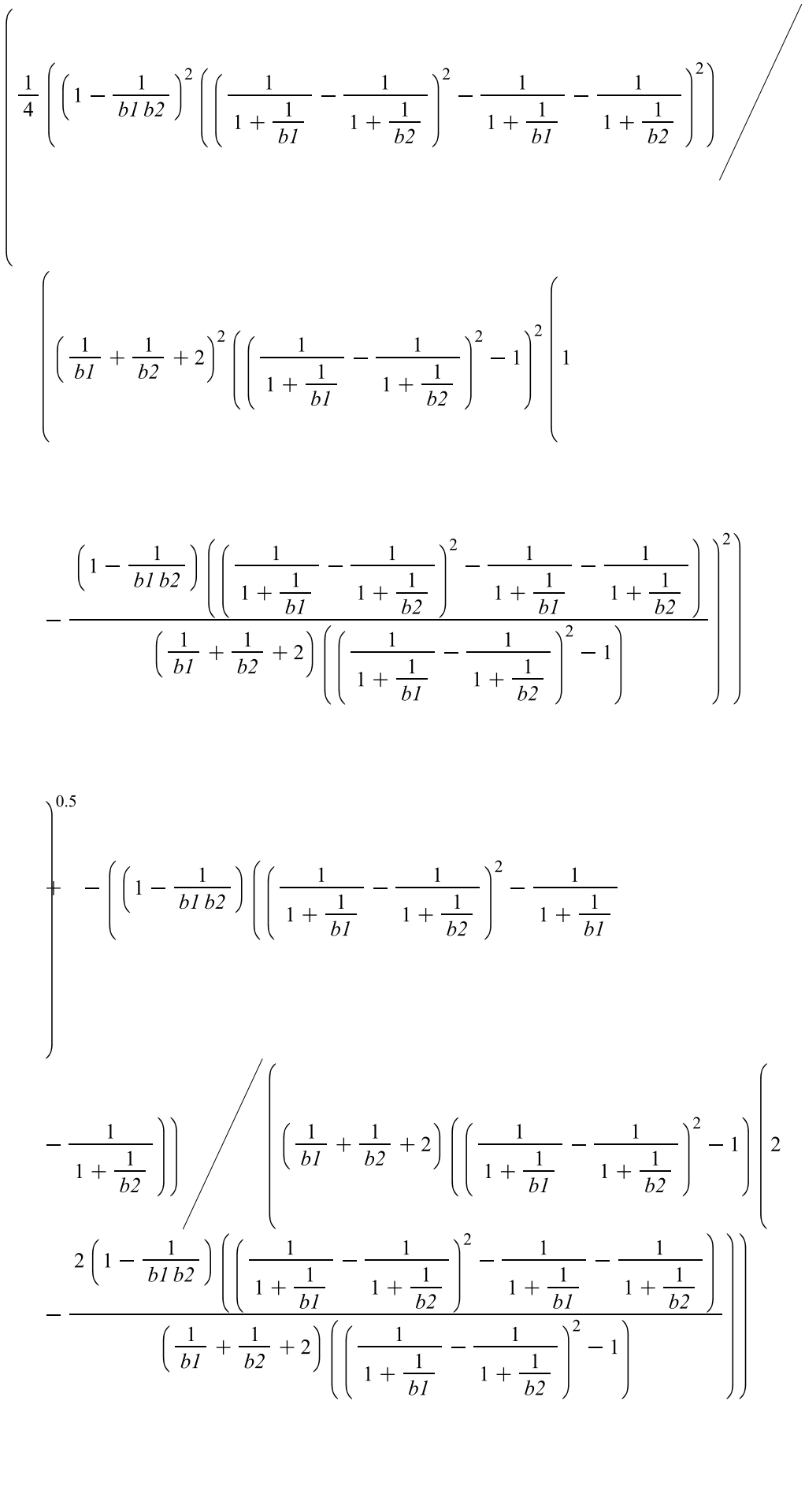
With non-standard money lines, where the line is negative for both the favorite and the underdog, define $\pi_F = b1/(1+b1)$ and $\pi_U = b2/(1+b2)$. Define the standard commission as:

$$c = \frac{1 - \frac{1}{b1\,b2}}{\frac{1}{b1} + \frac{1}{b2} + 2}.$$

Substituting π_F, π_U, and c into Shin's z yields:

$$z = \frac{\left(1 - \frac{1}{b1\,b2}\right)\left(\left(\frac{1}{1+\frac{1}{b1}} - \frac{1}{1+\frac{1}{b2}}\right)^2 - \frac{1}{1+\frac{1}{b1}} - \frac{1}{1+\frac{1}{b2}}\right)}{\left(\frac{1}{b1} + \frac{1}{b2} + 2\right)\left(\left(\frac{1}{1+\frac{1}{b1}} - \frac{1}{1+\frac{1}{b2}}\right)^2 - 1\right)}.$$

We then substitute z, π_F, and π_U into the Shin probability of the favorite winning to obtain:



Using Maple's simplify command, the Shin probability reduces to:

$$\rho_F^{SH} = \frac{1}{2}\,\frac{b1\,b2 + 2\,b1 + 1}{b1\,b2 + b1 + b2 + 1} = \rho_F^R$$